Francesco Romeo - Marco Dall'Aglio - Marco Giacalone

Algorithmic Conflict Resolution

Fair and Equitable Algorithms in Private Law

IUSTITIAM COLIMUS

G. Giappichelli Editore

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Content

Chapter 1

The pioneering work

Steven J. Brams, Alan D. Taylor

Adjusted Winner: Individual Disputes *

There are 1.2 million divorces in the United States alone every year. Though **hardly** typical, two of the most celebrated divorces in recent years were those of the Prince of Wales (Charles) and the late Princess of Wales (Diana) in Great Britain – a case only touched on in this chapter – and that of Donald and Ivana Trump in the United States. The Trump divorce, which we analyze in some detail here, mirrors the Camp David outcome in the sense of having an AW resolution that is quite dose to the settlement that the Trumps and their lawyers actually achieved on their own.

This was also true of the dispute between the 1996 Democratic and Republican presidential candidates about the composition and nature of their debates. This dispute was resolved only a few weeks before the November election; the AW result closely matched the compromise that the advisers of Bill Clinton and Bob Dole negotiated.

In retrospect, the settlements in both the divorce and debates cases are not particularly surprising. When two disputants have something to gain from a settlement, one would expect them to realize these gains, presuming they have sufficient good will and perseverance to see the negotiations through to a conclusion.

Unfortunately for other disputants, especially in divorce cases, these ingredients often seem to be sorely lacking. It is in these situations that AW can fulfill its most useful role, which is not that of just ratifying or speeding up an agreement that could have been made without it but in bringing an otherwise interminable or costly baule to some mutually acceptable resolution. It may be one that the disputants did not even contemplate prior to AW's use, illustrating the creative role that AW can play in dispute resolution.

Conflicts between individuals are ubiquitous both in the workplace and at home, especially with respect to the responsibilitiesand obligations of each person. At home, spouses frequently quarrel over the division of household chores,

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* *The Win-Win Solution: Guaranteeing Fair Shares to Everybody*, Chapter 7, P. 109-123, W.W. Norton & Co Inc, 2000.

which commends AW as a fair way to determine who does what: "Winning" on a chore would mean not having to do it – the other person would – which addresses the so-called *chores problem.* At work AW could be used to assign tasks, whereby each person would bid for the tasks he or she wants to do (if they are considered goods) or does not want to do (if they are considered "bads").

Indeed, goods and bads could be combined on the same list. Thus, if two company employees agreed that the task to "contact prospective customers" was a good but "handle complaints" was a bad, then both tasks could be put on the same "goods" list if the second task were written as "don't handle complaints". Using AW to assign tasks, each employee would then have to decide whether contacting prospective customers was worth fewer or more points than not handling complaints.

Divorce

Two divorces made the headlines in the 1990s: that terminating the 13-year marriage of Donald and Ivana Trump in 1991, and the final break-up of the 15-year marriage of the Prince and Princess of Wales, Charles and Diana, in 1996. These divorces were anything but typical: They involved mind-boggling assets, public-relations campaigns, and maritai infidelities replete with photos of the extramarital lovers splashed across the front pages of newspapers. Perhaps surprisingly, these divorces involved no dispute over the custody of the children in each marriage.

The more typical problems of fair division of goods or issues among divorcing couples, and how AW might alleviate them, are discussed later. Let's start, however, with the highly atypical problems of the royal couple.

The Prince and Princess of Wales fought over not only money and property, including jewelry and horses, but also titles. Would Diana stili be entitled to be addressed as "Her Royal Highness," regarded as a member of the Royal Family, or stripped of all association with Buckingham Palace? As for Charles, how much would he have to pay Diana to sente? Looming in the background was Charles's mother, Queen Elizabeth, who, in the end, probably bankrolled Charles's monetary payment of $22 million to Diana.

Indeed, Queen Elizabeth seems to have been the person who reluctantly decided that the couple's prior separation should be formalized by divorce. The couple's children, Prince William and Prince Henry (ages 14 and 11 at the time of the divorce), were not an issue, in part because they spent the better part of the year in boarding school. Also, they had been alternating holidays and school vacations with each parent after the latter were separated a few years before the divorce, which was an arrangement that had proved successful and was satisfactory to both sides.

Diana put great weight on receiving a lump-sum payment from Charles so as not to be dependent on him year after year, especially if she should choose to remarry. She also wanted to continue her role as "queen of the peoples' hearts" and "roving ambassador," whereas the prince and the queen wanted to minimize her association with the palate and for her "to pay her own bills" (her lifestyle had been an extravagant one).

Both sides seem to have won, relatively speaking, on the issues most important to them, suggesting an AW-type solution without the application of AW's formai apparatus. This is not surprising insofar as AW replicates, as in the Camp David agreement, what disputants – if they are successful – usually negotiate informally.

Turning to the Trumps, they were also successful in reaching a settlement, which principally involved contested property. Many couples, however, are less successful not only in dividing property, which often has sentimental value, but also in dealing with the children and money.

Donald Trump's authorship of *Trump: The Art of the Deal* (1987) augured well for his finding a suitable "deal" for ending his marriage with Ivana, whom he stili professed to love. However, the book is less advice on working out a deal and more a paean to Donald's success in getting his way in countless negotiations. As we will see, however, Ivana had her own cards to play, making her probably the equal of Donald and rendering AW, therefore, appropriate to apply to their divorce case.

It is worth noting in passing that Donald's next book, *Trump: Surviving at the Top* (1990), showed life not to be so rosy for someone so rich and famous. Perhaps he anticipated his 1991 divorce, not to mention the numerous setbacks in business that befell him after the booming 1980s. In 1997, Donald decided to separate from his second wife, Maria, in part, it seems, for financial reasons. He professed that he wanted to be "fair" to Maria but, alluding to their prenuptial agreement, indicated that "Hey, a deal was a deal". Donald describes his purported recovery from disaster in his third book, *Trump: The Art of the Comeback* (1997).

The lawyers on both sides of the divorce from Ivana were pessimistic that Donald and Ivana could reach a satisfactory agreement on their own. A long and costly court battle seemed inevitable, especially in light of the prenuptial and postnuptial agreements made by the Trumps that were later contested by Ivana.

In the Trump's prenuptial agreement, which was revised three times during their marriage, Ivana was entitled to a settlement of about $14 million and the couple's Greenwich, Connecticut, mansion. In addition, she waived her right to maritai property in the first three agreements. Her lawyers argued, however, that she never did in the fourth agreement, which was the basis of a lawsuit she initiated in early 1990.

In this lawsuit, she demanded haif of all maritai assets, which she estimated to be about $5 billion, under New York State's 1980 Maritai Equitable Distribution Law. In late 1990, however, after "The Donald" (as he was derisively called) con-

firmed the precariousness of his financial situation and the $5 billion figure was seen to be far off the mark, Ivana said she only wanted to be "fair" and abandoned her lawsuit.

Donald's main difficulty was that he had built his real estate empire largely on borrowed money and junk bonds, which tumed sour in the late 1980s as the economy plummeted into recession. Near bankruptcy, he tumed to the banks that had lent him money to stay afloat, and together they developed a rescue plan. But this pian, within a few months of its initiation, was declared "dead," and Donald was forced into a new agreement, which included the forfeiture of several of his properties.

Understanding Donald's serious risk of bankruptcy, Ivana realized that if she did not settle quickly, she might be no more than just another person in a long line of creditors. But Donald, not ready to strike one of his vaunted deals, saw no reason not to abide by the postnuptial agreement, which his lawyer contended was "more than ironclad, it's steel-wrapped"; Ivana's attempt to get around it would be like "trying to break down a steel door with a feather".

Nevertheless, Donald insisted that he, too, wanted to be "fair". He hoped he could reach an agreement with Ivana outside the courtroom, not incidentally avoiding all the negative pubiicity that would attend their struggle and put a spotlight on his financial woes.

When Ivana abandoned her lawsuit and said that she was willing to negotiate a settlement in early 1991, Donald was only too willing to comply. (It is interesting to compare Ivana's posture a year earlier: "Gincane the Plaza, the Jet and the $150 Million, Too" screamed the headline of the *New York Post* on February 13, 1990.) Disregarding the assets most likely to be taken over by the banks and the business properties to which Ivana was not entitled (including the Trump shuttle, a 282-foot yacht called the Trump Princess, and a personal Boeing 727 jet), the real estate in dispute comprised:

- A 46-room estate in Greenwich, Connecticut;
- A 118-room Mar-a-Lago mansion in Palm Beach, Florida;
- A Trump Plaza apartment in New York City;
- A 50-room Trump Tower triplex in New York City.

The couple also had to reach an agreement on a fixed sum of cash to be paid to Ivana by Donald, and child support for their three children (ages 12, 8, and 6 at the time of the divorce), over whom Ivana would retain custody and with whom Donald would have visiting rights.

Except for Ivana's estimated $1.2 million in cash and jewelry, there were no cash or receivables in the divorce; Donald was barely solvent, let alone liquid. Most of his businesses were either in the red or just breaking even. He lived off a monthly $375,000 "allowance" from one of the banks to whom he owed money.

It seems plausible that Donald and Ivana would allocate the following points to the different pieces of real estate:

Property	*Donald*	*Ivana*
Connecticut estate	10	40
Palm Beach mansion	40	20
Trump Plaza apartment	10	311)
Trump Tower triplex	40	10
Total	*100*	*100*

Hypothetical Point Assignments by the Trumps

Notice that Ivana places great importane on the Connecticut estate, which had been the primary family home. Her acquisition of this property is consistent with all four marital agreements that the couple had signed.

On the other hand, Donald is far more interested in the Palm Beach mansion, which had been a vacation home. His intention was to divide its surroundings irto eight large development areas, to be called the "mansions at Mar-a-Lago", while keeping the 118-room main house for himself.

Ivana has a greater interest than Donald in the apartment at the Plaza Hotel, where she was living with the children during the couple's separation. He had moved to the triplex at Trump Tower and had a correspondingly greater interest in retaining it.

Applying AW, Donald is awarded initially the Palm Beach mansion and the Trump Tower triplex, giving him 80 of his points, whereas Ivana receives the Trump Plaza apartment and the Connecticut estate, giving her 70 of her points (these initial winnings are underscored in the hypothetical point assignments). But now the equitability adjustment demands that Donald give back some of his points on the smallest-ratio item he wins, namely, the Palm Beach mansion (40/20 = 2, which is less than 40/10 = 4 on the Trump Tower triplex).

Let *x* dente the fraction on the Palm Beach mansion that Ivana will get. Equalizing the points of Donald and Ivana requires that

$$70 + 20x = 80 - 40x.$$

Solving for *x* gives

$$60x = 10$$

$$x = 1/0 = .17.$$

Thus, Ivana gets about 17% of the mansion and Donald keeps about 83%. This adjustment results in each spouse's receiving 73.3 of his or her points, which is nearly three-fourths of the total value.

There is little reason to believe that the point assignments Donald and Ivana would make, had AW been used, would be other than sincere. Donald repeatedly expressed his desire for an amicable settlement, even going "a step beyond [the last postnuptial agreement] because I happen to love Ivana." (As an aside, one might ask: "Love her more than Maria?," who was Donald's girlfriend at the time,

prior to their marriage.) Because Donald probably could have successfully pursued a court case to have this agreement enforced if he had wanted to, all indications are that he looked with favor on the outcome.

Ivana, coming out of a relationship in which she had often felt dominated, wanted "nothing more than a fair and equitable share." Although she indicated a desire to "get even" early in the breakup, and later expressed this sentiment in a cameo appearance in a movie, the truth seems to be that she stili loved Donald: "Many friends insist Ivana would stili take Donald back. So does she. She directed one person to cali me back alter our interview to say she was 'stia madly in love with him'". After her breakup with Donald, Ivana remarried but got divorced shortly thereafter.

The actual settlement almost exactly matched the hypothetical AW settlement. Ivana received the Connecticut estate and the Plaza apartment, and Donald received the Trump Tower triplex. As for the Palm Beach mansion, it was not physically divided or sold, but Ivana could use it as a vacation home one month a year to be around her socialite friends, which was important to her.

In addition, Donald was obligated to pay Ivana $14 million in cash and $650,000 in annua) child support, which had been stipulated in their originai maritai agreement. This settlement mirrors well the 17% equitability adjustment on the Paim Beach mansion.

Most divorcing couples, of course, are not swimming in the riches of Charles and Diana or of Donald and Ivana. While the latter couples undoubtedly experienced some emotional distress, it was surely lessened by their Jack of concern for material necessities. By contrast, the *informai* processes most couples use in divorce lead to settlements that are, in the view of some analysts, "often contentious, adversarial, and beyond the perceived control of one or both parties".

AW could provide many divorcing couples with rapid closure of their often endless haggling over money, physical property, and the children. Like the Trumps, each spouse, because of personal circumstances or for sentimental reasons, may attach different values to different items. By indicating this with their point allocations under AW, they could both end up with what they perceive to be two-thirds to three-fourths of what they want, as did the Trumps, according to our calculation, and probably Charles and Diana as well.

If how much one item is valued depends on whether one wins another item, the items are not separable and undermine the application of AW. However, there are ways to handle this. Assume that one contested item is the house, but one spouse does not have the means to maintain it. Because winning it alone in a settlement is of little value to that spouse, the house and its maintenance are not separable.

In a case like this, possession of the house might be combined with a maintenance allowance if one spouse wins, whereas if the other spouse wins, he or she

might get only the house (without a maintenance allowance). Thus, instead of treating the house as a single object in which winning means the same – possession of the house – for both spouses, it can be treated as an issue in which winning gives a different outcome for each party.

Dividing up the children of divorce is often a major problem, though joint-custody arrangements are increasingly common. If both spouses attach significant value to getting their proper "share" of them, this is likely to be the smallest-ratio item when it comes time to making an equitability adjustment.

Of course, one hopes that the issue of custody, as it was in the Trump divorce, can be resolved prior to dealing with the other issues, though this will not always be possible. If not, AW may stili help a couple avoid the constant and often harmful conflict over custodia) and visitation rights that is the unfortunate accompaniment of many divorCes.

If there is an equitability adjustment on the children, there will typically be one party that wins more than the other. Once, this breakdown is made known to the parents, but not who the relative winner and the relative loser are, one agreement will suffice: The relative winner will get primary custody, and the relative loser will have visitation rights. As we saw in Chapter 6, the two sides are more likely to reach a settlement if they do not know which side they will end up on, but in this case only one agreement need be negotiated because both parents want the same thing (primary custody).

There are three advantages to using AW in divorce settlements. First, the impersona) assignment of points will help a couple separate the strong emotions and bittersweet feelings that often accompany divorce from the actual division of the maritai property, including children, that will be part of the settlement. Second, AW will induce each spouse to think carefully about what he or she most values and wants out of a settlement. Thus, the assignment of points to items will weaken one's desire to spite the other person, because to do so would be to give up points on something one may value more.

Finally, having to give up "hard" points rather than "soft" positions will minimize posturing in the negotiations prior to applying AW. For example, a wife is likely to see through a husband's threat to put a lot of points on the children or on child support if she knows he really wants to win on alimony. This threat might be effective in negotiations without AW, forcing the wife to give up alimony, for example, if she very much desires to keep custody of the children. With AW, this threat will tend to be seen as a bluff if the wife believes, in the end, that the husband will not match her points on the children.

Consider another problem that may occur when AW is used: One parent may not want a child to know that he or she is worth fewer points to that parent than the other. This problem can be solved by using a trusted referee or mediator to implement AW, who would not reveal the disputante' point allocations but simply announce the assignments. Because of the equitability adjustment, these assign-

ments will not be a sure-fire guide to which parent allocated more points to any item.

In the end, with or without revelation of the point assignments, AW determines the final settlement. It is likely to foster compromises on the issues if, in fact, the parties are encouraged by the procedure to be truthful about what they most value.

Presidential Debates

Since 1976, live televised presidential debates have been a staple of U.S. presidential campaigns. Although the first debates between presidential nominees were held in 1960, when John F. Kennedy, the Democratic party nominee, debated Richard M. Nixon, the Republican party nominee, there was a lapse of three campaigns (1964, 1968, and 1972) before they were resumed.

In 1960, it was widely believed that Kennedy "won" in these debates. His superior performance, especially in the first of the four debates with Nixon, was regarded by many analysts as decisive in swinging this exceedingly dose election in his favor. Alter this close cali, it is not surprising that Nixon decided not to debate his Democratic opponent, Hubert H. Humphrey, in the 1968 election, which this time swung narrowly in Nixon's favor, or George McGovern in the 1972 election, which Nixon won in a landslide.

The 1996 presidential debates between Bill Clinton and Bob Dole extended the unbroken tradition of debates to six campaigns over 20 years. But the format and composition of these debates were themselves debated, as they had been in the past, before they became a reality. The four issues over which the candidates and their advisers sparred were the following:

- Inclusion or exclusion of Ross Perot, the Independence party, candidate;
- Number and timing of debates;
- Length of each debate;
- Format of the debates.

Let's discuss each of these issues in tura

1. *Inclusion/exclusion of Perot.* President Clinton, who had a large lead over Senator Dole at the beginning of the general-election campaign in September 1996, wanted Perot included, apparently believing that Perot would hurt Dole more than himself. However, the nonpartisan Commission on Presidential Debates, established in 1987 as the sponsor of the debates, recommended Perot's exclusion, arguing that Perot had no "realistic chance" of winning the presidency. (Perot in fact received 8% of the popular vote in 1996, compared with the 19% that he received in 1992 alter he was included, with Clinton and Ggorge Bush, in the 1992 debates.)

Aides to the president warned that Perot might not accept the Commission's recommendation and that another sponsor, more amenable to including Perot, might be sought. John Buckley, Dole's communications director, saw this as a blatant political ploy: "The Clinton people are not cleaving to Mr. Perot out of excessive zeal for the democratic process. They are making a cold, purely political decision that his inclusion in the debates helps this president".

Dole's advisers believed that Perot would be nothing more than a distraction in the debates, and would hurt Dole's poli standings, which they desperately needed to raise. Curiously, the strength of their feeling on this issue is perhaps best conveyed by Mickey Kantor, the head of Clinton's negotiating team: "The Dole camp took the position they just wouldn't debate with him. They made it clear they would have gone with no debates rather than have Perot in".

Was this bluffing? In the view of one analyst, "Bob Dole could not afford the elimination of the debates; he would be forced to show up for a three-way debate. He is so far behind that he has few bargaining chips".

2. *Number and timing.* Clinton from the start had said that he wanted only two debates, preferably on October 6 and 13, whereas Dole desired three, the last occurring on October 20. A third debate would bring the last debate closer to election day on November 5, which would give Dole more opportunity to publicize his positions against the better-known positions of the president. Also, a third debate would give Dole more opportunity to try to whittle away at Clinton's Iead as the election approached.

3. *Length.* The president, considerably younger than his challenger and already well known as an accomplished debater, wanted each debate to last two hours, reportedly believing that his opponent would not hold up as well as he would. Dole preferred a length of sixty to seventy-five minutes, in part because a third debate would make up, in total time, for the reduced length of two shorter debates.

Presumably, the preferred length of a debate is related to the number of debates and their timing, making these two issues, at least to some degree, nonseparable. One way to counter this problem would be to define a single issue that allows choices on both of the present issues at once. Thus, the single issue might be the package "two debates, two hours each," which would be the outcome if Clinton won, versus "three debates, one hour each," which would be the outcome if Dole won. But in this analysis, these issues will be kept separate.

4. *Format.* In the first debate, Clinton wished to have only a moderator, but in the second he wanted the format of a town meeting, with audience participation, a format he had shown substantial skill in managing in the 1992 debate. Dole, not as convivial as Clinton, was less comfortable with this format, although it offered him the chance to prove himself to be a "regular guy" in a more participatory setting.

The following gives hypothetical point assignments by the candidates on the four issues:

Issues	*Clinton*	*Dole*
Inclusion/exclusion of Perot	40	50
Number and timing	20	16
Length	20	18
Format	20	16
Total	**100**	**100**

Hypothetical Point Assignments by Clinton and Dole

The key issue for both candidates was the inclusion/exclusion of Perot, but Dole, being in the weaker position, probably felt more strongly about getting his way on it than did Clinton. Accordingly, let's assume he assigns 50 points to this issue, compared with Clinton's 40 points. As for the other three issues, it is hard to say how much more important one, rather than another, was for the candidates.

Thus, let's simply assume that each candidate gives more or less equal weight to each issue. Notice that if the candidates can make only integer (whole-number) assignments, then it is not possible for Dole to do so – after he has allotted 50 points to the first issue – because $^{5}93. – 16.67$ is not an integer. But if we assume that the issue of length is slightly more important to Dole than the other two issues, then it would be reasonable for him to assign 18 points to ihis issue and 16 points each to the other two.

These assignments by both candidates enable Dole, initially, to win on the exclusion of Perot (50 points for him) and Clinton, initially, to win on the other three issues (giving a total of 60 points for him). Clinton, therefore, must transfer a fraction x of points back to Dole on the smallest-ratio item (length).

Setting Dole's points equal to Clinton's yields

$$50 + 18x = 60 – 20x.$$

Solving for x gives

$$38x = 10$$
$$x = {}^{5/19} \approx .26,$$

so Clinton must give about one-fourth back on the length issue. As a result, each candidate receives 54.7 of his points, a relatively small number compared with the 65.8 points that each party derived from the Camp David settlement and the 73.3 points each party derived from the Trump divorce.

This relatively poor showing of both candidates stems from their viewing the presidential-debate issues in almost "zero-sum" terms. That is, the magnitude of the win of one candidate is almost the same as the magnitude of the loss of the other candidate on each of the four issues, making them sum, more or less, to zero. Consequently, there is not much "added value" that can be derived by both candidates – what one wins, the other mostly loses – compared with the situations engendered by the more divergent positions of the disputante in the previous cases: They did not want the same things, so both sides could be satisfied more easi-

ly. Nonetheless, both candidates come out ahead, if only by a small amount, with about 55 points each.

As matters turned out, the outcome of the debate negotiations was dose to what it would have been, using AW, under our postulated point assignments. Dole, whose position on Perot was certainly strengthened by the report of the Commission on Presidential Debates, managed to get Perot excluded, alter which Perot vociferously attacked Dole on grounds of not only being unfair but also suppressing his legitimate right to be heard on the issues of the day. This attack might have cost Dole some Perot supporters, who otherwise would have voted for him as Perot's candidacy faded in the campaign.

Clinton won on the other issues, except for length, on which there was a compromise (the debates were 90 minutes). This compromise was not exactly according to the equitability adjustment, but such exactitude in matching the actual outcome probably cannot be achieved.

It is possible to "cook" the point assignments to do betterthat is, so that they more closely approximate the actual outcome. Making perfect predictions (or, more accurately, "retrodictions", sine the predictions are about how something in the part might have turned out) is not the purpose of this exercise, however; rather, it is to ground the point assignments in the expressed preferences of the parties. Only by maintaining independence between the parties' preferences and the known outcomes of the negotiations can legitimate comparisons be made between the presumptive results of AW and the observed outcome.

Questions frequently asked in real-life cases like the ones just assayed are: Did Ivana Trump value the Connecticut estate more than the Trump Plaza apartment, where she lived (40 versus 30 points in our scenario)? Given that Bob Dole indicated he would not debate President Clinton if Ross Perot were included, would he have put "only" 50 points on this issue? These questions are vexing, even to the parties involved.

There is no magic formula for making comparisons or tradeoffs. The assigning of points is certainly easier if the items in dispute are goods that one side or the other will receive, and only one of which need be split or shared.

But negotiations often involve less concrete and more inchoate items – issues on which winning and losing are far from evident. Applying AW to issues requires first determining what these are and then deciding what winning and losing on each issue mean for the two sides, which may require arduous negotiations.

Is this effort worthwhile? The potential rewards of using AWan efficient, envy-free, and equitable settlement – can be gleaned not only from històrical cases but also from controversies that are recurrent or not yet resolved. These are illustrated in Chapter 8, in which larger organizational interests play a significant role. Indeed, conflicting corporate, national, and intemational interests often create logjams that individuala in personal disputes are better able to avoid.

Steven J. Brams [*], D. Marc Kilgour [**] and Christian Klamler [***]

How to Divide Things Fairly [****]

Dividing a set of indivisible items, such as the marital property in a divorce, between two people can be a tricky business when the husband and wife rank the disputed property the same or similarly. But problems can arise even when they rank each item differently, as in this example:

Husband : Sports Car > SUV > Boat > Desk > Couch > Painting

Wife : SUV > Boat > Sports Car > Couch > Painting > Desk

At the outset, things look easy. Because we can give both spouses their first choices, it seems evident we should do exactly that, awarding the sports car to the husband and the SUV to the wife. It also seems clear that we should avoid giving them their last choices, which we can accomplish by awarding the painting to the wife and the desk to the husband.

But that leaves the boat and the couch, and we now have a problem: Both spouses prefer the boat to the couch, so who should get the boat? (Both also prefer the couch to the painting, but precluding the spouses' worst choices took care of this problem: The painting went to the wife, so the couch could go to the husband.)

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An alternative approach to dividing marital property would be for the husband and wife to apply an algorithm, such as *alternation*, whereby the spouses take turns, each choosing one item when it is his or her turn. If the spouses are *sincere* – choose in the order of their rankings – and the husband starts, he gets {sports car, boat, desk}, and the wife gets {SUV, couch, painting}. If the wife starts, she does better, obtaining {SUV, boat, couch}, and the husband does worse, obtaining {sports car, desk, painting}. In the latter case, the husband does particularly badly, getting stuck with his worst item (the painting).

The items to be allocated need not be physical goods. For example, they could be committee assignments or project tasks, in which it is stipulated that each person is required to have three. Or they could be chores ("bads" instead of goods), in which case we could ask each person to rank them from least to most burdensome.

Can we go beyond the ad hoc criteria that we began with, such as giving the players – who need not be people but could be larger entities, such as organizations or nation-states – their best items and not their worst? This may not always be possible. If we use an algorithm, is there one that avoids the first-chooser bias of alternation, and is it applicable to more than two players?

Properties of Fair Division

The fair division of items, especially if they are indivisible or cannot be shared, is an age-old problem. In this article we describe a simple sequential algorithm, called SA, which seems to have been overlooked in earlier studies [**2, 3**], for carrying out this division when the players strictly rank items from best to worst. It is less demanding in the information it elicits than are algorithms that ask players to indicate their utilities for items [**10**], to rank bundles of items [**11**], or to apply the classic procedure of "divide-and-choose" [**9, 10**].

We begin by specifying the properties of an allocation for two or more players that it would be desirable to satisfy. Although SA uses only players' rankings, one of the four properties we describe below uses the *Borda score* of an item as one measure of its utility to a player: A lowest-ranked item receives 0 points, the next-lowest 1 point, and so on. A player's Borda score is the sum of its points for the subset of items it receives, which may be thought of as one possible cardinalization (into utilities) of the ranks. In the absence of the players' actual utilities for the items, which we assume to be additive, it is the only one we use in the subsequent analysis.

The properties of allocations that we analyze are the following, whose two-letter abbreviations we also use as adjectives in describing allocations:

- *Efficiency or Pareto-Optimality (PO)*: There is no other allocation that is at least as preferred by all players and strictly preferred by at least one.

- *Envy-freeness (EF)*: Each player values the set of items it receives at least as much as the set of items received by any other player.
- *Maximinality (MX)*: The allocation maximizes the minimum rank of the items received by any player.
- *Borda Maximinality (BMX)*: The allocation maximizes the minimum Borda score of the items received by any player.

MX ensures that the rank of the least-preferred item that any player receives is as high as possible [**7, 12**], whereas BMX ensures that the Borda score of the player with the lowest score is as high as possible [**8**]. As we will show, different allocations may satisfy each of these properties.

Because SA requires only that players rank items, we need a definition of envy-freeness that enables players to compare the value of their items with the value of the items received by the other players. We say that a player, say *A*, does not envy another player, say *B*, if and only if there is an injection (a 1-1 mapping) from *A*'s items into *B*'s items such that *A* prefers each of its items to the item of *B* to which it is mapped [**8**]. An allocation is *item-wise envy-free* (EF) if and only if no player envies any other.

To illustrate this definition in the two-person example we discussed in the introduc-tion, assume that the husband receives {sports car, boat, desk} and the wife receives {SUV, couch, painting}. Then we can map item-wise the husband's items into the wife's such that he prefers each of his items to his wife's:

sports car > SUV; boat > couch; desk > painting.

Although there is a mapping for the wife such that she prefers two of her items to two of her husband's,

SUV > sports car; painting > desk,

it is not true that couch > boat for her. Indeed, no allocation of three items to each spouse makes possible a 1-1 mapping such that each spouse item-wise prefers each of his or her items to the items of the other spouse. Thus, this example does not admit an EF allocation, based on item-wise comparisons.

The Sequential Algorithm (SA) and Examples

SA works in stages. We illustrate it with four examples in this section and also discuss its properties. In the following section we will prove more general results.

We assume that there are n 2 players and $m = kn$ distinct items to be allocated, where k is a positive integer. If m is not a multiple of n (e.g., if $n = 2$ and m is odd), the "extra" items might be distributed to the players at random – with a maximum of one to each player – after SA has been applied.

SA produces an *equal* allocation, in which each player receives the same number k of items. If the allocation is not equal, it is not possible to make item-by-item compar-isons, which our definition of EF assumes. We recognize that unequal allocations may be envy-free – based on the utilities that players have for their subsets of items, com-pared with the utilities they attribute to the subsets of items of the other players – but we cannot make this comparison based only on players' ranks.

The allocation rules of SA, which give one item to each player on each round, are the following:

(i) On the first round, descend the ranks of the players, one rank at a time, stopping at the first rank at which each player can be given a different item (at or above this rank). This is the *stopping point* for that round; the rank reached is its *depth*, which is the same for each player. Assign one item to each player in all possible ways that are at or above this depth (there may be only one), which may give rise to one or more SA allocations.
(ii) On subsequent rounds, continue the descent, increasing the depth of the stopping point on each round. At each stopping point, assign items not yet allocated in all possible ways until all items are allocated.
(iii) At the completion of the descent, if SA gives more than one possible allocation, choose one that is efficient (PO) and, if possible, EF.

The process of descent is the same as that of "fallback bargaining" [**4**], but its purpose is the fair division of items, not reaching an outcome acceptable to some (e.g., a simple majority) or all of the players.

We next give examples that illustrate rules (i)-(iii) when $n = 2$; later we analyze an example in which $n = 3$. The players are *A, B,* . . ., and the items they rank are 1*,* 2*,* . . .. Players rank items in descending order of preference.

Example 1:

A: 1234

B: 2341

The stopping point of round 1 is depth 1, where *A* obtains item 1 and *B* obtains item 2. At depth 2 we cannot give different items to the players, because item 2 has already been given to *B*, so in round 2 we must descend to depth 3 to give the players different items (item 3 to *A* and item 4 to *B*).

We have underscored the items that each player receives. Because this exhausts the items, we are done, which yields the unique SA allocation of (13, 24) to (*A*, *B)*. Henceforth, we list the players in alphabetical order, and their items in the order in which the players rank them.

Observe that on each round, each player prefers the item it receives to the item that the other player receives (for *A*, item 1 > item 2 and item 3 > item 4; for *B*, item 2 > item 3 and item 4 > item 1). Hence, there is a 1-1 mapping of *A*'s items into *B*'s, and *B*'s items into *A*'s, such that each player prefers its items to the other player's items. Therefore, the allocation (13, 24) is EF.

This allocation does *not* depend on a player's utilities for items, which we assume are consistent with their rankings (i.e., higher-ranked items have greater utility than lower-ranked items) and additive. Other two-item allocations, such as (12, 34), are not item-wise EF, because there is no 1-1 mapping of *B*'s items to *A*'s such that *B* prefers each of its items to the items to which it is mapped. In particular, notice in *B*'s ranking that items 2 and 1 bracket items 3 and 4, so *B* may prefer the combination of items 2 (best) and 1 (worst) to the combination of items 3 and 4 (two middle-ranked items).

For example, if *B*'s utilities for items 1, 2, 3, and 4 are 1, 5, 3, and 2, then *B*'s utility for its subset of items, 34, is 5, and its utility for *A*'s subset of items, 12, is 6, so *B* will envy *A*. But if *B*'s utilities are 1, 6, 5, and 4, then it values its subset at 9 and *A*'s subset at 7, so in this case *B* will not envy *A*. Only allocation (13, 24) is EF for all possible utilities of the players consistent with their rankings.

It is easy to see that (13, 24) is PO, because there is no allocation that is at least as preferred by both players. We say that (13, 24) is *Pareto-superior to*, or *Pareto-dominates*, another allocation—say, the "reverse" allocation, (24, 13)—because 1 > 2 and 3 > 4 for *A*, and 2 > 3 and 4 > 1 for *B*. In Example 1 no allocation is Pareto-superior to (13, 24), which means that (13, 24) is PO.

In general, an allocation is PO if and only if it is the product of a *sequence of sincere choices* by the players [**8**], whereby each player chooses its best available item on its turn. Thus, if the players choose items in the order *ABAB*, they obtain (13, 24); if they choose in the order *AABB*, they obtain (12,34), so both these allocations are PO. By comparison, no sincere sequence yields the allocation (24, 31), so it is not PO.

The allocation (13, 24) is also MX; the only other allocation of two items to each player that gives neither player a worst item is (12, 34), rendering it also MX. However, (12, 34) is *not* BMX, because it gives Borda scores of (5, 3) to (*A*, *B*), making *B*'s score less than the score of 4 that each player receives from (13, 24).

If SA gives two or more allocations, only one may be MX. This is true of the two SA allocations – one on the left, the other on the right – in our next example (the vertical lines are explained below):

Example 2:

A: 12345|678	*A* :	12345|678
B: 34567|812	*B* :	*34567|812*

In the first two rounds, SA gives (12, 34) to (*A*, *B)*, reaching depth 2 in both allocations. In round 3, the stopping point is depth 5 for both the left and right allocations, as shown by the vertical lines, but now there is some choice in the items we give to *A* and *B*. In particular,

(i) the left-hand allocation gives items (5, 6) to (*A, B*) at depth 5, followed in round 4 by items (7, 8) at depth 7, resulting in (1257, 3468);
(ii) the right-hand allocation gives items (5, 7) to (*A*, *B*) at depth 5, followed in round 4 by items (6, 8) at depth 6, resulting in (1256, 3478).

Clearly the right-hand allocation, with a maximum depth of 6, is MX. Note that all allocations must have depth 6 or greater; otherwise, item 8 would not be assigned to either player.

The right-hand allocation (1256, 3478) is also BMX, giving (A, B) Borda scores of (18, 18) – a minimum score of 18 – whereas the left-hand allocation (1257, 3468) gives the players scores of (17, 19) for a minimum score of 17. (An exhaustive search shows that no other allocation, even among allocations in which players receive differ-ent numbers of items, gives a greater minimum than 18.) Thus, while both allocations are EF and PO, only the right-hand allocation is MX and BMX.

Example 2 illustrates how SA can result in more than one allocation that is PO and EF, but only one is MX or BMX in this example. Our next example illustrates that if both players rank the same item last, there cannot be an EF allocation.

Example 3:

A : 123456	*A* : 123456
B : 235416	*B* : 235416

In rounds 1 and 2, with stopping points at depth 1 and depth 3, the left-hand and right-hand allocations coincide, giving 13 to *A* and 25 to *B*. In round 3, the stopping point is depth 6 for both allocations, the lowest possible, but items 4 and 6 are assigned in two different ways.

The player who receives item 6 must be envious, because no 1-1 mapping can map item 6 to a less-preferred item. Thus, the allocation of items in Example 3 is not EF, although the partial allocation of the first four items to both players at depths 1 and 3 is. Both complete allocations are MX (maximum depth of 6) and BMX (minimum Borda score of 8).

Our final example in this section, which duplicates the rankings of the husband (*A*) and wife (*B*) in the introduction, illustrates that an SA allocation may fail to be EF even when the players rank every item differently (we illustrated this failure earlier for alternation but not for SA):

Example 4:

A : 123456	*A* : 123456
B : 231564	*B* : 231564

Here the problem arises in round 2, with the stopping point at depth 4, where the left-hand allocation gives 14 to *A* and 23 to *B*, whereas the right-hand allocation gives 13 to *A* and 25 to *B*. Whoever receives the lower-ranked item in round 2 will be envious, because

no 1-1 matching can map every item of that player to a lower-ranked item of the other player. Despite the fact that each player ranks the six items differently, and no player receives a worst item in either SA allocation, neither allocation in Example 4 is EF.

In Example 4, both allocations are MX (in the descent, the depth reaches 5 when the allocations are complete); moreover, they are equal according to BMX, with Borda scores of 10 to the advantaged player and 8 to the disadvantaged player, so the mini-mum Borda score of each allocation is 8. According to both MX and BMX, therefore, the two allocations in Example 4, unlike Example 3, are equally fair to the players.

When $n = 2$, there is a simple condition, called "Condition D" [**6**], for determining whether an EF allocation can exist: For every odd i, the two players' sets of top i items are *not* identical. In Example 3, A's and B's top 5 items are identical, and in Example 4, their top 3 items are identical, so neither example yields an EF allocation. This is not true for the top 1 and top 3 items in Example 1, nor the top 1, 3, 5, and 7 items in Example 2, so in both of these examples an EF allocation exists which, as we showed, SA finds.

To summarize, SA may give a unique PO-EF allocation (Example 1) or multiple PO-EF allocations (Example 2), only one of which – possibly not the same one – is MX or BMX. In addition, SA may not produce an EF allocation (Examples 3 and 4), even when the players rank all items differently (Example 4), because no EF allocation exists.

Properties of SA

If all items can be allocated in an EF way, we say there is a *complete EF allocation*. For $n = 2$, Brams, Kilgour, and Klamler [**6**] provide an algorithm, AL (for "algorithm"), which finds at least one complete PO-EF allocation if one exists (though not necessar-ily all of them, as we will see). Furthermore, when there is no complete EF allocation, as in Examples 3 and 4, AL finds the largest and most preferred subset of items that can be allocated in an EF way. Items that cannot be so allocated (e.g., items 4 and 6 in Example 3; items 3 and 6 in Example 4) are placed in a "contested pile," to which another algorithm, called undercut, can be applied [**1, 5**].

By contrast, SA always allocates all items. As illustrated in Examples 3 and 4, SA yields an allocation that may be EF only on some rounds, rendering the allocation only partially EF.

Although SA may not give a complete EF allocation, it always produces at least one PO allocation. Moreover, this is true however many players there are (i.e., for all n 2).

Theorem 1. *SA rules (i) and (ii) produce at least one allocation that is PO.*

Proof. Under SA, all items are allocated one at a time to the players and ranked at or above each stopping point in the descent process. Because each allocation gives equal numbers of items to the players, a non-SA allocation must give at least one player an item

it ranks below some item that it would receive under SA. This proves that no non-SA allocation can be Pareto-superior to any SA allocation. Because Pareto-superiority is irreflexive and transitive, at least one of the SA allocations – say, X – must be maximal with respect to Pareto-superiority within the set of SA allocations. Because no non-SA allocation can be Pareto-superior to X, X must be PO. ■

Example 5:

A :	12345|678	A :	12345|678
B :	78123|546	B :	78123|546

At the completion of round 2, SA gives (12, 78) to (A, B), stopping at depth 2. The next stopping point is at depth 5, indicated by the vertical lines, where B must receive item 3 in round 3. There is, however, a choice for A: The left-hand allocation gives item 4 to A, and the right-hand allocation gives item 5 to A.

Continuing, the left-hand allocation gives items 6 and 5, respectively, to A and B in round 4, with the stopping point at depth 6, whereas the right-hand allocation gives items 6 and 4 to A and B, respectively, in round 4, with the stopping point at depth 7. Because both players prefer the last two items they receive in the left-hand allocation to those that they receive in the right-hand allocation, the left-hand allocation Pareto-dominates the right-hand allocation, so only the left-hand allocation is PO.

Interestingly enough, both SA allocations in Example 5 are complete EF alloca-tions, even though only one is PO, showing that EF does not imply PO. The converse also fails because, for example, an allocation that gives one player only its top items will generally make another player envious. Thus, PO and EF are independent properties.

In Example 5, the left-hand allocation (1246, 7835) is MX (its maximum depth is 6). An exhaustive search of equal and unequal allocations shows that it is also BMX, with Borda scores of (19, 18), compared with Borda scores of (18, 17) for the right-hand allocation.

But SA does not invariably find a PO-EF allocation that – based on the properties of MX or BMX – is superior to a non-SA allocation, as our next example illustrates.

Example 6:

A :	12345678	A :	12345678
B :	87632154	B :	87632154

The SA allocation is shown on the left. In the first three rounds, at depths 1, 2, and 3, SA allocates (123, 876) to (B). On round 4 and at depth 7, A and B receive, respectively, items 4 and 5, producing the allocation (1234, 8765).

But the non-SA allocation (1245, 8763) on the right is of depth 5. Moreover, it is not only MX but also BMX, giving Borda scores of (20, 22), compared with (22, 19) for the

SA allocation. Both the left-hand and the right-hand allocations are EF and PO. We will return to this example later to show that there are seven distinct complete EF allocations, but only the aforementioned two are PO.

Both the left-hand (SA) and the right-hand (non-SA) allocations in Example 6 are complete EF and PO (prefers the former, and *B* the latter, when each player obtains its four best items). AL gives only the SA allocation, so like SA, it does not always find all PO-EF allocations – including those that might be MX or BMX (e.g., the non-SA allocation on the right in Example 6) – as we incorrectly stated in [**6**].

Although at least one SA allocation is PO by Theorem 1, it may not be EF, as we showed in Examples 3 and 4. But if there is an EF allocation when $n = 2$, we have the following result.

Theorem 2. *Let n = 2. If an EF allocation exists, then SA will give at least one allo-cation that is EF and PO.*

Proof. We earlier mentioned Condition D (see Example 4) – that an EF allocation exists if and only if, for all odd k, at least one of A's k most preferred items is not one of B's $k$most preferred items. Another necessary and sufficient condition for an allocation to be EF is that the item that each player receives on the j th round is among the player's top $2j - 1$ items [**6**].

Assume Condition D holds. Taking $k = 1$, it is clear that A's and B's most pre-ferred items are different, so on round $j = 1$, SA must allocate to each player its most preferred item, and the stopping depth d_j is $d_1 = 1$.

Now assume that, up to the completion of round j, SA has allocated j of each player's top $2j - 1$ items, and the stopping depth on round j is d_j $2j - 1$. Consider round $j + 1$. Combined, the preference orderings of A and B account for either 2, 3, or 4 distinct additional items at depth $2j$ or $2j + 1$. Therefore, to assign an additional item to both A and B from their top $2j + 1$ items, it is necessary to increase the stopping depth to at most $d_j + 2$.

If there is a choice, ensure that a player does not prefer any unassigned item to the item it receives. It follows that d_{j+1} $2j + 1$, and that the $(j + 1)$st item received by each player is among its $2j + 1$ most preferred items. Therefore, the resulting SA allocation is EF. Moreover, it is PO, because it is the result of a sequence of sincere choices (as discussed after Example 1). ■

When $n = 2$, it is relatively easy to determine whether a given allocation is EF, PO, MX, or BMX. It is considerably more complex to find *all* allocations that are, say, EF.

To illustrate this calculation, recall from Example 6 that we gave a non-SA equal allocation that improved upon the SA allocation in terms of MX and BMX, but we did not prove that it was the only such allocation, or that there was not another allocation that better satisfied one or both of these properties. To analyze Example 6 in detail, we list all possible item-by-item allocations at each odd depth.

Example 6 (repeated):

A: 12345678
B: 87632154

At depth 1, (*A*, *B*) must receive items (1, 8). Then, at depth 3, (*A*, *B*) must receive, in addition, one of (2, 7), (2, 6), (3, 7), or (3, 6). Finally, at depth 5 and again at depth 7, (*A*, *B*) must receive pairs of items that depend on the items already received. The details are shown in Table 1, which includes all EF allocations for Example 6, as well as their MX depths and Borda scores, illustrating that the determination of all EF allocations and their properties may be combinatorially complex.

As Table 1 shows, there are seven EF allocations, labeled *a*, *b*, *c*, *d*, *e*, *f*, and *g*, which we call *classes*, that can be reached in a total of 21 different ways. Specifically, there are 7 *a*'s, 4 *b*'s, 2 *c*'s, 1 *d*, 1 *e*, 5 *f* 's, and 1 *g*. The MX depths and Borda scores depend only on the class, not on the way it was obtained. These scores are shown only for the first member of each class.

The MX depths of the *b*'s and the *f* 's are minimal (i.e., 5), but only the *b*'s have a maximin Borda score (20). This verifies that allocation *b* (1245, 8765) is indeed MX and BMX. It and the unique SA allocation (allocation 1 in class *a*) are the only PO allocations.

So far we have not illustrated SA with examples in which $n > 2$. While its applica-tion to the division of items among three or more players is straightforward, if more tedious, SA no longer ensures that if there is a complete EF allocation, it will be chosen by SA when $n > 2$.

Example 7 (repeated):

A: 12345<u>6</u>789

B: <u>58</u>126<u>7</u>349

C: <u>349</u>125678

SA allocates items (1, 5, 3) to (*A*, *B*, *C*) at depth 1; then (2, 8, 4) at depth 2; and finally (6, 7, 9) at depth 6. Notice that *B* may envy *A* for obtaining items {1, 2, 6}, which fall between *B*'s two best items (items 5 and 8) and its sixth-best item (item 7). Because *A*'s items bracket *B*'s, it follows that there is no 1-1 mapping of *B*'s items to *A*'s such that *B* always prefers its own item to the item of *A* to which it is mapped. Thus, this allocation is not EF.

However, by switching items 6 and 7 between *A* and *B* in the SA allocation, we obtain a non-SA allocation, as demonstrated below.

Table 1. – **EF Allocations, MX Depths, and Borda Scores for Example 6**

Allocation	Depth 3	Depth 5	Depth 7	Complete	MX Depth	BMX Score
1	(2, 7)	(3, 6)	(4, 5)	(1234, 8765)-*a*	7	(22, 19)
2		(4, 6)	(3, 5)	(1243, 8765)-*a*		
3			(5, 3)	(1245, 8763)-*b*	5	(20, 22)
4		(4, 3)	(5, 6)	(1245, 8736)-*b*		
5			(6, 5)	(1246, 8735)-*c*	7	(19, 18)
6		(5, 6)	(4, 3)	(1254, 8763)-*b*		
7		(6, 3)	(4, 5)	(1264, 8735)-*c*		
8	(2, 6)	(3, 5)	(4, 7)	(1234, 8657)-*a*		
9		(4, 3)	(5, 7)	(1245, 8637)-*b*		
10			(7, 5)	(1247, 8635)-*d*	7	(18, 17)
11	(3, 7)	(2, 6)	(4, 5)	(1324, 8765)-*a*		
12		(4, 6)	(2, 5)	(1342, 8765)-*a*		
13		(4, 2)	(6, 5)	(1346, 8725)-*e*	7	(18, 17)
14			(5, 2)	(1345, 8762)-*f*	5	(19, 21)
15		(5, 6)	(4, 2)	(1354, 8762)-*f*		
16		(5, 2)	(4, 6)	(1354, 8726)-*f*		
17	(3, 6)	(2, 7)	(4, 5)	(1324, 8675)-*a*		
18		(2, 5)	(4, 7)	(1324, 8657)-*a*		
19		(4, 2)	(5, 7)	(1345, 8627)-*f*		
20			(7, 5)	(1347, 8625)-*g*	7	(17, 16)
21		(5, 7)	(4, 2)	(1354, 8672)-*f*		

Note: At depth 1, the 21 complete EF allocations give items (1, 8) to (*A*, *B*). At lower depths, they fall into seven classes (7 *a*'s, 4 *b*'s, 2 *c*'s, 1 *d*, 1 *e*, 5 *f* 's, 1 *g)*, each of which gives the same complete allocation but different items at different maximum odd depths. The MX depths, and the BMX scores, are shown only for the first member of each class. The MX depths of the *b*'s and the *f* 's are minimal (5), but only the *b*'s have a maximin Borda score (20). The *a*'s and the *b*'s are the only two classes that yield PO allocations, with the first *a* allocation (allocation 1) being the unique SA allocation.

Example 7 (cont.):

A: 123456789

B: 581267349

C: 349125678

To show that the allocation *(*127*,* 586*,* 349) is EF, observe that *C* gets its three best items, so it cannot do better and, therefore, will not be envious. But now it is easy to check that the required 1-1 mappings of *A*'s items to *B*'s, *B*'s to *A*'s, and *A*'s and *B*'s to *C*'s, all exist, confirming that the allocation is EF.

As illustrated in Example 1, we can similarly demonstrate that the SA allocation in Example 7 is PO with the sequence of sincere choices, *ABCABCCAB*. To demonstrate that the non-SA allocation is also PO, we can use the sequence of sincere choices, *ABCABCCBA*.

Although not EF, the SA allocation in Example 7 has the advantage of being both MX and BMX. It gives *A* and *B* at worst a sixth-best item, whereas the non-SA allo-cation gives *A* a seventh-best item. Similarly, the SA allocation gives Borda scores of (18, 18, 21) to (*A*, *B*, *C*), whereas the EF allocation gives the players Borda scores of (17, 19, 21), so the SA allocation gives a higher minimum. Clearly there are trade-offs among our properties, and which should take priority may be open to debate.

As a final property of SA, we consider its vulnerability of manipulation. Not sur-prisingly, if $n = 2$ and one player (say, *A*) has complete information about the prefer-ences of the other player (*B*), and *B* is sincere, *A* can exploit *B*, as shown in our next example.

Example 8:

A: 123456

B: 635421

The SA allocation is underscored, with *B* receiving its three top items and *A* not doing quite so well. But now assume that *A* insincerely indicates its preferences to be those shown below, with *B*'s preferences remaining the same.

Example 8 (cont.):

A: 312456

B: 635421

This SA allocation shows that *A*'s insincere preferences turn its original disadvantage into an advantage by giving it its three top items, whereas *B* now does worse.

Although not strategy-proof, SA seems relatively invulnerable to strategizing in the absence of any player's having complete information about its opponent's or oppo-nents' preferences. The manipulator's task is further complicated if the other players are aware that an opponent might try to capitalize on its information and, consequently, they take countermeasures (e.g., through deception) to try to prevent their exploitation.

Summary and Conclusions

To summarize, we have shown that if n 2, SA always yields at least one PO allocation and, if $n = 2$, SA always yields an allocation that is PO and EF, provided an EF allocation

exists. Although, initially, SA may produce some allocations that are not PO, these will be eliminated by invoking SA rule (iii). The set of PO-EF allocations that SA produces, however, may not include one that satisfies the properties of MX or BMX, although our examples suggest that it probably will not be far off.

If $n > 2$, SA may fail to yield an EF allocation when one exists. In such a case, however, an SA allocation may have redeeming properties, such as be MX or BMX. While SA is not strategy-proof even when $n = 2$, in most real-life cases it is unlikely that one player would have sufficient information about another player's preference rankings – not to mention be able to formulate a strategy that would exploit such information – to manipulate it successfully.

SA seems most applicable to allocation problems in which there are numerous small items, which need not be physical goods, as we noted earlier. If there is one big item that two players desire (e.g., the house in a divorce), it may not be possible to prevent envy, especially because SA specifies that each player must receive the same number of items. (This stipulation may be viewed as essential to achieving fairness in some, but certainly not all, situations.) In such a case, the most practical solution might be to sell the big item – in effect, making it divisible – and divide the proceeds.

Other modifications in SA might include not restricting the allocation of items of one to each player on every round, and relaxing the assumption that the number of items is an integer multiple of the number of players. These modifications would change the fair-division problem fundamentally, however, because properties like EF, MX, and BMX would have to be redefined to take into account that players may not receive the same number of items.

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Summary. We propose an intuitively simple sequential algorithm (SA) for the fair division of indivisible items that are strictly ranked by two or more players. We analyze several properties of the allocations that it yields and discuss SA's application to real-life problems, such as dividing the marital property in a divorce or assigning people to committees or projects.

Chapter 2

The legal and ethical framework

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Equitative Algorithmic Justice. Use, Innovation and Limits in Law

Abstract

Equitative algorithms refers to a set of algorithms that can be used in the legal field for the resolution of conflicts in which it is possible for the parties to freely assess their own interests and values to be protected. The article is an introduction to the research of the CREA-Project on the subject and discusses some issues that arise in the legal field from the application of the Fair Division Theory to legal dispute resolution. The article aims to introduce the theoretical-practical legal basis that can make justice via equitative algorithms an instrument of general application in the legal field. Two main issues are discussed: the identification of law with a text and the identification of law with justice.

1. Equitative algorithmic systems and the law

The expression 'equitative algorithms' refers to a set of algorithms that can be used in the legal field for the resolution of conflicts in which it is possible for the parties to freely assess their own interests and values to be protected (Equitative Algorithms Justice, EAJ). The freedom of assessment is addressed both to the algorithm and to the legal system, as well as to the other parties involved. In other words, we are faced with EAJ whenever a dispute is algorithmically resolved and the parties, freely and independently of each other, have established their own order of values with respect to a set of assets and rights. It is possible that there are external limitations, coming either from the market or from the law or from the *de facto* relationships between the parties, but these can sometimes be taken into account by the chosen algorithm.

Equitative is a neologism in the English language, created by the CREA group, which launched the project with the same name [1]. The neologism helps for pur-

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[1] The project CREA has received funding from European Union's Justice programme 2014-

poses of univocity of meaning. The words equity and equitable have a very long history behind them, which intertwines, and sometimes knots, with that of law and justice [2]. Since the second half of the last century, their meaning has been enriched with new dimensions thanks to the studies carried out in Decision Theory, Game Theory and Economic Analysis of Law. We considered it appropriate to find a new word to focus and bound this new branch of legal studies.

EAJ is daughter and debtor of fair division theory and algorithms [3] (FD), as outlined by Steve J. Brams and Alan D. Taylor (Brams 1990; Brams & Taylor 1996; Brams 2012). FD has an original approach, which naturally leads it to budding branches and secondary theoretical suckers in many disciplines. In fact, according to them, its methodological approach "involves

– setting forth explicit *criteria*, or *properties*, that characterize different notions of fairness;
– providing step-by-step *procedures*, or *algorithms*, for obtaining a fair division of goods or, alternatively, preferred positions on a set of issues in negotiations; and
– illustrating these algorithms with *applications* to real-life situations" (Brams & Taylor 1996, 1).

Beyond the considerations specific to each of the scientific fields involved, from Decision Theory to Social Sciences, from Economic Analysis of Law to Political Theory and Legal Science, Brams and Taylor's FD is important for bringing these three steps together in a single theoretical moment.

The FD is applied to conflict resolution, but lacks a theoretical-practical legal basis that can make it an instrument of general application in the legal field.

The problems that arise when comparing these procedures with traditional legal dispute resolution procedures are manifold. Among others, it is necessary to clarify immediately those arising from the meaning of fair and equity, which have, in the legal field, strong theories dating back a considerable length of time and which cannot be ignored.

EAJ, therefore, intends to apply the FD in the legal field, analysing and hopefully solving the legal problems related to its application.

As said before, the choice of the neologism, equitative instead of equitable or

2020, under grant agreement No. 766463. This book provides some of the most important achievements of the research, www.crea-project.eu.

[2] The history of the idea of aequitas is well investigated in literature. This research regains the connection between the emotionality of the parties and the rationality of the legal order in the formation of the judgment. In this insight, it is perhaps a return to the Roman concept of aequitas, before Irnerio and the school of glossators in Bononia.

[3] The origins of the FD, like every origin, are discussed and can be traced back to the starting point of European philosophy: the philosophers of ancient Greece, in particular Aristotle. (Moulin 2004).

equity, serves, in the legal field, to separate the concept from the traditional connection with ethical or historical legal issues, avoiding misunderstandings. It also avoids misunderstandings with the rigorous definition adopted by Brams and Taylor for equitable in the FD: "[a]n allocation is equitable for two players if each player thinks that the portion he or she receives is worth the same, in terms of his or her valuation, as the portion that the other player receives in terms of that player's valuation. If the two players have different entitlements, equitability means that each player thinks that his or her portion is greater than his or her entitlement by exactly the same percentage" (Brams & Taylor 1996, 241).

A proposed dispute resolution that can be accepted as fair by the litigants requires conditions that are often less stringent or limiting than those imposed by Brams and Taylor's equitability, since those required by envy-freeness are sufficient: "An allocation is envy-free if every player thinks he or she receives a portion that is at least tied for largest, or tied for most valuable and, hence, does not envy any other player" (Brams & Taylor 1996, 241).

2. What are these writings and this book for and who they are directed to?

This paper, as well as the whole book that the paper introduces very briefly, is mainly addressed to mathematicians and to those, among jurists and economists, who want to deepen and spread in law the equitative algorithmic systems.

It aims, therefore, at being an interface between different worlds and sets of knowledge, in which even methodological principles diverge, as well as basic and powerful concepts such as truth and reality, but also efficiency and justice.

It may sound bizarre that, in different branches of knowledge, the concepts of truth are different. Truth should be one, and only one, but we should always bear in mind that theory of law is a normative discipline or science, whose *raison d'être*, we might even say axiology, is the resolution of social conflicts, affording social peace in the society to which it is addressed. This is true in every legal system.

A normative science, such as legal science, therefore, provides guidelines and solutions for settling conflicts. Any reality is seen in this peculiar teleology and the decision on what is the legal truth about the past in which the conflict originated is, indeed, a decision, not a simple recognition. It is a decision with particular features because it comes from a single authority empowered to rule on it: the judge.

In the legal field, one may argue about the true interpretation of a provision of law or the truth about an event of the past, but the only true interpretation, for the law, valid for all citizens, is the one coming from the judge. Truth and authority

come together in an act, the judgment, which puts an end to uncertainty and creates social truth. In law, there is no truth without authority that establishes it.

As odd as it may seem to common sense, if you reflect with your mind clear of *prêt-à-porter* ideas, wondering how a judge thinks and how he or she reaches the conclusion, you may be able to approach this reality. Let us ask ourselves, for example, how to distinguish the judgment of a corrupt judge from that of a judge who, instead, honestly tries to judge according to objectivity, truth and exactitude. By no way can we do this, we can only induce it probabilistically by gathering evidence of that judge's corruption. But this means that we have no way to establish the truth of the judgment by analysing the judgment itself. Even the judgment of an honest judge can make a distorted use of the facts or an ideological use of the law, as well as that of a corrupt judge. These judgments are all true, because they all have the authority to sentence. Social truth is there, in the judgment, not in facts or laws. The judgment of a Supreme Court, in Italy of the Corte di Cassazione, is true and the judgment that has been overturned is false because of a principle of authority, nothing else. The truth is the product, not the origin nor the find out of the process.

This could lean to consider that no space is open to the algorithmic decision in the legal field, because it is not a decision coming from a judge and because the algorithm has no authority. But the question that should be correctly asked is which authority, if any, should be given to an algorithmic decision. The question of the authority is a matter of order and regulation, or a matter of social recognition.

This is a point of reflection on which legal theorists are currently confronting, not without contortions and pains of various kinds, but still looking for a solution that takes into account the existence of these new possibilities of decision. In fact, they prefer, rather, to talk about support for the decision.

In this representation, the term ‘law’ refers to the result of a decision (the judgment) taken by a certain authority (the judge) of a certain legal order, and effective in a certain society.

But the term ‘law’ may also have other references. The citizen will look to the law in search of an unambiguous answer to predict or organise his or her own and others’ behaviour. In this case, the term law refers to a fictitious reality, existing and arising from the set of rules and judgments and legal acts and facts existing and in force in a given society. Here the law is not a decision, but a meaning contained in general and abstract descriptions or in behaviours. Also, from this path it is possible to reach authority: a social behaviour shared and repeated over time, a consolidated custom, or an interpretation confirmed by behaviours can be considered law because they rely on the authority (effectiveness) of social consensus.

3. The game of law

The judge's verdict is the result of a procedure, sometimes very long, in which the lawyers of the parties and the parties themselves, among themselves and with the judge, face each other. Here there are different roles that the parts play and purposes to which they tend; it is also different as to what they refer to as law. For the lawyers, the truth of the trial, the law, is built in the defence of the client; for the judge it is in the exact reconstruction of the fact, in the correct interpretation of normative texts, in deciding according to justice. The judge has a greater need for truth, objectivity and exactitude than the lawyer, who will need persuasion, not by any means, but almost.

But the law does not limit its scope to the trial, it is a point of reference in the activity of the Public Administration and of the citizen; the legal scholar, then, considers the analysis of law his job.

If it were possible to turn the behaviour of lawyers, judges, parties, citizens and administrators into game theory rules, there would be a set of different rules defining different games [Chiassoni 1999], some, for example, being games aimed at achieving knowledge and others at achieving a practical result [Chiassoni 1999, 89]. In all of them, however, it is a question of determining 'the law'. The concept of law also will depend on the role of the players and on the type of game being played.

The lawgiver too has a role to play in the game of law. The lawgiver should be the equivalent of nature in the natural sciences; he sets the laws that the player-judge must take into account, interpret and transform into acts modifying reality. But, while the scientist of nature is constrained by mathematical laws, chosen by him as a chain of inferences, in defining the laws of nature, the player-judge is constrained by human laws in establishing law. As everyone sees the two procedures are not isomorphic, there is no match between laws and realities observed in the two worlds; human laws do not take the place, in this metaphor, of the laws of nature, but of the inferential rules of mathematics.

4. The commonplace in law

This introductory paper is also intended to help avoid some commonplaces and misunderstandings about the law, facilitating a possible use of the EAJ, replacing or complementing some established legal procedures.

The commonplace is a shared, and frequent in use, opinion on something, which, while allowing a quick exchange of speeches and ideas, nevertheless hides and overlooks a number of issues. It means, therefore, an inadmissible inaccuracy in the scientific field. Commonplaces also differ between various cultural groups, so, for example, the commonplaces of mathematicians with re-

gard to law are often very different from those of engineers or economists.

There is little kinship or understanding between scholars belonging to different fields of knowledge, especially those who belong to a field considered as among the humanities hardly understand and, in turn, are poorly understood by those who deal with mathematics. The 'jurist' is often accused of carrying on arguments that do not hold from a formal point of view, or of not being reliable. How much these *clichés* are anchored in reality and how much in prejudice is difficult to assess.

However, mistrust is a sign of prejudice, where there are deep-rooted aversions that the individual, even of a good cultural level, is unable to remove; this happens because he or she would not like to remove them. They come from the personal emotionality, so are normally little known and recognised by the individual himself. Prejudices cradle and reassure people and removing them is like removing a part of oneself, extracting them just like extracting a tooth. One may think that this writing has the intent to disarm these 'rooted' convictions, aversions, prejudices and judgments.

Of course, jurists dress in a strange way, absolutely so here in Italy, with a tie, even in summer, and there could be the danger that a mathematician, in a scientific meeting with jurists, would be forced to wear a jacket and tie too, which would prevent him from wearing comfortable flip-flop sandals: which would not be efficient for an economist and would not be rational for a mathematician, but would give great authority to the jurist. Instead, a judge in flip-flops would need detailed investigation, including psychiatric ones.

Instead, I do not want to disarm anyone in his convictions and habits, I want here to analyse, for a brief moment, the law from a different point of view, and different also for the jurists. I want to just check if and where it is possible to insert the EAJ in an efficient way for the legal systems and advantageous for the users of them and justice in general and under what conditions. Equitative algorithms are new procedures that need to be ‘jurified’; this requires looking at the law with new eyes.

Everybody should keep their prejudices about jurists, economists and mathematicians, but let us now look at the law through Galileo's lenses.

What good is this book to a mathematician? To see what needs there are in the legal field, and what operating conditions, and, consequently, to identify the areas in which mathematical research can be usefully employed.

What can this book be used for as a jurist? To take a look at the law outside the schemes and trivialities of our days spent on algorithms, to afford a vison of what will probably be some future developments of legal systems. It could also be used by the jurist to restore the veritative function in the world that other sciences are taking away from him today.

Every theory of law gives it only a partial look, closed within the limits useful for the methodology's validity. The use and development of future EAJ systems is

possible only by giving the law an overall view; however, while new and wider boundaries are valid for EAJ, it still has to take all the necessary steps to transform FD systems into legal procedures. We cannot limit ourselves solely to the utterances, neither to their meanings, nor to their validity nor to the effectiveness alone. We need all of that.

Let's get the trumpeters and flag-waving flag bearers of immutability away from us and from the scientific desk. The law is far from immutable, it has manifested itself in human societies in the most varying ways, linguistic and not, rational and not, but always designated by the purposes that constitute its reason for existence in human societies: the resolution of conflicts.

The pivotal point in EAJ's systems, which differentiates them from other legal algorithmic systems, is the possibility of leaving it to the parties in establishing the order of interests, or, in general, of the values that they most prefer. Western legal systems have frequently taken away from the citizen the possibility to intervene in the process to modify the order of values established in the law, often even when this was not necessary for reasons of protection of the weaker party or for other constitutionally guaranteed reasons. Reasons of streamlining and speed of proceedings have supported this choice, or even the principle of uniformity of law. The judge, after hearing the parties, after hearing the experts, assesses and decides, attributing assets and rights according to his own evaluation, together with that of the experts.

EAJ systems, instead, allow a new kind of stating law or giving justice, in which the individual and subjective emotional and value part, different case by case, is present and often diverging from the one contained, as standard, in the legal texts.

In the representations of jurists on how Artificial Intelligence or even, simply, algorithms, would be inserted in the trial, the image that arose was always that of a replacement of the different actors of the trial, from the judge to the lawyer, with artificial systems able to carry out those mental operations that, until then, were considered peculiar to man. In our case, such representations don't hit the mark, they are misleading. The change can be much more radical. These algorithmic systems do not follow the legal solutions already socially and politically shared, but, instead, they create new ones. These systems do not simulate human action or the human mind artificially; they are not a copy of the human being, whether of the judge, the lawyer, the legal advisor, or the administrator. They find new solutions tailored to the parties, their needs, wants, interests and values.

Here the parties do not delegate to the legislator how to protect their own interests, because they decide, scale, order interests and values, remaining the legislators in their own right. It is immediately clear to the jurist that much discussion is needed here about the admissibility of these systems in Western legal systems. Justice would again become a justice of the individual case, where different orders of interests and values will lead to different legal solutions.

The contemporary state's paternalistic role would crumble, granting a generalised 'age of majority' to its citizens. At the same time, the right, the solution of the case, would be charged with all that emotionality that the parties, in the current procedures, must remove. It is an enrichment and an enhancement of the citizen to the acceptance of the solution.

5. First commonplace: the law is always in the wording, however formulated

A commonplace, perhaps the most deeply rooted, consists in identifying the law with a text, either of law or of previous case law. But these texts are only means of communication, forms in which, in the history of human societies, the legal ought has manifested itself. Other is the law itself, which closes the dispute, every specific controversy, determining new truths in the world of facts with the behaviour of the addressees of those laws and sentences. The ought to be of the law, is a different moment. It is that which communicates to people certain conditions that are desirable to respect in order to be part of what a future possible sentence, or decision, may decide as law. The first is a factual truth, the second is a hypothetical evaluation [4].

The law and the precedents used to decide a new dispute provide conditions for the decision of the judge. The judge's judgment becomes law for the case decided and in respect of everybody, the law is then manifested in the new order of interests that follows the judgment. In the example given above, an erroneous, ideologically distorted judgment or one from a corrupt judge still forms the law until it is reformed by another judgment and succeeds in causing social change. A newspaper article or an authoritative ethical or economic opinion is not enough to 'declassify' the judgment as an opinion. The ruling of the judge remains the formant of law, that which gives shape to law, and, with the good peace of opinion makers, their opinions remain as such.

Considering all the facts and words that compose the law, it is easy to see that EAJ's systems can be introduced at various stages and in very innovative ways. They can be placed, for example, between the texts of law and the judgment, replacing the ought to be of the law and the judge with another ought to be, which comes from the litigants. The algorithm would still allow the decision to be reached in a completely rational way. The emotional, or irrational, part is present in the representation of the parties. It replaces the ought to be of the law as interpreted by the judge, but it represents, much more than the law and the judge do: it

[4] The difference between the two moments is generally recognized in legal literature, but the different schools do not agree on which part to recognize as their object of investigation. We need all the various moments that make up the law, both the factual and the evaluative ones. [Kelsen 1934].

represents the individual sense of the just, of what each subject, unlike any other, considers just and claims for himself.

Current procedural models give the judge a role of conclusion and synthesis of all the different moments, and, in this conclusion, he must use both his rationality and his emotionality. The values and principles present in the legal system are taken into consideration, but the graduation between them and the choice depend on the emotionality of the judge. On the other hand, he has to come to the decision on a rational path, which must be communicated in the judgement. The judge is, therefore, a third party sui generis, partly a normal human being and partly a legislator, as the bearer of personal and ordinamental values. This has been a necessary union in the history of law; the judge has always been an irreplaceable part of the process.

Now, in my opinion, the situation has changed. In EAJ, fair, true and rational come together in a new way for the law. In an EAJ system, the parties quantify or order their values and value judgments on conflicting assets and rights. It is an order that, in contemporary legal systems, is replaced partly by the assessments contained in the legal texts, to which the judge refers in order to arrive at the decision, and partly by the subjectivity and order of values of the judge himself.

The standardised assessment provided by the law has multiple reasons in its favour. It evolved in all human societies because it had a considerable evolutionary advantage: it made the behaviour of other individuals predictable, favouring cooperation between individuals within the group itself. A cooperative social group is a group that is more likely to replicate, or pass on, its cultural traits.

There are examples of standard legal forms that, in the history of the whole mankind, have crossed thousands of different legal systems unscathed. Sometimes, the conqueror in battle or war has seen his or her own legal system, or parts of it, replaced by parts of the won legal system. This is the case, for example, of the legal form of the contract in Roman law, today a universal instrument of private autonomy, which has come down to us almost unchanged, through dominances and empires over centuries of history.

This increased opportunity for cooperation, however, implies the need to ensure predictability in other people's behaviour. The provisions of law and previous case law fulfil this main function (Romeo 2010; Romeo 2012a; Romeo 2012b).

The linguistic form of all contemporary legal systems is grounded on this, but it cannot take the place of the entire legal phenomenon in human societies. Perhaps the most widespread theories on law today are those linked to the methodology of linguistic analysis.

However, this partialisation of scientific research excludes important parts of human judgement that also contribute to the decision, such as, for example, the entire emotional part. This limit is recognized and accepted theoretically, but its inclusion in the scientific analysis makes the results of this questionable. Instead,

EAJ offers a possibility of rational management of individual emotionality and personal value scales.

The introduction of EAJ's new tools should also be considered with regard to the functions performed by the regulatory provisions that would be superseded by the parties' disposition.

The greater predictability of citizens' behaviour is transformed into a greater possibility of restricting their freedom to choose and decide for themselves on their own behaviour or even on the values and interests to be pursued. Not everything can be attributed to the free evaluation of the individual without affecting that possibility of directing individual behaviours typical of legal systems.

Today these limitations to individual freedom are built up in the constitutional and political history of the community, and the different constitutions restrict their extent. For example, the realisation of the principles connected to egalitarian policies involves a series of normative interventions that may also impose limitations on the freedom of the citizen necessary to achieve an equality that otherwise the same citizen would not be pursuing (Machan 2002).

The constitutional values dimension is realised in the ordinary regulation and this fulfilment is also the realisation of a political-legal project, which the constitutions theoretically outline as rightful. This is a characteristic moment of contemporary Western legal systems and must be kept in mind in algorithmic formalisation. Some evaluations have a necessary bias deriving from the constitutional order and must be kept in mind. This is not an absolute barrier to formalisation and quantification, but it requires a preliminary step that can be solved, not with the FD itself, but with tree decision procedures, or other artificial intelligence tools, combined with the EAJ.

Here, however, we're talking about wording, be it laws or sentences. Instead, the thread that links the decision of the judge of a contemporary Western legal system with the peacemaking decision of a group of hunter-gatherers, at the origins of human history, and with EAJ, is the peacemaking and convincing capacity of the decision, not necessarily accompanied by a text, but expressed in all the imaginative ways of human communication: smiles, songs, hugs, representations, dances, cave drawings, output of an algorithm (Goodall 1983; de Waal 1990; Frolik 1999; Guttentag 2009; Romeo 2010). Whatever it may be, whether this ability goes back to social sharing or to subjection to an authority, its only meaning is identified by the elimination of conflicts.

6. Second commonplace: the law is just or must pursue justice

Nor can law be confused with the sense of justice, individual and personal as well as the interests that animate it. My starting point fully accepts that of Hans Kelsen: “The lew as a moral category is tantamount to justice, the expression used

for a social ordering that is absolutely right, that fully achieves its objective by satisfying everyone. The longing for justice, considered psychologically, is the eternal longing of man for happiness, which he cannot find as an individual and therefore seeks in society. Social happiness is called 'justice' […] From the standpoint of rational cognition, there are only interests and thus conflicts of interests, which are resolved by way of an ordering of interests rhat either satisfies the one at the expense of the other, or establishes a balance, a compromise between the opposing interests. That only one ordering of interests has absolute value (which really means, 'is just') cannot be accounted for by way of rational cognition. If there were justice in the sense in which one usually appeals to it when one wants to assert certain interests over others, then the posirive law would be completely superfluous, its existence entireiy incomprehensible. Given an absolutely good social order emerging from nature, reason, or divine will, the activity of the legislator would be as foolish as artificial illumination in the brightest sunlight. The usual objection, however, is that although there is indeed justice, we cannot define it, or, what amounts to the same thing, we cannor define it unequivocally. This objection is a contradiction in terms, masking in typically ideological fashion the all too painful truth: justice qua absolute value is irrational. However indispensable it may be for human will and action, it is not accessible to cognition." (Kelsen 1934, 16-17).

Justice and irrationality accompany the claims of individuals and the choices of individuals are justified as just by the individuals themselves, by others as unjust; however, at the basis of individual behaviour and decisions there is a large irrational component.

The most convincing example is provided by the divisions of family assets in the event of divorce or inheritance. Family relationships are built on love and lack of love. Two strong emotions of human nature which lead to decisions that often conflict with the interests of the individual taking them. They are surrounded by a galaxy of interdependent emotions, feelings and behaviour. Altruistic behaviour, for example, is difficult to explain rationally and the problem of the evolution of an 'altruistic' gene is still one of the greatest challenges in evolutionism, but, in the family environment, altruism is definitely one of the drivers of behaviour (Boehm 1999).

The division of goods within the family is the best example of the emotional and irrational forces that can be triggered. It often marks the end of a relationship, which severs all ties in that emotional galaxy. The individual's behaviour becomes unpredictable and his decisions non-rational. Feelings such as reproach or resentment take over and lead to difficulties in the peaceful resolution or settlement of the dispute. It is not uncommon for one of the litigants to accept a solution, that is disadvantageous to him/her, only in order to harm other litigants. In such cases, what the party intends to obtain is not calculated on the assets obtained by the party itself, but on those obtained by the other parties. Vetoes and restrictions on the

circulation of assets are among the most common examples in such cases. One of the necessary prerequisites of game theory doesn't seem to apply, i.e. the necessary will to win by the players.

But, if we conceive the irrational attitude as part of the game when defining the rewards, then irrationality itself can be redefined as a kind of rationality updated on the subject (s), on its scale of values, an s-rationality. Now, the problem shifts only in the definition of the different s-rationalities for the definition of the payoffs, but this is a problem that has to be analysed with assumptions other than fairness.

I believe that the presence of widespread irrationality should not discourage and lead to think that they are irreducible to EAJ; however it may be, I firmly believe that EAJ systems can help even in these cases. While the positions of the parties can afford any irrationality and the legal decision may not be fair to many, it can gain a rational basis when it also includes the irrationality of the parties, calculating the optimum on this point.

Emotionality about objects is another example of the distance between the value attributed by the law to goods and the subjective value. The affective value is non-existent for the sake of law, but a subject can attribute immense value to things of little importance for his/her own affective reasons. The legal systems generally fail to take adequate account of these individual needs, which, however, contribute to forming in the subject the perception of the just and unjust with respect to the specific act attributing the asset, as well as the approval with respect to the judgment.

That is the reason why many researchers argue that right or justice, tout court, is measured by social happiness, not just social wellbeing (Kelsen 1934, 16). Widespread social wellbeing can be accompanied by widespread social unhappiness; an increase in wellbeing does not imply a necessary increase in the sum of individual satisfaction, of social happiness. Even if we do not know any rigorous demonstration of this, the observation of the growth of radical dissent movements in the richest Western societies leads us to reflect on the differences between social wellbeing and social happiness. The one is measurable, the other only statistically observable and, as irrational, not easily to be discussed. The reduction of the subjective irrational to algorithmic procedures is and will be one of the main crossing points from FD to law. The greatest reflections on the point can be found on the mathematical front, rather than on the legal one (Moulin 2004).

EAJ try to encapsulate individual irrationality without diminishing practical decisive power and theoretical relevance. Individual irrationality establishes the assumptions of the calculation, algorithmic rationality does the rest. After all, even this can be considered a fair division between the two.

There is an oft repeated question in our field, which asks whether it would be preferable to be judged by a judge or by an algorithm. Personally, if I were guilty, I would answer by a judge and, if I were innocent, I would answer by an algo-

rithm. In the former case, I would look for human complicity or empathy, in the latter for rationality; in the first case I'd look for the judge's emotions, in the second I'd look for his rationality while maintaining my values and emotionality, avoiding those of the judge. In civil matters it doesn't change that much.

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Rimantas Simaitis * and Milda Markevičiūtė **

Introducing equitative division algorithms into the legal realm

Introduction

For more than a couple of decades equitative division algorithms are being developed. Many mathematical models have been created. Some prototypes of software where such models are implemented are available as free samples or integrated already into web applications offered for users. Nevertheless, these achievements of mathematics, economy and informatics sciences are barely known to the legal science and practice. It was among CREA's [1] project aims to investigate field and scope of possible use of fair division algorithms, create new type of algorithmic division models that would be more easily adopted by legal practice. In this article we focus on examining of main roadblocks for introduction of CREA and analogous types of algorithms into legal realm and elaborate on methods and means to overcome them. This analysis is a part of a more complex attempt to pave a way for innovation transforming legalistic methods of division of assets, increasing their efficiency and user-satisfaction.

We will start our analysis by comparing aims, criteria and methods that are applied in the legal way and the algorithmic way of division of assets. Analysis is furthered discussing benefits that may be used for advancement of legal methodology and instruments related to division of assets. In the third part we expose types of key issues and their solutions to integrate algorithms-based property division methods into legal field. After this analysis we draw the conclusions underlining main ideas of the analysis.

Both authors have legal background. Therefore, they use legal analysis methodology in this paper.

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[1] For more information please see http://www.crea-project.eu/.

Differences of legalistic and algorithmic division

For purposes of this part of analysis it is sufficient to compare one example of division of assets in one jurisdiction and one type of disputes how this problem would be settled by legal means and in contrast by algorithmic means. Let us take a case of division of assets owned jointly by spouses. We will base our analysis on Lithuanian legislation and legal practice.

Division of such assets in accordance to Lithuanian legislation may be achieved by parties making agreement on the subject though a pre-martial or a post-martial agreement, or by a judge handling a dispute in case of parties' disagreement [2]. Judicial control and a power to decide over the matter in case of disagreement of parties is established as a mandatory rule. Lawyers, mediators, experts and/or other professionals might be invited to assist, but the last word on the final resolution of a problem of division of assets lies within discretion of a judge. We can draw an important conclusion from such a legal setup. A power of qualified independent judge to rule upon a legal problem of division of assets is perceived as a fundamental guarantee safeguarding fairness and legality of this exercise.

From substantial point of view division of assets shall satisfy certain criteria established in the statutory law. Articles 3.116-3.127 of the Civil Code of Lithuania [3] set up a number of relevant criteria and rules. Below we will point out the most significant ones for our purposes of comparison of legal and algorithmic division of property:

1. assets are divided in kind if this is possible not damaging items that have to be divided, taking to account value of items and shares of spouses in joint ownership;
2. value of goods is determined as a market value;
3. if it is not possible to divide in kind, monetary compensation will be established;
4. shares in join ownership are presumed to be equal, but it is possible to deviate from this on the basis of important circumstances, such as interests of minor children, health condition or wealth of one of the spouses or other relevant factors.

These criteria put a legal, logical and economic framework to a discretionary power of judge to rule upon the issue. Nevertheless, some rather significant questions remain unanswered by the law. To what extent parties' preferences have to be taken to account? Is maximisation of utilisation of assets after division by the

[2] Article 3.116 of the Civil Code of Lithuania. Available via Internet: https://www.e-tar.lt/portal/lt/legalAct/TAR.8A39C83848CB.

[3] Available via Internet: https://www.e-tar.lt/portal/lt/legalAct/TAR.8A39C83848CB.

parties is relevant? How to determine market value? Needless to say, such soft questions as emotional side of the distribution is not considered as legally relevant. If parties agree on these additional hard and soft criteria, they may apply them. If not, judge's subjective assessment of formal hard criteria established in the law and of all other criteria which judge finds relevant will determine the final result of division.

Legal style solution of division problem aims at ensuring legality, equality, efficiency and fairness of division within a reasonable time frame and at a reasonable cost. Intervention of a judge with decisive power ensures that the division task will be efficiently accomplished even in situations of stalemate of parties or default of one of them. Authority vested by law in judicial decisions out-weights possible disagreement of any party. In exercising its decisive authority judge has to provide motives for particular result explaining and substantiating it. Obligation to give motives acts both as a tool to convince parties to accept the decision on one hand, and to justify the decision in eyes of society or any outside observer on the other hand.

In case of algorithmic division of assets parties' preferences are the most significant factors. Algorithm designed in the CREA project allows to express such preferences by numeric value of bids in a predefined range or by qualitative value of five stars. Possible range for bids might correlate to approximate marked value of particular items, but do not have to match it exactly. Parties place their bids or stars individually on each idem included in poll of assets for distribution. To increase efficiency, fairness and reduce manipulation as well as envy, values of preferences are assigned confidentially, not disclosing them to the other party before calculation of the final result. Allocation is performed by mathematical calculation and comparison of values of preferences allowing for fair, *Pareto* optimal, envy-free and manipulation-free allocation. [4]

Let us give more examples of fair division algorithms designed earlier. In the end of the last century Steven J. Brams ir Alan D. Taylor created an algorithm called the *Adjusted Winner*. It enables fair division of any number of items between two persons. First of all, goods or issues in a dispute are designated. Then parties indicate how much they value obtaining the different goods or "getting their way" different issues by distributing 100 priority points across them. This information might be or might not be confidential. Priority points are used determine winners of specific assets/issues who gave more priority points. In the adjustment phase transfer of items or fractions of them is made to achieve equitable allocation until points of both parties put on items of parts of them become equal. The order in which items are transferred is determined by certain fractions corresponding to items that the initial winner has and may have to give up. They start

[4] CREA Handbook containing the result of the research. *CREA project materials*, 2019.

adjustment transfer from the item with a smallest fraction and continue until both parties get items corresponding to equal number of points. This algorithm is characterised by authors as ensuring fairness, efficiency and elimination of envy. [5] Software prototype is available online. [6]

B. Knaster and H. Steinhaus developed the *Sealed Bid* procedure, which consists of distributing various quantities of indivisible items among any number of participants based on who offers the highest price for each particular item. The discrepancies between values are mathematically smoothed by monetary compensations. [7]

The *Nash Product Maximizer for Divisible Items* is developed by A. Bogomolnaia, H. Moulin, F. Sandomirskiy, and E. Yankovskaya. It is designed to divide any number of divisible items between any number of participants on the basis of the public price assigned to each product (such as the market price) and the portion of the total budget allocated to the participants in the distribution. The distribution is made to the participants in the process using their assigned budget, indicating the cost of the items or parts they wish to receive, and the allocation is made to each participant based on the maximum *Nash* social welfare function calculated by *convex* programming techniques. This asset allocation ensures result which is *Pareto* optimum, proportionate, envy-free, manipulation safe and *Competitive Equilibrium with Equal Income (CEEI).* [8] There is also the *Nash Product Maximizer for Indivisible Items*. [9]

By contrast to the legalistic approach, algorithmic approach to division of assets has several important distinctive features:

1. Active inclusion and participation of parties to evaluate and express their preferences;
2. Identification of individual utilities of particular asses to the parties;
3. Parties preferences is the most significant factor affecting allocating of items;
4. Significance of ensuring fairness by maximizing utilization of goods after distribution, reduction of envy and safeguarding from manipulation;
5. Application of mathematical transparent methods for allocation, that are based on achievements of mathematics, economy and informatics sciences.

The algorithmic division method revolves around parties' choices, preferences

[5] Adjusted Winner Website – NYU. Avalaible via Internet: http://www.nyu.edu/projects/adjustedwinner/.

[6] Ibid., *Try It* chapter.

[7] DALL'AGLIO, M.; DI CAGNO, D., FRAGNELLI, V. *Report on State of Art f the Game Theory Tools. CREA project materials,* 2018, pp. 16, 17.

[8] Ibidem, p. 26.

[9] Ibidem, p. 26, 27.

and sympathies. It is built around parties and try to satisfy their needs instead of pleasing a judge's. Parties are the ones who control it. Such method is focused on facilitating parties' agreement on solution of the problem. This is quite significant difference between the two analysed approaches, focus may shift from judge-centric to parties-centric when you replace legalistic method by algorithmic.

In current stage of evolution algorithms themselves are not capable enough to provide a finite set of rules and procedure on how to divide assets from the start of solution of related issues to the very end. They can offer only one piece in the larger spectrum of actions that have to be performed by parties and other persons. The legalistic method is more elaborated. It offers full-cycle of comprehensive procedures to resolve division of assets issues from beginning to the end.

As was mentioned in the introduction of this article, fair division methodologies and prototypes are developed for many decades already. By now this movement is still alien to legal realm. Techniques and procedures used on an industrial scale do not include algorithmic division element.

The landscape of practicing law in recent years started to change dramatically. Digitalisation and automation of practicing law became and remains hot topic in field of transformation of legal practices. This tendency is growing. Fair division algorithms represent one type of algorithmic decision-making tools. In this classification it is relevant to note, that algorithms as described above are pre-defined and transparent. On one hand, it means that they are less flexible and adaptable. On the other hand, such features create less issues of ethics and legitimation connected to lack of transparency, explainability and human control.

Potential benefits of algorithmic approach

Use of algorithmic tools for division of property as discussed above are aimed at improvement of satisfaction of parties to a dispute and transforming a division problem into a mathematical task. Among other things, it aims at achievement of more optimal distribution of utilities. On the other hand, such mathematical exercises strive to reduce envy, ensure better efficiency and safeguard from manipulation. These features create better user experience in the process of distribution and higher reliability of the final result. Undoubtedly, such features can be regarded as benefits.

Another meaningful angle to discuss here is combating uncertainty. Algorithmic way of dealing with a problem provides for a higher level of definability, certainty and parties' control. In general, these are the same aims legalistic division of assets pursues as well. Clients of legal system resort to legal means to overcome uncertainty and provide them clear predictable answers to their claims, but in conventional context of distribution of assets certainty quite rarely might be offered. Let us take a closer look at that.

In litigation process parties are expected to combat over proving their rights and different positions in front of a judge. Decisions on the substance of a matter is made by a judge, not by parties themselves. And these decisions do not necessarily match propositions of decisions and offerings brought by the parties. This in reality leads to escalation of tensions, growth and deepening of a dispute. Fighting-mode result is generated depending on multitude of variables. Many of them, such as subjectivity and discretion of a decision maker, does not give up for exact calculations, predictions and control by parties as ultimate beneficiaries.

In conventional negotiation process which is positional bargaining parties play a game of "selling" discounts. Some commentators call this as a manipulation or "dancing" over discounts. Uncertainty, unpredictability and distrust normally accompany such interaction. Mediation processes and interest-based negotiations try to mitigate these negative aspects by amplifying positive emotions and focusing on true interests and needs of the parties. Providing parties more control and understanding about their dispute create higher level of certainty. This at the same time relieves tensions and empower settlements.

Distribution of assets via predefined algorithmic tools such as designed in CREA project presuppose users' agreement to apply mathematically exact rules. By enabling this algorithmic way of solution of a problem it has *a priori* higher level of certainty. Parties actively participate placing bids or preferences so triggering application of formulas for calculation of a result. They are more involved in shaping final decisions. Their choices directly affect the result. These links can be traced down and exposed whenever needed.

Disputes' avoidance and de-escalation are other positive features that deployment of the analysed tools may bring. When parties' interaction for solving of a problem is channelled to application of exact rules and procedures, this by itself close a gate for a conflict to grow. Establishing clear rules, procedures, introduction of institutions, active involvement and common interaction are classical measures applied for prevention of conflicts. Direction of parties' energy towards using procedures and user-friendly engaging tools of constructive resolution of a problem may solve a problem in the very initial stage of its evolution. One the other hand, the same effects can heal an already existent conflict.

Finally, cost-saving and time-saving effects can also be achieved. Properly selected and balanced algorithmic tools may offer simple and fast procedure to resolve complicated issues. If successful, parties will spend less time and money. "More for less" and "time matters" are significant pressure factors on all providers of services in a modern XXI century context. In this environment introduction of algorithmic means would be very timely to answer that pressure.

Challenges and risks

There are many roadblocks in bringing any innovation into life. They might be of economical, technical, psychological and legal nature. We can distinguish three groups of main challenges that stand on a way of introduction of algorithmic division of property into the legal field:

1. Technical;
2. Legitimation;
3. Integration.

The first set of challenges - technical issues. In our perception as law experts this group encompass defining the field of use of using algorithms (a "playground") for fair division, designing suitable and efficient algorithms, their adaptation, solving issues with automation, creating software, funding of these R&D activities and maintenance of new tools.

As was elaborated in the first part, methodologies and criteria significantly differ between legal and algorithmic division tools. Therefore, one of the main tasks is identification of a scope in which algorithmic division would be valid and meaningful. Issue on definition of a "playground" for algorithmisation as a legally adding value means may be elaborated in two directions.

The first one – application of algorithms in the field where parties have wide discretion for an agreement and where they may be interested to opt in for using algorithmic tools. In private law sphere such field is quite wide. In the example analysed above about division of joint property owned by spouses mandatory, rules normally are switched on only in situations of parties' disagreement. In this context algorithmic tools would be valid as a smart assistance technology in negotiations, mediation and conciliation. *Ex post* adjustments of results of algorithmic division would be necessary only if parties would be dissatisfied with them, if they would create results deviating from previous parties' agreements (e.g. pre-marital or post-marital) or if such results will contradict to a few mandatory rules limiting parties' free discretion.

Another direction – possible integration of algorithmic methodologies as assistance tools for judges and parties in litigation. This task is more complicated because of the existent tight legal framework. Algorithms are still obscure to laws on division of assets. Both procedural rules and substantial rules do not prescribe them.

Legal analysis performed in the context of the CREA project uncovered that there are significant differences in legal regimes among European Union countries of distribution of property even in quite narrow fields, such as division of property owned by spouses and division of inherited assets. On top of that, regulatory regime prescribing set of default rules to follow might be modified *a priori* by such instruments as pre-marital and post-marital contracts. This creates more

complex legal framework for deployment of any automation methodologies and technologies. Results generated by the CREA algorithms have a much higher chance to deviate from this complex of mandatory statutory law and contractual special rules. Thus, in very restrictive substantial law framework applicability of algorithms such as CREA's tools or analogous in their pure current state would be quite limited. *Ex post* adjustments performed by a judge to the algorithmic allocation results seem to be unavoidable. What methodology should be applied for these adjustments, to what extent they could fix problems or create more of them is still untested and unclear.

Another option of using fair division algorithms in courts – to treat results of algorithmic allocation as a part of relevant material among multitude of other facts that may be considered and assessed by the judge making his/her own discretionary decision. In this case legal framework and existent patterns of courts' work would require less modifications. Piloting may be performed without any legal modifications. At an initial integration stage, it seems more appropriate and easier to handle.

Designing, testing and adaptation of fair division algorithms for many years was a non-legal task. In a context of the CREA project the CREA team tried to consider this issue as an interdisciplinary one, including legal dimension. In course of the project decision was made that in order for the algorithms not to lose their positive implications they should not be squeezed into a narrow framework of rules for legalistic division. More prospective vector was to improve existent algorithms following the same pattern of their design based on game theory and behavioural economy rules. CREA algorithms were intended to be more user friendly and flexibly applied in variety of contexts. Therefore, they can be used for division of property among two or more persons/agents and can cope with division of both divisible and indivisible items. Such methods of division of goods might fit into the legal playground as elaborated above in this chapter.

Needless to say, designing of workable automation patterns, creation, upgrading and maintenance of the software, R&D and piloting activities require substantial funding. Recent attempts to launch commercially sustainable IT products were not very successful and, to the best of knowledge of the project team, they do not extend to Europe. It would seem more promising that at least part of the total cost comes from public funding, allocated for designing and upgrading of e-justice and ODR platforms.

Another challenging dimension is legitimation. Dispute resolution in courts is a domain where public law rules apply. One of the fundamental principles here stipulated that allowed are only actions that are permitted by law. As a consequence direct prescription is necessary in procedural and substantial laws of all of the procedures, tools and rules that may be used for dispute resolution.

In substantial law it would be useful to establish that preferences of parties and better/maximised utilisation of items after distribution should be added to a list of

criteria for allocation of property. Such modification would not be very significant on a large scale, but it would be sufficient to open a gate to introduce tools and methodologies to extract this information on a scientifically reliable basis. Further transformation of laws might come as automation and modernisation of this field will develop. It would be difficult to speculate what changes would be reasonable before thoroughly testing and piloting algorithmic property division tools.

From a procedural point of view judges in conventional civil procedure setup do not have right and power to invite or obligate parties to use algorithmic division tools. Unless in pilot projects courts and individual judges would lack sufficient procedural legal ground to introduce use of any algorithms in their cases. Rules and procedures shall be created enabling judges to invite or obligate parties to express their preferences by using special tools such as CREA or analogous tools.

In negotiation and mediation processes techniques and tools might be used more freely and flexibly. On the other hand, in order to avoid any doubts or speculations about possible manipulations or any other flaws in using game theory-based methodologies and tools it would be purposeful to allow their use in special laws or bylaws.

Risks connected to potential breaches of fundamental human rights shall be considered and dealt with. Automated decision-making systems has to be transparent, explainable, safe from manipulation and discrimination. One of the ethical principle for use of AI in judicial field, recently promulgated by the bodies of the Council of Europe, is "under human control" [10]. This principle by analogy may be extended to non-AI decision making systems and methodologies as well. Fundamental rationale behind "under human control" principle is equally valid for all automatic decision-making tools.

In a long run accreditation schemes on certifying of algorithmic tools for use in official proceedings may be useful. At current stage when only a small handful of prototypes exist and their use in practice come as an experiment, such accreditation would create unnecessary burden. But if these practices will grow in variety and numbers, accreditation may ensure reliability and trust.

The third group of challenges is connected to issues of integration into ways how legal procedures of division of assets are organised and managed. Automation brings disruption of existent *modus operandi* of legal professionals. It inevitably faces doubts or resistance in critical cases. These problems are not easy to deal with. Change of attitude and transformation of patterns of professional activities into new ones requires creativity and time.

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[10] CEPEJ European Ethical Charter on the use of artificial intelligence (AI) in judicial systems and their environment. 3-4 December, 2018. Available via Internet: https://rm.coe.int/ethical-charter-en-for-publication-4-december-2018/16808f699c.

Creating demand and awareness for new technologies, as well as promoting of benefits cannot be concentrated on users only. True gatekeepers for transformation of ways how law works are judges and lawyers. They should be treated as equal targets for modernising ways how their business is organised and operate. If algorithmic tools will be integrated into daily activities as positive novelties in comprehension of legal professionals, these novelties can claim to take root. On the other hand, tools and methodologies extinguishing participation of lawyers and judges, taking down their control over matters traditionally entrusted and valuable to them have many risks and chances to be blocked.

Training, education and piloting with an aim to create and transpose new models of work comfortable for judges and lawyers shall accompany introduction of algorithms into legal field. On top of education various incentives for users and legal professionals to start and continue using algorithmic tools might be necessary to ensure success.

Efficiency of such strategy to apply complex measures of incentivizing, raising awareness, training, creating motivation targeted on the first hand at legal professionals, creating their new modern work patterns has not only theoretical roots. It is also confirmed by a real success' stories. Let us briefly discuss one of them. In the middle of 2013 Lithuanian courts launched an e-filing service connected to the Lithuanian e-courts IT system LITEKO. No obligation was imposed on any private litigant or their legal representatives to file documents digitally. Instead of that, start of the system was marked by introducing a statutory right to submit any procedural document to court in civil and administrative proceedings digitally through this new at that time service of LITEKO system, a statutory right to get documents from courts digitally through access to e-files kept in the LITEKO system and corresponding obligation of courts to digitalise all procedural documents received in courts in civil cases within three working days.[11] Litigants were incentivized to file documents digitally by introducing 25 percent discount on state filling fees in case of pure digital filling. At the same time obligation to receive service of procedural documents from courts was introduced to "professional" litigants - lawyers, notaries, bailiffs, financial institutions and insurance companies. In designing, tuning and piloting of the new e-services judges and lawyers were heavily involved. Launch of the system was preceded and followed by massive training and awareness raising campaign targeted at all judges, court staff, lawyers, notaries and other "professional" litigants. This led to success rates designers and administrators of the system had only in their brave imagination. Number of civil cases filed and handled only digitally reached 33,91 percent in 2014, 50,35 percent in 2015, 65,59 percent in 2016, and stabilized at figures over 72

[11] Article 37[1] of the Law of courts of the Republic of Lithuania. Available via Internet: https://e-seimas.lrs.lt/portal/legalAct/lt/TAD/TAIS.5825/asr.

percent in 2017 and the following years. [12] Initial resistance and doubts mainly expressed by the same target group in quite short period of couple of years shifted radically to high support to the e-filling and digital files. Analysed e-services of LITEKO swiftly became recognized not only as formal statutory rights, but as indispensable tools in everyday work of every Lithuanian lawyer acting in courts.

Conclusion

Bringing equitable division algorithmic tools into legal realm might invoke significant benefits for users. It can ensure more optimal distribution of utilities, better satisfaction of parties' interests and needs, reduction of envy, safeguard from manipulation. On top of that, allocation of assets based on scientific methods and last achievements of economics and mathematics can reduce uncertainty, increase reliability, save costs, time and prevent from escalating of conflicts. Nowadays all service providers are highly concerned with improvement of user experience. Introduction of algorithms into division of property can offer this.

However, recognition of such benefits for one group of payers active in a field – users/parties – is not sufficient to make this transformation happen and bring all the benefits to life. There are number of multifaceted roadblocks to deal with on a path to innovation of legal practice. Besides challenges of pure technical and legal nature there are issues of attitude, inert *modus operandi*, and possible resistance of legal professionals to any disrupting of their patterns of work.

Creative and sensitive style of dealing with these issues can pave a way for transformation of legal reality. Authors suggest that legal professionals should be actively included into testing, piloting and bringing to life of new tools that will be transforming their activities in the future. Judges, lawyers and other legal professional are strong gatekeepers for any innovation in dispute resolution field. Providing opportunities to adapt methods of how judges, lawyers and other legal professionals create value and earn for living is essential element for success. It would not be wise to introduce new algorithmic tools as job-takers. Such initiatives have high chance to end up with nothing or lead to a long-lasting distress which would be even more embarrassing. Enabling legal professionals to maintain in the loop, perform more efficiently with help of modern technologies would create right environment for fair division algorithms to root into legal reality.

[12] LITHUANIAL NATIONAL COURTS ADMINISTRATION. Annual reports of performance of Lithuanian courts of 2014-2018. Available via Internet: https://www.teismai.lt/lt/visuomenei-ir-ziniasklaidai/statistika/4641.

Nikos Stylianidis

Use of Algorithms in Dispute Resolution: Assumptions and Methodological Comments

1. Introduction

Use of artificial intelligence (hereafter AI) (in the form of algorithms or other) in legal processes is gradually acquiring an overriding significance; in particular, use of AI techniques in order to resolve disputes between parties (a task traditionally and strictly reserved to judicial organs only) and not only as a tool, e.g., for legal research, affects our deepest convictions and long-standing practices about what the law is and how it is applied; and it is certain that for a wide part of legal theorists (especially between the ones that insist on the close connection between evaluative or moral principles and the law) and practitioners this sounds somehow shocking and worrying; in the present, and with particular reference to the "CREA project", we will try to "de-mystify" such processes and alleviate such worries, drawing from theories and practices of this very same legal tradition: in our view (and though we initially share the same worries), use of such techniques, with appropriate caveats, can be fruitfully accommodated within legal theory and philosophy, while providing an important assistance to the implementation of law's operational framework.

2. Preliminary remarks: Artificial Intelligence (AI) and law

2.1. The expression of concern and worries on the potential application of AI in resolving legal disputes, tacitly presupposes that the application of law cannot be "automatic" or mechanical; it rests, i.e., on the assumption that legal reasoning is part of practical reasoning in general; and that this latter, as expressing a fundamental reflective or quasi-reflective relation between thought and (human) action cannot be reduced to a mechanical, automated process: human affairs, the domain of application of practical reason, "could always be otherwise", and the changing circumstances and particularities of every decision do not allow their

standardization and, a fortiori, formalization in mathematical terms and models: usually, when it rains, we go out with an umbrella or wearing a raincoat; this is the "rational" thing to do, also according to the relevant predominant practice; but, on the other hand, for any reason (equally rational), maybe because I love the purity of the rain or because I want to be "singing in the rain", I decide to go out with no umbrella or raincoat; in a similar way, relevant-to-law actions present individualized, particular characteristics; this is particular acute in penal law cases; but even in more simple cases, e.g. while sharing goods in the process of a divorce, one party may value, e.g., a book or a record (of a petty market value) more than a luxurious sport car; and, accordingly, the judge or the mediator has to understand these choices and particularities in order to reach a relevant decision and resolve the dispute. In general, as law has to do with human action and practical reason, it inevitably necessitates the understanding of human action from an internal to the actor, **"hermeneutic point of view"** [1]; obviously, the first, prima facie objection, to the application of AI in resolving legal disputes stems from the view that human action cannot be "understood" (and, a fortiori, "empathized') by a machine and practical reason cannot be classified with the use of mathematical models.

2.2. A second, prima facie objection, is related to the so called "**defeasible character**" of legal rules and concepts (also connected to their "open texture") [2]: legal concepts cannot be analyzed and defined in terms of necessary and sufficient conditions of their application and the word "etcetera" is indispensable in "any explanation or definition of them" [3]; equally necessary in this effort, is the word

[1] Cf, totally indicatively, Hart (1961), Mac Cormick (1978) and (1981), Ricoeur (1977), Winch (1958).

[2] I would like to thank Andreas Takis, Assistant Professor of Philosophy of Law in The University of Thesssaloniki, for pointing out this contradiction between the defeasibility of legal rules and the binary logic used by computers. Indeterminacy of meaning as related to open texture of relevant linguistic rules is denoted by various concepts: "cluster concepts", e.g., are concepts that cannot be smoothly applied even in ordinary situations; their sense (if equivalent to their applicability conditions) cannot be analyzed in terms of necessary and sufficient conditions - for such an analysis of law as a cluster concept, see Sartorius in Gavison (1987); this vagueness is, strictly speaking, to be distinguished from the "open texture" of a concept, that denotes the possibility of the presence of a doubt concerning its application on novel situations (i.e. the potential vagueness of words), not yet present (hypothetical) and maybe extreme but foreseeable or imaginable; despite that difference the presence of open texture has the same result, namely, it does not allow for an analysis in terms of necessary and sufficient conditions – for the notion of open texture and the related notion of "porosity" of a concept, see Waissman (1945).

[3] See Hart (1949), p. 174. H.L.A. Hart seems to use the term of "open texture" – cf. Hart (1961) – especially pp. 120-132 – in order to denote both vagueness and open texture (Moore 1981); see also Bix (1991) for an original approach to the relation between open texture and judicial discretion in Hart's work; nevertheless, it seems that there are two sources of vagueness to which Hart refers

"unless": though the "positive conditions" for the existence of a contract, e.g., are an offer, acceptance and consideration, a contract may still be void or voidable due to some not known "defence" (or "*exception*") such as fraud or incapacity; and a legal decision that declares the validity of a contract is subject to these "defences", as legal rules are subject to being defeated in particular circumstances or under particular conditions. This "defeasibility"[4] of legal rules and concepts, is prima facie incompatible with the **binary logic** predominantly used in AI; from another point of view, law is not identified to a "set of rules" but, more importantly and fundamentally, to a coherent unity of principles; and principles, contrary to rules, do not function in an "all-or-nothing", but in a "more-or-less" fashion[5], which is, again, distinct from computer's binary logic; and, though recent developments in AI enables computers to apply "quantum computing", expressing outcomes statistically and not in a purely binary form, still statistical reasoning is mathematically construed and distant from practical reasoning and elaboration of complex interpretative judgments.

2.3. A fortiori, computers lack the distinctive human skill of accumulating experiences (and not only "raw data"), of combining various elements of different nature in the process of deliberation (practical, sentimental, utilitarian, evaluative, political etc.); paradigmatically through "age" and "maturity", living in interaction with others develops a certain sense of a proper way of understanding and

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without discrimination: first, an "intensional vagueness" (close to the notion of open texture, with the difference just noted) due to the inherent ambiguity of any criterion that could be used for the determination of the sense of a concept ("intensional ambiguity"); second, an "extensional vagueness", an absence of clarity due to the (extensional) ambiguity of the denoted objects; this second vagueness is tied to the fact that "fact is richer than dictum", according to J.L. Austin's expression: language is incapable of apprehending the totality of factual situations (in principle, infinite) whether they are present or future. The concept of "family ressemblance" (cf. Wittgenstein 1988, par. 65-71) at least shares with the "cluster concept" the idea that it is impossible to assign essential properties to a certain concept; nonetheless, Wittgenstein seems to indicate that every concept is a concept of "family ressemblance" (Moore 1981) or leaves the question open (Baker 1980), while only certain concepts are "cluster"; Hart uses the similar concept of "defeasibility" thus denoting the impossibility of an analysis of legal concepts in terms of necessary and sufficient conditions (as someone cannot exclude the possibility that conditions that "defeat" the relevant application of the concept do arise) – see, also, Baker (1977); Hart's difficulty in demonstrating the irreducibility of legal concepts to empirical ones via their supposed particular, "defeasible" character probably stems from this Wittgensteinian obscurity: if every concept is "family ressemblant" or "defeasible", then no concept is particularly intelligible as such (according to Wittgenstein's principle of significant negation) and, consequently, this characteristic cannot demonstrate their particularity; along these lines, we could analogously distinguish between two sorts of defeasibility (intensional and extensional) – see also Stylianidis (1994), especially pp. 379 ff.

[4] See also Baker (1977). From a logical point of view, the problem seems identical to the more general problem of induction.

[5] See Dworkin (1978).

acting, which is not identical to formal knowledge, but includes "practical" elements as well: an illuminating sense of "what is right", connected to the respective "form of life", which is not formal yet neither totally intuitive and, certainly not irrational; it is characterized by a certain **"reasonableness"** (a typical example of which is interpretation of dicta and appraisal of actions in the legal and judicial domain) [6], by a certain deeper sense of understanding of "how things are" that transcends formal logic and is guided by reflection on accumulated experiences; this particular ability of taking into account various elements in order to form a "wise" judgment through the Aristotelian "phronesis": practical (in general) and, more particularly, legal decisions do not solely rest on formal criteria, but on multifaceted considerations and interpretations that would not mechanically subsume a particular case to a legal rule, even if relevant formal conditions are satisfied, also in order to protect higher-order rights or the so called "common interest": besides penal law (where the complex estimation of the accused personality is vitally important), this is often the case in other fields of law, as, e.g., public law, where a "contra legem" interpretation has to be advanced, in order to best serve crucial public interests. This largo sensu comprehensive faculty, probably grasped by the Aristotelian term of «σοφία», allows for the formulation of considered judgments or assertions that transcend the computer's formal – even cognitive – capacities.

2.4. In this line of thought, we should distinguish between two separate questions, often conflated in the relevant discussion:

a) the factual ("is") question: can computers think (act, decide) like humans?
b) the normative ("ought") question: should computers think (act, decide) like humans?

On the first question, we should succinctly note that, at least for the moment (and for the mid-term future), AI cannot reach the level of human reflection: computers are still dependent on the information provided to them (by humans), do not recognize existence and external objects as such (as particular, individualized objects), lack self- awareness or self-recognition [7], do not experience "consciousness" [8], cannot use their intelligence for an infinite, non pre-determined number of issues or problems and the exact way in which they reach a certain conclusion is yet unknown. In more technical terms, it seems that current status of AI is still at the level of "artificial narrow intelligence" [9] and that computers do

[6] On the relation between "rational" and "reasonable" see, indicatively, Aarnio (1987), Perelman (1979).

[7] The "know thyself" («γνώθι σαυτόν») dictum - maxim (presumably) of Socrates.

[8] See, e.g., Wikipedia, Artificial General Intelligence (AGI).

[9] Kaplan and Haenlein (2019) structure artificial intelligence along three evolutionary stages: 1) artificial narrow intelligence – applying AI only to specific tasks (ANI); 2) artificial general intelli-

not have the faculty of "understanding" (in Kantian terms) or of "high level perception" (in contemporary terms) [10].

Given that the answer to the first, factual question is, at least according to the current state of affairs, negative, further examination of the second (and, possibly, even more crucial and debatable) question, seems quasi-redundant; in the legal domain, a fortiori, assigning a decisive (and, not, simply, an assisting) role to computers in deciding cases faces even more acute, substantive and procedural obstacles and constraints, at least within the current legal and institutional framework of the European Union [11]; at least for the moment, computerized techniques are used in order to help, and not replace, judges or lawyers [12].

2.5. Computers in the (future) stage of AI denoted by the term "artificial general intelligence" could appear as candidates for occupying, apart from humans, an overall cognitive - epistemic standpoint, i.e. the (transcendental or empirical) standpoint from which knowledge is possible; nevertheless, and independently of whether computers could ever acquire the necessary reflective capacity and understanding just described and of whether they could be seriously denoted as "intentional" "beings" (something which is extremely problematic) [13], any such computer would still be *the creation of a human being/mind, a human artifact*; consequently and fundamentally, from a purely abstract point of view, the ultimate cognitive subject would still be human.

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gence – applying AI to several areas and able to autonomously solve problems they were never even designed for (AGI); and 3) artificial super intelligence – applying AI to any area capable of scientific creativity, social skills, and general wisdom (ASI).

[10] See Chalmers et al. (2006), p.5: "Corresponding roughly to Kant's faculty of Sensibility, we have low-level perception, which involves the early processing of information from the various sensory modalities. High-level perception, on the other hand, involves taking a more global view of this information, *extracting meaning* from the raw material by accessing concepts, and making sense of situations at a conceptual level. This ranges from the recognition of objects to the grasping of abstract relations, and on to understanding entire situations as coherent wholes".

[11] See relevant contributions of distinguished colleagues in this collection of papers.

[12] It seems, though, that in some states of the U.S.A., computers do defer "decisions" on prima-facie "typical", minor offences, also based on data and record of the offender, with a considerable "success" (estimated at 70% of the cases); the issue is vital and obviously involves crucial questions of eventual violation of fundamental civic and human rights, on the basis of utilitarian considerations and calculations; adequate discussion of this extremely important, multi-level issue exceeds the scope of the present.

[13] As, among others, P.M.S. Hacker has rightly pointed out (Athens' lecture, 2014); things could be different in case computers were capable of autonomous reproduction; even this, from the point of view of the present, would not prima facie signify a crucial, significant difference; in any case, matters of artificial intelligence and relevant recent developments cannot be adequately explored within the limits of the present; probably R. Dreyfus' (1972) is also still pertinent.

3. Project

In view of the previous remarks (of a more general scope), it is necessary to concisely examine the main relevant, more particular features of the CREA project:

3.1. The CREA project aims at facilitating the reaching of an agreement between two parties through the use of algorithms, in certain, limited areas of law's intervention, i.e., for the moment, in civil law and, in particular, on issues related to divorce and inheritance; *so, it is not an overall proponent of wholesale application of AI in dispute resolution in every field of law*.

3.2. The algorithm under elaboration mainly tries to assist in the resolution of disputes arising in the process of distribution of goods in divorce and inheritance; based on models known from economic theory (such as Pareto optimality models), *it respects the volitions and rights of the parties, while, at the same time, being realistic*: parties do attach values on the goods to be shared according to their personal preferences, but these "personal" values have to somehow conform with current market values; this, quasi objective measure of these (otherwise "mythical") [14] volitions, allows for their de-mystification.

3.3. Most importantly, the model *fully respects the existing legal framework in force*: aspiring to its application all over the European Union, the project carefully examines, in its first phase, the set of relevant mandatory rules and respective requirements in these fields in the legal systems of a variety of European countries: the model will be a complementary tool within these common frameworks; it does not purport to ignore or modify them, but only to assist in their more quick and efficient application, according to the existing procedural and substantive rules.

Consequently, the CREA project advances the application of a **limited purpose-specific algorithm**; it is a technique of ad hoc application of mathematical logic to facts within a given framework of thought and rules [15] (and not, properly speaking, a creation of Artificial Intelligence); it assists judges, parties and other actors in the dispute resolution process, does not replace them; it does not "understand" the parties' intentions, but only helps then in clearly expressing them; it does not issue a rational or reasonable judgment by itself; but it contributes in its quick, non-ambiguous, fair elaboration.

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[14] In A.J. Ayer terms, cf. Ayer (1973).

[15] And it does not seem to have the ambition of "machine learning".

4. Theoretical /Methodological Assumptions and Affinities

In this context, it is further useful to concisely but more closely examine some of the project's implicit or explicit theoretical and methodological affinities with relevant traditions in legal and social theory.

4.1. Legal realism: law in action

An obvious source of inspiration and theoretical basis of the CREA project is legal realism; especially in its U.S.A. version, American Legal Realism [16] identifies the "core" of the legal process with the outcome of legal decisions and legal theory with "predictions" about judge's behavior in particular cases [17]. This view is backed up by:

a) Emphasis on the indeterminacy and "open texture of rules" [18] that does not allow rules to provide adequate, binding guidance for their smooth, definite application; rules are, in extremis, assimilated to "pretty playthings" [19] in the hands of judges that, contrary to the prevailing formalist doctrine, actually decide cases according to their personal preferences, idea of fairness or, even, feelings of the moment [20].
b) Adoption of a form of behaviorism: human action is assimilated to behavior, to complex socio-psychological facts that can be observed and asserted from an external point of view; accordingly, it is possible to predict behavior of officials, judges, parties etc. involved in dispute resolution.
c) Adoption of a type of "pragmatism" that values rather the efficiency of legal operations than law's coherence (which is, in any case, unattainable in view of the open texture of rules) or the protection of rights and conformity of decisions to moral/evaluative standards. [21]

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[16] "American Legal Realism" is used in order to denote a very wide variety of views and motivations, ranging from Karl Llewellyn to J. Frank; in the 1970's, the Critical Legal Studies Movement partly continued this tradition, mainly by equally severely criticizing legal formalism. See, indicatively Twining (1973), Unger (1983).

[17] Cf. the famous dictum of Oliver Holmes: "The prophecies of what the courts will do in fact, and nothing more pretentious, are what I mean by the law". American Legal Realism mainly focused on the judicial practice, while the so-called Scandinavian Realism systematically exposed an "ontology of law", roughly assimilating law to complex socio-psychological facts. See Olivercrona (1939), Ross (1958).

[18] See supra, footnote 3.

[19] In Llewellyn's expression – see Twining (1973).

[20] The so-called "breakfast theory of law".

[21] A non-cognitivism in ethics/values is thus presupposed.

The above theses have been (often justly) criticized along the following lines:

Independently of the importance of the judiciary for the overall operation of the legal system, any such system would still be in need of a Rule of Recognition that would recognize and identify judges and officials as such (the so-called 'realist paradox") [22]; besides, predictive theories of law can, maybe, explain the outcome of ad hoc disputes, but cannot account for the continuity of the legal order. In addition:

a) Despite their open texture, facts and history show that rules have a core of meaning that cannot be reasonably questioned in practice, allowing them to provide definite solutions in the vast majority of (de facto) "easy" cases; the judge or mediator of the dispute "strikes on its own" (but, again, not arbitrarily but based on general principles and values embodied in the legal system and recognized as part of this system) only in "hard", "penumbra" cases.
b) Identification of law to feelings of compulsion or socio-psychological facts would ignore the distinctive legal normativity, grasped from an internal to the actor/speaker point of view and linguistically evidenced in the existence and difference in meaning between the expressions "I am obliged" and "I have an obligation"; normative legal statements (as the latter) cannot be reduced to empirical statements about law, i.e. to statements that would constitute "prophecies" about the future behavior of legal authorities.
c) Priority in the efficiency of legal operations is itself an evaluative judgment; further, efficiency presupposes the principled unity of the legal order and is rather assisted than threatened by the coherence of the legal regulative structure [23].

Notwithstanding the plausibility of such critiques, legal realism as a descriptive (and not normative) theory of law, often accurately describes legal practice; it is true, e.g., that from a practitioner's point of view [24] what is crucial is the "success in the courtroom", the issuing of a decision that would be favorable to her (his) client's interests; and it is also true that judges often decide a case according to their general idea of what the law is, in conformity to their feeling of justice and fairness, political preferences or even out of sympathy for the one or the other party; and it is only ex-post that they subsume their (already formed decision) to the regulative era of a certain valid legal rule; "pragmatic" as it may be, this picture is often confirmed by the methodological flaws in the reasoning and justification of legal decisions, so

[22] In Hart's version of positivism, rules are legal because they are established, "enacted" according to criteria provided by an ultimate rule which is (meta) legal and social at the same time: this rule allows for the identification, the recognition of the legal realm that is thus rooted in social (and not natural or moral) facts alone; that is why it is called "Rule of Recognition" (R of R).

[23] For these critiques see Hart (1961), Dworkin (1986).

[24] And it is not accidental that many of the legal realists were practicing lawyers.

familiar to legal practitioners; from this point of view, legal realism is a honest, "down to earth" theory, that reveals "brute" legal reality, often obscured and hidden behind the veil of formalism; it rightly draws the attention of legal theory to this pragmatic, often chaotic reality of legal practice, which is quite distanced from the "heaven of concepts" of traditional perceptions of law; it thus provides useful tools for the understanding and systematization of law in action.

Further, legal realism reminds us that the law is a practical tool of smooth societal life and interaction, it is a mechanism of dispute resolution, of performing what Llewellyn named "law-jobs" in order to regulate societal life; law in context, in the vast majority of cases, is not concerned with abstract problems of justice, duty or morality, nor with complex interpretative arguments; it is a significant part of a social regulative network that tries to accommodate opposed interests, resolve contradictions and provide solutions that crucially affect everyday life and status of citizens [25]. In executing these tasks, dispute resolution mechanisms (stricto sensu judicial or other) have to be fair, equitable, quick and efficient; while the the art of legal argumentation is long, life is short – "ars longa, vita brevis".

The CREA project shares exactly these very concerns: it purports to facilitate dispute resolution, assist the parties in finding non-biased, fair and mutually accepted, viable solutions to their practical problems, to avoid long, time and money-consuming processes; simultaneously, it innovatively introduces new computing techniques in order to serve these practical legal functions in action.

4.2. Algorithms and Preferences in practical reason

The introduction of algorithms in the process of practical reasoning and decision making is related to another, long-standing debate in the domain of the theory of action: as also analyzed supra, human action is distinctively normative and prima facie irreducible to mathematical calculations; there is always a "normative gap" between motivation, intention and action; humans are not like robots, they are driven by a variety of motives, sentiments, evaluations that are unique and impossible to predict; preferences cannot acquire a definite mathematical value and any such preference scale ignores the richness and normativity of human agency, being, by definition, imperfect and methodologically erroneous [26].

However, models of preferences have long been widely elaborated (at least since the beginnings of the 20th century) and used (with relative practical success), especially in economic theory; in this classical view, "**preference** is the order that a person (an agent) gives to alternatives based on their relative utility, a process

[25] See, e.g., the establishment of an almost immediate eviction process for tenants in the U.S., with the assistance of relevant data bases and computerized records.

[26] On this distinctive character of human agency see, indicatively, Taylor (1985).

which results in an optimal "*choice*" (whether real or theoretical). Instead of the prices of goods, personal income, or availability of goods, the character of the preferences is determined purely by a person's tastes. However, persons are still expected to act in their best (that is, rational) interest" [27].

Despite the prima facie plausibility of such critiques, in the modern technological era, we all use (consciously or not) such elementary preference "scales", e,g, when bidding in Amazon or e-bay for a book or good; such scales, in spite of their theoretical deficiencies, do have an undeniable practical usefulness; and it is practice and technology, the developing necessities of real life that impose their widespread acceptance and use; besides, the CREA model is very careful in being applied in the repartition of quantifiable goods, where the process of "quantification of preferences" is relatively safe; in this line of thought, a further categorization of goods to be divided within the CREA model could be elaborated, as well as relevant "filters" and a method of determining a "valid" range of preferences that, while respecting the parties' needs and wants, could "objectify" their criteria, also in order to exclude "malicious" choices impeding agreement [28]. In any case, the smooth function of the CREA algorithm presupposes that both parties share the intention and sincere volition of finding a mutually acceptable, beneficial solution; and that they do not intend to impede or block the agreement process.

4.3. Models of Rational Utility

As mentioned in the above definition, scales of preferences are related to and presuppose that the parties involved are capable of rational choices, based on utilitarian calculations; obviously, creators of such models are aware of their inherent limitations: by default, models are perfect, while reality (especially social, non-natural reality) is imperfect; no human being, e.g., disposes of a flawless rationality, and parties lack the perfect equal amount of information required for the absolute success of such "optimal" **models**; nevertheless, these defects do not make them useless; on the contrary, even deeply normative theories on the creation of the social bond of Kantian inspiration, as, e.g. and famously, Rawls' "Theory of Justice", do apply "models" in order to establish the principles that should govern social structure and are presupposed by its current status; Rawls' famous original position is such an abstraction and, in this line of thought, all social contract theories constitute largo sensu "models".

Similarly, the perfect **rationality** of the involved individuals – parties which is crucial for the function of such models should best be considered as an optimal (and quasi-evaluative) standard of behavior, as a sort of "regulative idea" of proper so-

[27] See Wikipedia, "Preferences" (with further references).

[28] In the line of thought already incorporated in the CREA model and reified in the important role of the "market value" of goods in the negotiation process.

cial action, presupposed by and necessary for the assessment of actual action in real life; it is interesting to note that, while probably oscillating between a purely transcendental and an empirical foundation of his major argument, Rawls himself has initially characterized his decision-making model as a "model of rational choice"[29].

This rational choice is presumably based on the relative **utility** of goods and related alternatives; even against the prima facie most plausible view of "rule-utilitarianism"[30], numerous critical arguments have been raised: even if we assume that "pain and pleasure" (according to the classical, Bentham's utilitarian principle) could be assessed by using complex, even psychoanalytically informed, structured criteria, it is always an open question whether these are "quantifiable", measurable entities and whether the "good" (either individual or collective) could be "measured", in view of the normative complexity of human agency and volition succinctly described supra; the priority attached to the "good", as the desired consequence of action is itself an evaluative position, that is in need of further justification (and, on pain of circularity, on non-utilitarian grounds); "good" is not a value measurable only in material (e.g. financial) terms; for competing moral theories, an act (or rule) should be judged on its per se value and character, independently and in abstraction of its particular consequences; last but not least, utilitarianism seems to presuppose non-necessarily utilitarian values (e.g. liberty, equality etc.) as constitutive of this very fundamental decision-making framework (e.g. of public argumentation) in which the utilitarian position is also, inevitably inserted[31].

Despite, again, these often plausible critiques, it is undeniable that utilitarianism has a prima facie, common-sense plausibility which is vital for practical purposes; independently of the soundness of its theoretical foundations, it is a model of decision-making that can enjoy a wide acceptance by the vast majority of the community members of our form of life; and it is not accidental, again, that Rawls' normative model "takes utilitarianism seriously", his critique being "internal" to the utilitarian standpoint: in view of the veil of ignorance of individuals in the "original position" the choice of the principles of justice that should govern future social structure is also based on calculations of a quasi-utilitarian type; in slightly different accounts, law as type of regulative social contract is imposed by "natural necessities"[32] and older contractarian theories clearly stand on a utilitarian basis[33].

In this line of thought, utilitarian considerations are have to be taken into ac-

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[29] See Rawls (1999), Gaus (2015), a characterization that he later disavowed.

[30] According to which the rightness or wrongness of a particular action is a function of the correctness and consequences of following the rule of which it is an instance – as opposed to act-utilitarianism that focuses on the utility of ad hoc actions.

[31] That cannot be themselves justified solely on utilitarian grounds, also on pain of circularity. On utilitarianism see, indicatively, Sen and Williams (1982).

[32] See, e.g., Hart (1961), Chapter IX.

[33] As, e.g., the classic Hobesian exposition of social contract.

count in legal reasoning and efficiency of legal operations in action; at least, they have a significant practical relevance and they are not to be totally rejected; as just stated, refinements of the CREA algorithm would facilitate both parties in assigning relatively fair values to the goods to be divided; and, ab initio, the CREA model adopts an advanced and qualified version of utilitarianism, allowing for a wide range of motives and values, of personal preferences, of factors of "pain and pleasure" to be incorporated in the decision-making process.

4.4. Source, Validity and Normativity of "CREA rules"

In Hart's, predominant version of legal positivism, legal rules **are rules primarily because of their origin** (and not obligatorily because they are themselves directly and per se respected in view of their content), i.e. because they are established, "enacted" according to criteria provided by an ultimate meta-rule (the Rule of Recognition) [34]. However, the rules that govern such processes of "mediation" (as the process exposed in the CREA project), as well as the CREA algorithm itself and the bilaterally binding rules that emanate from the agreement – via the use of the algorithm – between the parties, seem apt to regulate aspects of societal affairs, not so much because of their source, "pedigree" or authoritative origin, but because of their **content,** because **they are actually accepted** as appropriate, binding standards of decisive action within a communal, public sphere of human activity; these rules (that we could name "CREA rules" – **CRs**), neither have a particular source, nor are they enacted according to a particular formal procedure; they seem to lack such strict formality and they are not always "posited' in the above sense; as opposed to "proper" legal rules, that are prima facie state-driven, CRs' main source is **the volition of the parties themselves**, as the parties voluntarily subject themselves to the CREA procedure, proposed to them by their legal advocates; this characteristic of the CRs roughly assimilates them to new conceptions of regulation, such as "governance rules", that could have a wide variety of informal sources (such as civil society organizations, international organizations and associations, NGOs, or totally informal societal formations as "pressure groups", "groups of interest" or even individuals – e.g. through internet and social media); such modes of regulation of societal affairs, are emerging in relatively recent years: non-state sources of (as, at least "soft" law) are gradually recognized, even within traditional conceptions of law, as, e.g., the so-called "law of contracts", the source of which is rather to be found in the prevailing legal practice (and developed jurisprudence), e.g. of big, international, legal firms; despite their private source, relevant rules have acquired a certain compulsory character, in virtue of their continuous application and acceptance by participants in the relevant

[34] See Hart (1961).

practice, though they are not, stricto sensu, enacted by state(s) or "enforceable"; in this line of thought, legal theory can have the flexibility of conceiving, as part of its subject-matter, rules of a non purely state origin (as the CRs): apart from the theory of *"legal pluralism"* [35], a positivist, "sociological" theory of law could conceptually accommodate the existence of non-state legal rules by considering that, in different societal contexts (as, e.g. in the field of contractual relations), different Rules of Recognition of legally binding rules can be actually followed and accepted; and it is crucial to note again, that the CREA model *fully respects "proper" legal, mandatory rules in force, rather aiming at facilitating their efficient application, than contesting their binding, authoritative status.*

Besides, legal theory distinguishes between different types of validity (formal, substantial, de facto …) of legal rules [36]. A legal rule, e.g., can have a formal validity (as being enacted according to formal, pre-determined criteria), without being actually applied in social life (cf. the various examples of the so-called "black letter laws") [37], without, i.e., being "de facto" valid. On the other hand, though CRs may seen as usually lacking "formal" validity, they do possess a "substantive" and de facto validity, as their content is accepted by the involved parties they are thus de facto applied in the relevant practice and "govern" relevant relations. No doubt, the ensuing normativity of such CRs seems thin or "weak", as compared to the strict normativity of formally enacted legal rules, within a supposedly closed, hierarchically structured legal order; in the famous, Kelsen's conception of the legal order as a pyramid [38], the top of which is occupied by constitutional rules (followed by laws, administrative acts etc.), CRs would probably occupy the lower positions; nevertheless, as legal realism (as supra analyzed, especially in its US version) has persuasively argued, this de facto normativity of CRs, even if not inspired by deep moral considerations but by practical necessities of regulation, is crucial for the effective operation of the societal relational system in its actuality.

4.5. Substantial Affinities

In spite of its "technical" character, certain affinities of the CREA model with substantive claims of justice and fairness can be depicted.

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[35] The term refers to the presence of multiple **legal** orders within one social field, such as state **law** or customary **law** based on culture or religion or other value systems; the theory was tied to German romanticism, according to which law's source is the "geist", the "spirit" or "soul" of people and cultural traditions within a certain community; it is also backed-up by anthropological remarks on the operation of relatively primitive societies, where co-existence of multiple regulatory systems can be observed.

[36] See, e.g., Weinberger (1984).

[37] As, e.g., at least till recently, the rule for the prohibition of smoking in public establishments in Greece.

[38] See, indicatively, Tur and Twining (1986).

Notwithstanding its impartial character, the CREA algorithm is inherently designed in order to provide optimal and simultaneously fair solutions that correspond to the (rational) choices, mutual interests and expectations of the parties [39]; these views suggest solutions of the general type "first cut, second chooses" to the problem, e.g., of the optimally fair division of a cake; it is, thus, in an important sense, committed to fairness and equal treatment of parties; similarly, it is connected to classical, but still pertinent substantive conceptions of "distributive justice", as, ultimately, the parties seem to get a fair share of goods, and "what they deserve", in view of their particular preferences. Further, "technical" application and systematization of already provided data by the CREA algorithm, promotes "procedural justice", as, at least to an extent and according to their external characteristics, it assists in treating "like cases alike", in conformity to the fundamental common law maxim.

While not contesting the validity of mandatory rules in force, the CREA model secures and promotes the (individual) rights of the parties, and, actually, to a "deeper than the usual" level, as even their personal preferences (and not only their formally protected possibilities of action) are being taken into account (with proper caveats, though, as, e.g., the "market value of goods"); in this aspect, the model coincides with rights – based theories of law, that consider "rights" rather than rules as the ultimate sources of the legal order; this view assigns a proper, interpretative coherence to the legal system and the ensuing legal decisions, otherwise left to the officials' arbitrary discretion when the clear guidelines – solutions engendered by rules "run out" [40]. In a related manner, the model seemingly reflects the fundamental idea that each member of the society has a sphere of "autonomy" and, certainly, the right to choose her own path toward happiness, toward what she considers a "good life"; and that this choice, as a fundamental expression of the idea of the autonomy of a person/individual, should not be impeded by any form of external restrictions.

Conclusive remarks

The CREA model is best characterized as an instrument of technical assistance, complementary to the existing regulatory framework; it fully respects legal mandatory rules in force, rather aiming at enhancing their efficient application, than contesting their binding, authoritative status. It does not make foundational, justificatory claims and is limited to disputes that are quantifiable, constituting a fair, impartial, technique of distributing goods at stake; it does not intervene in the

[39] Following relevant traditions, as analyzed supra under 4.2, 4.3.

[40] Paradigmatically in the so-called "penumbra cases". See Dworkin (1978), (1986).

legal, real or substantive data of a case (procedural rules, values of the parties, nature of the case etc.) but provides a method of systematizing these data in order to quickly reach an efficient, mutually beneficial agreement; it draws from the reality of legal practice, aiming at satisfying practical needs of the legal agents; it purports to facilitate relevant practical decisions, and not to impose the decision itself.

Especially in consideration of these characteristics and caveats, the CREA project can be fruitfully accommodated and inserted in the continuum of the relevant legal and political theory tradition; these affinities (as supra analyzed) endow the project with a secure epistemological and theoretical foundation, also thus providing a prima facie plausible defense (if not quasi-immunity) against potential accusations, particularly from the standpoint of normative legal theories.

Obviously, the present is to be considered rather as an attempt to sketch, from a detached, unbiased point of view, a comprehensive theoretical and methodological framework of the CREA project (also in view of its future, more extended application in other fields of law) than as an exhaustive treatment of the major particular issues involved.

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Marco Giacalone * and Seyedeh Sajedeh Salehi **

Online Dispute Resolution: The Perspective of Service Providers ***

Abstract

The current study examines the application of Online Dispute Resolution (ODR) with the specific focus on resolving civil disputes. For that purpose, this paper initially provides an introduction to the concept of ODR clarifying the major distinguishing characteristics which differentiate ODR from ADR. Next, the study will provide a thorough overview on two types of ODR namely, E-Negotiation and E-Mediation followed by arguing the applicability of these two principles into the ODR field in the global context. Finally, this paper will analyse the establishment and implementation of the EU ODR platform which was launched by the EU Commission to improve the efficiency of conflict resolution between traders and consumers through providing them with access to quality dispute resolution mechanisms procured by dispute resolution bodies.

Keywords: *Online Dispute Resolution (ODR), E-negotiation, E-mediation, ODR Platforms, EU*

Introduction

The past twenty years have seen increasingly rapid advances in the field of Online Dispute Resolution (hereinafter, ODR). With our society embracing tech-

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*** §§ Introduction, I, II & Conclusion are attributed to both authors. §§ II, IV & V are attributed to Marco Giacalone.

nology so thoroughly, many concerns have been raised regarding, how these technologies should assist parties in resolving their disputes.

Initially, the first platforms of online dispute resolution simply replicated face-to-face dispute resolution approaches online. According to Frank Sander, since the Pound Conference in 1976 [1], we may envision a courthouse with several doors, each leading us to a resolution process appropriate for a different kind of dispute. Doors that can be customized to individual disputes on demand.

Additionally, Colin Rule the founder of Modria platform, in one of his written contributions in 2015 emphasizes that:

> "*Experience quickly demonstrated that online dispute resolution required new approaches to reach its full potential. For example, ODR is pushing practitioners to break down some of the silos we have constructed within the face-to-face dispute resolution field. Instead of bright lines between diagnosis, negotiation, mediation, arbitration, and ombuds (terms parties often don't understand), many online disputants prefer a seamless progression from communication to evaluation, perhaps within hours*". [2]

However, one should bear in mind that Online Dispute Resolution is not a good fit with every type of dispute. Hence, it remains crucial for the dispute resolution professionals to correctly realize how to use technology when it is appropriate and avoid using it when it is not.

Taking into account the rapid growth in practicing ODR, during the last decade, by large private international entities – such as e-Bay – also by courts and government agencies – such as Civil Resolution Tribunal in Canada – this clearly indicate the willingness of some private and public entities to provide an opportunity for citizens to raise complaints and seek redress through online dispute resolution.

To procure Online Dispute Resolution services, while many of ODR providers have automatic algorithms – such as Fair Outcomes.Inc – others provide access to live mediators, arbitrators or even adjudicators – such as Rechtswijzer 2.0 – with varying templates for form complaints or opportunities to customize negotiations and complaints directly with companies and government agencies.

As many online dispute resolution experts have urged, technology will be a positive force for enhancing access to justice and broaden opportunities to negotiate for redress. [3]

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[1] Traum, Lara, & Farkas, Brian. (2017). THE HISTORY AND LEGACY OF THE POUND CONFERENCES. Cardozo Journal of Conflict Resolution, 18(3), 698.

[2] Rule, C., Technology and the Future of Dispute Resolution, Dispute Resolution Magazine, 2015, 21(2). (Retrieved on 20 November 2019) from www.americanbar.org/publications/dispute_resolution_magazine/2015/winter/technology-and-the-future-of-dispute-resolution.html.

[3] Susskind, R., Susskind D., The Future of the Professions: How technology will transform the work of human experts, Oxford University press, 2015.

Online negotiation allows the disputants to engage in a direct negotiation with a "hosted" negotiation partner . For instance, a trader from which the consumer has purchased an item or a service or the government agency with whom the consumer is willing to negotiate over a benefit or an obligation.

Moreover, the newly emerged online dispute resolution platforms provide opportunities for users to negotiate with others as the negotiations are taking place in the format of non-in person negotiation. As prominent examples of such platforms, the old algorithm driven Cybersettle for monetized civil and insurance cases as well as Skype – in both synchronous and asynchronous time – can be mentioned.

It should be pointed out that, despite the existing advantages of using ODR for resolving disputes, there are not sufficient data available on the outcomes of the disputes solved through the channel of ODR service providers. It is necessary for these service providers to provide non-confidential data regarding the outcomes of the disputes resolved by the means of using online dispute resolution systems and about the potentials of these platforms in handling legal or customary matters. By publishing such data, medical-disease affinity groups, online and hosted "legal problem" sites can enhance bargaining power by providing data and sharing stories of successes. [4] Another advantage of releasing the non-confidential data of success stories of disputes resolved online, the ODR providers can use their settlement agreements in previous similar cases to give possible remedies and recommendations – through using ICT tools to identify recurring patterns of disputes or categorising complaints – as well as preventing the conflicts, [5] through providing disputants with opportunity to reconsider their behaviours. [6]

For a long period of time, the e-supported communication systems were considered as being inferior to face-to-face interactions in resolving disputes between the parties. Such sceptical approach particularly in cases such as the arrangement of a divorce – a specific type of e-supported mediation can be successfully applied to mediate family disputes with outstanding results of high settlement rates and high scores, reaching not only an agreement, but also a high level of justice perceptions. [7] Specifically, starting from the beginning stages of a mediation an asynchronous communication may offer parties a protected atmosphere to voice their emotions, share their opinion and talk without any sort of negative emotional

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[4] Menkel-Meadow, C., Schneider, A.K. & Love, L., Negotiation: Processes for Problem Solving. New York: Aspen Wolters Kluwer Law and Business, 2014.

[5] Cortes, P., A supra note 160.

[6] Susskind, R., The End of Lawyers? Rethinking the nature of legal services, Oxford University Press, 2010.

[7] Poitras, J. & Le Tareau, A., Quantifying the quality of mediation agreements. Negotiation and Conflict Management Research, 2009.

prejudice.[8] After all, it is the way parties perceive the mediation that will affect their current and future behaviours, feelings and thoughts.[9]

An appreciation of these ODR systems is important as this study sets out to develop a normative position on the use of ODR methods by citizens and to look at the ability of ODR platforms in its various guises to meet this normative approach.

With this end in view, the current study explores the status and applicability of the ODR in the form of six Sections. The first part deals with the state of the art of Online Dispute Resolution followed by discussing the ODR benefits, which may help the parties in different ways to reach an agreement. Next, the paper will move on to focus on the two major developed ODR systems specifically examining the online negotiation in Section III and the online mediation in Section IV. Section V is concerned with the analysis of the existing ODR platforms in a worldwide context. The final Section will focus on the EU platform as established by the European Commission to assist consumers and traders to settle their disputes through ADR bodies.[10]

I. The State of the Art Online Dispute Resolution

The Internet and cyberspace in general are continuing to rapidly evolve with the high rate of using them, enormously affecting various dimensions of our societies. Similar to all other evolutions, the application of technology into the various aspects of human lives does not merely bring positive aspects.

By all means, communication technologies and speed are soared, and it is almost possible to virtually communicate synchronously and asynchronously, everywhere. Nevertheless, this will not indicate that the web will be always a harmonious place.

The World Wide Web (WWW.) was invented in 1989. Despite this, he online population at that time was not [i]willing to generate a range or a quantity of conflicts that would suggest something different from the physical and concrete dispute resolution methods and entities.

E-commerce disputes, privacy and copyrights are currently among the most common disorders characterised by the online field and people need to have an easy and safe mean to solve eventually their disagreements.

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[8] Yen, J.Y., Yen, C.F., Chen, C.S., Wang, P.W., Chang, Y.H., & Ko, C.H.,
Social anxiety in online and real-life interaction and their associated factors. CyberPsychology, Behavior and Social Networking, 2012. http://dx.doi.org/10.1089/cyber.2011.0015.

[9] Bollen, K., Verbeke, A. & Euwema, M., Computers work for women: Gender differences in e-supported divorce mediation, Computers in Human Behavior, 2014.

[10] https://ec.europa.eu/consumers/odr/main/?event=main.home2.show&lng=EN. (Retrieved 20 November 2019).

Traditionally, the online population often tend to find ways to avoid the solution of the potential conflicts which could be raised in the web domain just in the physical world entities. On the other hand, with the aim of safeguarding these needs, it is becoming inevitably significant to seriously take into account the progression of Online Dispute Resolution multifaceted phenomenon.

Even if the term ODR was coined in the mid-1990s what is lacking is a cogent and univocal theoretical base. On account of that, ODR can be broadly defined as the use of Alternative Dispute Resolution techniques over the Internet. [11] Since its origin, ODR was essentially focused on disputes related to online activities, however, currently this method is also applied in offline disputes.

As remarked by Ethan Katsh, "The marketplace for ODR is now offline disputes as well as those originating online and public sector conflicts besides those originating in the private sector".

Indeed, it is barely preferable to make a distinction between proceedings that rely heavily on online technology and proceedings that do not. [12]

Alternative dispute resolution and online dispute resolution specifically form the EU perspective are considered as means of safeguarding more eminent access to justice. To put it another way, they both should be implemented to foster dispute resolution services to citizens.

Basically, ODR differs from ADR in one important distinguished aspect. While, ADR typically refers to processes, ODR is not merely dealing with the process of resolving disputes but it also often obscure issues. [13] In addition, ODR is a natural evolution of the previous ADR trend using alternative approaches across a wide range of civil domains. [14]

A considerable amount of literature has been published on Online Dispute Resolution especially in the last few years, but the ground breaking book still remains the "Online Dispute Resolution" by Katsh and Rifkin. [15] The both authors were the first in observing the fact that our society was creating a huge number of disputes born online without any practical method for the parties to engage in traditional face-to-face solutions. Thus, Katsh and Rifkin introduced the concept of Online Dispute Resolution as a "fourth party" that completes the other three par-

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[11] Pappas, B.A., Online Court: Online Dispute Resolution and the Future of Small Claims, UCLA Journal of Law & Technology, Vol. 12, 2, 2008, 7, 26.

[12] Hörnle, J., Cross-Border Internet Dispute Resolution. Cambridge: Cambridge University Press, 2009.

[13] Wahab, M., Katsh, E. & Rainey D., Online Dispute Resolution: Theory and Practice, Eleven International Publishing, 2012.

[14] Ebner, N., Zeleznikow, J., Fairness, Trust and Security in Online Dispute Resolution, Journal of Public Law and Policy. Available at: http://digitalcommons.hamline.edu/jplp/vol36/iss2/6.

[15] Katsh E. & Rifkin J., Online Dispute Resolution: Resolving Conflicts in Cyberspace, San Francisco: Jossey-Bass, 2001.

ties with Information and Communication Technology (ICT).[16] Considering ICT as the fourth party at negotiation table such as a 'friendly and patient robot', the first two parties are the two disputants while the third part is the human negotiator/mediator.[17]

The fourth party' theory is a clear metaphor which stresses how technology can be as influential as to change the traditional three-side model.[18] The fourth party embodies a range of facilities in the same manner that the third party does.[19] Whereas the fourth party may at times take the place of the third party – i.e. automated negotiation – it will frequently be used by the third party as a tool for assisting the process.[20]

There are a large range of activities that are conducted mainly through the use of this fourth party, such as organizing information, sending automatic responses, shaping writing communications in a politer and constructive manner (i.e. blocking foul language).

In addition, technological systems can monitor performance, schedule meetings, clarify interests and priorities and so forth.[21] The application of the fourth party will increase the use of technology, reducing the role of the third neutral party. Literally, ICT advance is occurring exponentially since it speeds up over the time.[22]

Although differences of opinion still exist, however some believe that Online Dispute Resolution stands as an alternative to traditional dispute resolution procedures. The acronym ODR embodies a multitude of concepts which Schultz and other commentators define it, firstly, as a mixture of a *sui generis* form of dispute resolution responding to the needs of Internet users. Secondly, as a different alternative dispute resolution form enriched with online capabilities.[23]

Specifically, ODR is the application of information communications technology to the practice of dispute resolution. In a general sense, ODR involves the application of dispute resolution techniques over the Internet. Thus, it is used to resolve

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[16] Cortes, P., A European Legal Perspective on Consumer Online Dispute Resolution. Computer Telecommunications Law Review, 2009, 1-28.

[17] Katsh, E. & Rifkin J., supra note 159.

[18] Katsh, E; Wing, L., Ten Years of Online Dispute Resolution (ODR): Looking at the Past and Constructing the Future, 38 University of Toledo Law Review 19, 2006.

[19] Gaitenby, A., The Fourth Party Rises: Evolving Environments of Online.
Dispute Resolution, 38 University of Toledo Law Review 101, 2006

[20] Bol, S.H., An Analysis of the Role of Different Players in E-Mediation: The (Legal) Implications. IAAIL Workshop Series – Second International Workshop. www.odrworkshoinfo/papers2005.

[21] Katsh, E. & Rifkin J., supra note 159.

[22] Katsh, E. & Rifkin J., supra note 159.

[23] Schultz, T., Online Dispute Resolution: The State of the Art and the Issues (with Gabrielle Kaufmann-Kohler, Dirk Langer & Vincent Bonnet), University of Geneva, 2001.

Internet-related disputes such as e-commerce, nonetheless ODR is also suitable to be used for resolving traditional off-line disputes. [24]

Petrauskas and Kybartiene argue that the key components of ODR can be listed as follows:

1. Like ADR, companies agree to resolve their disputes outside the courts, the difference being to use the Internet to enhance the process;
2. Experts guide the parties and apply their ADR experience to support the Internet process;
3. ADR rules and practices are adapted to the Internet environment, and
4. Software tools are used to enhance Internet exchanges. [25]

Moreover, they also focused on the main web-based services offered in ODR systems which have been tested and introduced.

These new methods will enable the main parties and the third ones to:

1. Meet online and work in shared and protected work spaces,
2. Have direct access to databases with precedents,
3. Retrieve and manage key documents, and
4. Hold meetings through voice and video-conferencing as desired and with translation services as needed. [26]

II. The Benefits of Online Dispute Resolution

Nowadays, Online Dispute Resolution has been integrated into familiar dispute resolution processes generating novel approaches to responding to and preventing conflict, approaches that is not possible to be developed in a traditional offline context. [27]

As earlier discussed about the role of the fourth party, it is crucial to emphasize the fact that the fourth element is still likely to be considered as a factor. Colin Rule, co-founder of Modria [28] – one of the most well-known ODR providers in Silicon Valley – believes that the distinction between online and offline is a false dichotomy:

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[24] Id.

[25] Petrauskas, F. & Kybartiene, E., Online Dispute Resolution in Consumer Disputes, Jurisprudencija, 2011.

[26] Id.

[27] Wahab, M., Katsh, E. & Rainey D., supra note 157.

[28] Modular Online Dispute Resolution Implementation Assistance, www.modria.com. The successful operation of Modria in offering comprehensive ODR services to public agencies (i.e. courts and tax-related corporations) encouraged the TylerTech Corporation to buy Modria platform in 201, with the aim of developing the justice solution services.

"*Most of us are comfortable using technology to communicate sometimes and at other times getting together face-to-face. We constantly navigate back and forth between our online and offline channels, sometimes in the space of just a few minutes. This is true in ODR as well. We may begin a process with an online filing form and move to telephone calls and then to face-to-face meetings before finalizing the agreement online. Joint sessions might be held in person, with in-between conversations happening over e-mail. This is the way our parties live their lives, and they expect to be able to resolve their disputes with similar fluidity*". [29]

Indeed, it is useless to pick online or offline dispute resolution, we can choose both.

There are several advantages making ODR in particular attractive to be applied for resolving disputes. These benefits include cost savings, the speed of resolution, convenience, and individually tailored processes.

In terms of money, ODR is mostly useful in cases where the attorneys' fees would exceed the likely award amount. ODR is faster than a typical trial or even ADR, since technology has the potential to shorten the distances which parties might otherwise need to travel. Furthermore, ODR does not depend on clearing time on a mediator's or a judge's calendar. Using e-mail, group discussion, online platforms and agreements that can be conveniently written and amended are among other advantages of using ODR. [30] Further, instead of a cookie cutter approach, each dispute process can be tailored to fit the disputants' individualized needs and it would be easy to avoid the possible distraction due to the presence of the emotional aspect of the conflict.

Generally, the key companies' and individuals' needs and concerns can be listed as follows: reputation, quality control, information strategies and mutual learning.

As a result, Online Dispute Resolution can often resolve disputes promptly, in an efficient way and with more participation and control over the outcome by the parties, who must work with each other to resolve the dispute. By using ODR, the parties can have more flexibility. The concept of flexibility here does not merely refer to a geographic flexibility, but it also means the flexibility in choosing the applicable laws when the parties are from different jurisdictions. In addition, companies – especially e-commerce ones – which showed on their homepages the possibility of solving possible conflicts directly online may attain more reliability and trust from web users, who want to trust online transactions and know a reliable dispute resolution process exists in the event a dispute arises between the consumer and trader. The existence of such trust will encourage more consumers to

[29] Rule, C., supra note 154.

[30] Krause, J., Settling It on the Web: New Technology, Lower Costs Enable Growth of Online Dispute Resolution, 2007.

frequently use the website. In fact, the key advantage of resolving disputes through an online system is that it avoids the matter of whether a specific court has jurisdiction over the dispute. [31]

Moreover, ODR could be the only feasible option for people unable to afford expensive travelling costs or for those involved in e-commerce negotiations regarding small amounts of money. [32] Many sceptic practitioners have argued that an online negotiation are less influential than face-to-face dispute resolutions since it is more difficult for both parties to understand each other's interests due to the absence of non-verbal signals, which in turn reduces the chance of achieving a satisfying agreement. [33]

Recently, researchers have shown that online communication is more unsharpened and rough than face-to-face communication and can therefore more easily lead to misunderstandings. [34]

Several studies have attempted to explain that it would be easier to increase the likelihood of finding an integrative agreement if negotiators know each other prior to the negotiation, hence they have more cues to interpret the other party's actions and motivations. Otherwise, misunderstandings can easily lead to negative communication, distrust and eventually impasse. As suggested by Diane Moore, for ODR to be triumphant it is accordingly important to have some insights into the other party's intentions which can "be achieved by shared group membership or mutual self-disclosure". [35]

On the one hand, it may consequently be argued that divorce cases are particularly suited for ODR since both parties know each other well enough to interpret each other's actions, and, on the other hand, especially during divorces parties could be distracted by the emotional sides of the divergence, but thanks to the distance, they could focus on the matter that need to be settled.

Another benefit of using online dispute resolution is definitely ODR's potential for growth as a means of dispute resolution. It is certain that ODR field is being changed by technological development. The only uncertainty that remains is whether this change will take one, five, ten or more years to elect Online Dispute Resolution as the normal way of solving disputes.

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[31] Lide, E., ADR and Cyberspace: The Role of Alternative Dispute Resolution in Online Commerce, Intellectual Property and Defamation, Ohio State Journal on Dispute Resolution, 1996, 921-941.

[32] Bordone, R., Electronic Online Dispute Resolution, 3 Harvard Negotiation Law Review 175, 1998.

[33] Bazerman, M., Negotiation, 51 Annual Review of Psychology, 2000, 279314.

[34] Sproull, L. & Kiesler, S., Connections: new ways of working in the networked organization, 1991.

[35] Moore, D., Long and short routes to success in electronically-mediated negotiations: Group affiliations and good vibrations., 77 Organization Behaviour & Human Decision Processes, 1999.

Online Dispute Resolution may be divided into two main classes:

The first class refers to the so-called Hard ODR – or traditional ODR – which covers procedures intending directly to resolve conflicts. While, the second class or the so-called Soft ODR seeks to prevent disputes, or to facilitate their resolution once conflicts have arisen, without actually adjudicating them.

As suggested by Thornburg – a US commentator – this distinction is supported by the idea of thinking ODR as encompassing not just traditional resolutive processes, but also newer preventative processes by content owners to forestall copyright infringement. [36]

Online Dispute Resolution systems may be classified according to the "hard systems" into three main categories:

a) Online negotiation, using expert systems to automatically settle financial claims.
b) Online mediation, using a website to resolve disputes with the aid of qualified and accredited mediators.
c) Online arbitration, using a website to resolve disputes with the aid of qualified and accredited arbitrators.

It is essential to emphasize that not all types of ODR are fully developed, yet. Online negotiation and online mediation are currently the most advanced systems. Therefore, this paper will now move on to discuss these two more developed ODR methods.

III. Online Negotiation (E-Negotiation)

Online negotiation may be divided into two main sub-groups of Automated Negotiation and Assisted Negotiation, however it may sometimes involve a combination of these two methods.

Whereas there are providers such as Cybersettle [37], who provide blind-bidding model also called automated negotiation, other platforms – such as eBay [38] and PayPal [39] – the offer so-called assisted negotiation by outlining, based on prior experience from similar cases, a number of possible remedies to the parties to a dispute. [40]

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[36] Thornburg, J., Going Private: Technology, Due Process and Internet Dispute Resolution, 2000.

[37] www.cybersettle.com (Retrieved 20 November 2019).

[38] www.ebay.com (Retrieved 20 November 2019).

[39] www.paypal.com (Retrieved 20 November 2019).

[40] Cortés, P., A new regulatory framework for extra-judicial consumer redress: where we are and how to move forward, University of Leicester School of Law Research Paper, 2013.

a) Automated Negotiation

Online Dispute Resolution uses a wide variety of ICT tools for negotiations. Thus, not only tools such as e-mail or videoconferencing are used, but also with providers which help parties in negotiating online through a process called "blind-bidding". During automated negotiation ICT takes over the negotiation. Blind-bidding is a negotiation process designed to determine economic settlements for claims in which liability is not challenged. Indeed, automated negotiation is ideal for online settlement of financial claims.

This is a notably simple and straightforward method without any necessity for further assistance.

Generally, automated negotiation involves two parts:

1. Offering Party, which is the party who makes the offer (the party who is going to pay).
2. Demanding Party, which is the party who makes the demand (the party who is seeking payment).

As a general rule, as blind-bidding example, it is necessary to consider a dispute between two insurance companies as to who pays out and in what proportions in relation to a car accident.

Typically, in blind-bidding, one party (hereinafter, A) contacts an ODR provider, presenting his/her case against the second party (hereinafter, B). The chosen ODR provider contacts B, who can accept or refuse to submit to the competency of the institution. The parties then enter the so-called "blind-bidding" procedure. Each party, in turn, enters their respective offer and demand. The proposed figures are confidential; they are neither made public nor communicated to the other party. The figures are kept confidential regardless of whether the case settles or not. The parties also choose a percentage range. The ODR algorithm computes a settlement amount between the offer range and the demand range provided that the figures fall within the given range. The number of bids varies between three and unlimited. If the two bids in any of the rounds come close enough to one another, the midpoint figure will be deemed as accepted.

Most of platforms offering automated negotiation also impose a time limit for the parties to reach an agreement.

Automated negotiation ODR is mainly applicable only to purely monetary disputes and cannot deal with factual or legal disputes of any complexity. Examples of such websites include Cybersettle, Smartsettle [41] and FairOutcomes, [42] all of them claim to have processed large amounts of cases successfully. [43]

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[41] www.smartsettle.com (Retrieved 20 November 2019).

[42] www.fairoutcomes.com (Retrieved 20 November 2019).

[43] Edwards, L. & Wilson C., Redress & Alternative Dispute Resolution in Cross-Border E-commerce Transactions, Briefing Note, (IP/A/IMCO/IC/2006-206).

By way of illustration, Cybersettle allows three rounds of bidding, using a simple and practical system based on double blind bidding. In such a procedure, both parties are unaware of the specifics offered by the other party, only that a negotiation is in process. The computer operates according to a formula for each round – compares the offers and counteroffers – and when the offers are within a specific range it announces a deal. If the software determines that a settlement has not been reached, then their offers remain confidential and future bargaining positions are unaffected. In Cybersettle, a settlement is reached if there is less than 20% between the offers in any of the rounds, and then the claim will settle for the average of the two amounts. [44]

In case of Smartsettle, this system uses a method called visual blind bidding. In Smartsettle – based on visual blind bidding – that is applicable to simple cases and scalable to complex multiparty cases, the visible suggestions are put forward by each party and the computer operating as an intelligent agent, but each side's acceptances are kept hidden from the other party. The computer announces a deal when hidden acceptances coincide.

In the end, Fair Outcomes Inc., is the most interesting one as a provider, since it provides parties with access to several proprietary systems that are grounded in mathematical algorithmic theories of fair division and of games – it is into this provider that Brams' algorithms of fair division are applied.

b) Assisted Negotiation

In order to describe the assisted negotiation, it is necessary to bear in mind what was previously stated about negotiation. Similar to the offline negotiation, the assisted negotiation, is indeed, a process where parties negotiate and settle their issues, disputes or grievances.

Negotiation is chosen by parties on a voluntary basis. Attorney may represent the parties during the process. Parties reach agreement without any external entity empowered to make a decision against their will.

However, the main difference is that in the assisted negotiation, it is the technology who assists the negotiation process between the parties to the dispute.

The ODR provider supplies facilities such as a secure website, communication facilities, and possibly storage for documents and other such facilities.

The main services provided are the following:

developing agendas, engaging in productive discussions, identifying and assessing potential solutions, writing agreements.

It is worth mentioning that the assisted negotiation procedures are designed to improve parties'communications through the assistance of a third party or a software.

[44] www.cybersettle.com/demo/demo_pf.asp (Retrieved 20 November 2019).

The major advantages of these processes, when used online, are their informality, simplicity and user friendliness. [45]

A well-known example is the Internet auction website eBay which has a business relationship with the ODR provider SquareTrade, [46] and thereby provides assisted negotiation to a large number of eBay users alongside other processes.

Thus far, we illustrated the difference between automated negotiation and assisted negotiation, therefore we can introduce the concept of E-Negotiation. It is a process that uses a negotiation support system including computers or other forms of electronic communications that enable parties to negotiate their own agreements.

"In its most advanced form, E-Negotiation is a form of artificial intelligence that fully automates mediation (perfectly neutral, super intelligent, and very secure). While in many cases unnecessary, E-Negotiations can include face-to-face meetings, if such meetings enhance the process". [47]

In 1980s the first E-Negotiation systems or computer-mediated negotiation systems emerged across the world. [48] Amongst the most prominent examples of the E-Negotiation systems, we may mention – in geographical order, Adjusted Winner [49], AutoMed [50], Cybersettle [51], Fair Outcomes [52], Genie [53], Genius [54], Persuarder [55] all were established in the United States. In Canada Ne-

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[45] Motion, P., Article 17 ECD: Encouragement of Alternative Dispute Resolution. Online Dispute Resolution: A View from Scotland, Hart Publishing, 2005.

[46] www.squaretrade.com (Retrieved 20 November 2019).

[47] Thiessen, E.M., Miniato, P. & Hiebert, B., ODR and eNegotiation, Online Dispute Resolution: Theory and Practice (a treatise on technology and dispute resolution), Eleven International Publishing, Wahab M., Katsh E. & Rainey D., eds, 2012.

[48] Id.

[49] See supra n. 54.

[50] Chalamish M. & Kraus, S., AutoMed – An Automated Mediator for MultiIssue Bilateral Negotiations", Journal of Autonomous Agents and MultiAgent System, 201, www.aaai.org/ocs/index.php/ICCCD/ICCCD09/paper/viewFile/1025/3320 (Retrieved 20 November 2019).

[51] www.cybersettle.com, (Retrieved 20 November 2019).

[52] www.fairoutcomes.com, (Retrieved 20 November 2019).

[53] Wilkenfeld, J., Kraus S., Holley, K.M. & Harris, M.A., GENIE: A Decision Support System for Crisis Negotiations, 1999. Available at www.sciencedirect.com/science/article/pii/016792369400027P, (Retrieved 20 November 2019).

[54] Lin, R., Kraus, S., Tykhonov, N., Hindriks K. & Jonker, C.M., Supporting the Design of General Automated Negotiators", in T.Ito, M. Zhang, V. Robu, S. Fatima, T. Matsuo, and H. Yamaki(eds.), Innovations in AgentBased Complex Automated Negotiations,", Volume 319 of Studies in Computational Intelligence. Available at http://u.cs.biu.ac.il/~linraz/Papers/linetalagentDesignBookChapter.pdf, (Retrieved 20 November 2019).

[55] Sycara, K., Machine Learning for Intelligent Support of Conflict Resolution, Decision Support Systems, 10, pp. 121-136, 1993.

goplan [56] and Smartsettle [57] were designed to handle disputes through online means of technology. Split-Up [58] and Asset Divider [59] (ex. Family Winner) emerged in Australia. In Europe, in Germany Negoisst [60] and Joint Gains [61] in Finland came into the scene of online dispute resolution.

These systems provided the stakeholders with opportunities to resolve their dispute just online, regardless of where the conflict was originated. [62] Some systems are designed also with the possibility of face-to-face interactions between the parties. [63]

Nowadays, a large number of case studies illustrate "how cross-cultural negotiations can be managed through modern channels of social influence and information-sharing and shed light on the critical social, cognitive and behavioural role of the negotiator in resolving on-line, cross-cultural, conflicts and disputes, and generally in bargaining and negotiation". [64]

As stated by Ernest Thiessen, Paul Miniato and Bruce Hiebert:

> "*the key difference with E-Negotiation is that the parties are in full control both during the process and in accepting or rejecting an outcome… and a well-designed E-Negotiation system will reduce the conflict or eliminate it by changing the fundamental nature of the interaction between the parties*". [65]

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[56] Matwin, S., et al., NEGOPLAN: An Expert System Shell for Negotiation Support, IEEE Expert, 4(4), 1989, 50-62.

[57] See supra n. 188.

[58] Stranieri, A., et al., A Hybrid-Neural Approach to the Automation of Legal Reasoning in the Discretionary Domain of Family Law in Australia, Artificial Intelligence and Law, 7(2-3), pp. 153-183, 1999.

[59] Abrahams, B., Bellucci, E. & Zeleznikow, J., Incorporating Fairness into Development of an Integrated Multi-agent Online Dispute Resolution Environment, Group Decision and Negotiation, published online 3 March 2010: vuir.vu.edu.au/6967, (Retrieved 20 November 2019).

[60] wi1.uni-hohenheim.de/negoisst.html, (Retrieved 20 November 2019).

[61] sal.aalto.fi/en/personnel/raimo.hamalainen/publications, (Retrieved 20 November 2019).

[62] Thiessen and Zeleznikow believe ODR systems face five main challenges as they attempt to present an effective medium for online dispute resolution: 1) Problem representation, 2) Preference elicitation, 3) Effective communication, 4) Neutrality provision and 5) Degree of automation.

[63] Thiessen E. & Zeleznikow J., Technical aspects of online dispute resolution challenges and opportunities. Conley Tyler, M., Katsh, E. & Choi, D., Proceedings of the Third Annual Forum on Online Dispute Resolution, 2004.

[64] Harkiolakis, N., E-Negotiations: Networking and Cross-Cultural Business Transactions, Gower, 2012.

[65] Lide, E., supra note 175.

IV. Online Mediation (E-Mediation)

The so-called E-supported mediation (or just E-Mediation) refers to mediations that are fully e-supported as well as the hybrid mediations, where they are partly computerized and partly face-to-face. Participation is voluntary and confidential.

During an e-supported mediation, the parties need to fill out an online intake prior to joining the mediation process. For responding to the intake questions asynchronous messages are used which are merely shared with the mediator, not even with the other party. Finally, once the data intake stage is completed, appointments are made for the face-to-face mediation and the mediation sessions are carried out accordingly. [66]

In ODR mediation, compared to a traditional mediation, asynchronous are opposed to real time discussion and common and private communication rooms are desirable, however not always available.

The main disadvantages are the lack of face-to-face contact that may inhibit development of trust, deny clients their chance to "tell their story"thus impede reaching into possible solutions.

A study conducted by Juripax has shown that adding an online element to workplace mediation processes greatly contributes to their effectiveness by eliminating asymmetry in hierarchal workplace disputes. [67]

There are several notable e-mediation projects [68]conducted aiming at analysing the efficiency of online mediation. The list below includes some of these projects.

- The Online Ombuds project, which was a pilot ODR program established in 1996, [69]
- The Maryland Family Mediation project, which was another early initiative funded by the National Center for Automated Information Research (NCAIR) in the United States, [70]
- The Cybertribunal project at the University of Montreal School of Law, which

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[66] Bollen, K. & Martin Euwema M., The Role of Hierarchy in Face-to-Face and E-Supported Mediations: The Use of an Online Intake to Balance the Influence of Hierarchy, Negotiation and Conflict Management Research, 2013.

[67] Bollen, K., Martin Euwema M., Angry at Your Boss or Fearing Your Employee? Negative Affect in Hierarchical Conflicts and the Moderating Role of E-Supported Mediation, Kyoto, IACM, 2009.

[68] Katsh, E., Rifkin, J. & Gaitenby, A., "E-Commerce, E-Disputes, and EDispute Resolution: In the Shadow of eBay Law", 28 Ohio St. J. on Disp.

Resol. 810, 2000.

[69] Katsh E. & Rifkin J., supra note 159.

[70] Getz, C., Closing the Distance with Technology: Report on Phase I of the Technology-Assisted Family Mediation Project, 2007.

later developed into e-Resolution, a commercial service provider providing e-mediation as well as arbitration for domain name disputes, [71]

- The SquareTrade, which picked up the eBay mediation project where the Online Ombuds left off, as a business venture. [72]

This last example – referring to eBay in conjunction with the Internet start-up SquareTrade – introduced an online dispute resolution system allowing buyers and sellers to settle various contentious issues in a structured format. [73] SquareTrade, as an e-mediation provider asks theparties to answer questions on a customised complaint form and provide supporting documentation for their claim. During this initial stage, the parties try to reach an agreement by communicating directly with each other through SquareTrade's Direct Negotiation tool, which is a completely automated web-based communication tool. SquareTrade will transmit the form to the other party encouraging that party to respond. If the parties fail to reach a compromise, through direct negotiation, then they have the option of requesting assistance from a mediator. SquareTrade is careful to explain that the mediator is not a judge or arbitrator, but merely seeks "to facilitate positive solution-oriented discussion between the parties… The mediator will only recommend a resolution if the parties request it". [74] Even then, the mediator's recommendation is not binding on the parties.

The dispute resolution mechanism established by Wikipedia works in a similar fashion and is considered as another prominent example of online mediation. [75]

V. Examples of Online Dispute Resolution Platforms in Action

As illustrated by Professor Julia Hörnle in 2011, an international ODR platform should have five separable basic functions.

First, it is mandatory to set minimum standards for ADR/ODR providers and only admits ADR/ODR providers complying with these (the so-called Clearing House Function).

Next, it is essential to enrich search engine data to enable consumers to find

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[71] A Katsh E. & Rifkin J., supra note 159.

[72] Abernethy, S., The Square Trade Experience in Online and Offline Disputes, Proceedings of the 2003 United Nations Forum on ODR 2003. [224] Abernethy, S., Building Large-Scale Online Dispute Resolution & Trustmark Systems, Proceedings of the United Nations Economic Commission for Europe, UNECE, 2003.

[73] Abernethy, S., Building Large-Scale Online Dispute Resolution & Trust-mark Systems, Proceedings of the United Nations Economic Commission for Europe, UNECE, 2003.

[74] www.squaretrade.com/cnt/jsp/odr/learn_odr.jsp (Retrieved 20 November 2019).

[75] Rabinovich-Einy, O., Katsh, E., Digital Justice, Reshaping Boundaries in an Online Dispute Resolution Environment, 1(1) International Journal of Online Dispute Resolution, 2014, 5-36.

competent ADR/ODR scheme in trader's country and tests whether dispute is in scope (the so-called Referral Function).

Third, it is fundamental to give consumer choice and info and to provide information about outcomes of other disputes (the so-called Transparency Function).

The fourth point to consider refers to allow consumer to file dispute (claim plus evidence and to transfers dispute to competent ADR/ODR (the so-called Transfer Function).

Finally, it is significant to maintain the records of ADR/ODR outcome. It means, to record whether the trade has complied with the mandatory standards or not and to compile statistics (the so-called Enforcement Function). [76]

In the previous paragraphs, we briefly discussed Cybersettle, Modria and SquareTrade/eBay system. Nonetheless, there are more ODR platforms which are already developed in providing citizens with online dispute resolution services. [77] Among them, in alphabetic order, the authors will discuss the Civil Resolution Tribunal, Financial Ombudsman Service, Nominet, Online Schlichter, Rechtwijzer 2.0, Resolver, Traffic Penalty Tribunal and Youstice, in more details. [78]

a) The Canadian Civil Resolution Tribunal [79]

The Canadian Civil Resolution (hereinafter, CRT) was initially established – in 2011- as an alternative pathway to the existing traditional courts in British Columbia (BC), Canada. [80] However, the successful implementation of this service led the BC government to enact the Civil Resolution Tribunal Act 2012 with the aim of applying algorithms in dealing with small claims and some other traffic accidents related disputes. In 2015, the CRT Act was amended by expanding the scope of its jurisdiction to wider categories of claims. Thus, the CRT has currently jurisdiction over most of small claims up to 5,000 CAD, Strata property disputes of any value, traffic accidents and injury claims up to 50,000 CAD, also disputes related to societies and cooperative associations of any value, through a more convenient and less costly process.

There are two systems being developed to support the CRT and increase ac-

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[76] Hörnle, J., www.law.ox.ac.uk/sites/files/oxlaw/dr_julia_hornle.pdf.

[77] For more information on a list of ODR service providers see Appendix I and II at pages 29-32 of this paper.

[78] Susskind, R., Report of Civil Justice Council's Online Dispute Resolution, Advisory Group, www.judiciary.gov.uk/wp-content/uploads/2015/02/Online-Dispute-Resolution-Final-Web-Version1.pdf.

[79] www.civilresolutionbc.ca, (Retrieved 20 November 2019).

[80] Raymond, Anjanette H., & Shackelford, Scott J. (2014). Technology, ethics, and access to justice: Should an algorithm be deciding your case? Michigan Journal of International Law, 35(3), 485-524, p. 505.

cess to justice in Canada. The first system, the Solution Explorer, is designed to provide people with the tools they need to assess their options and resolve their disputes themselves.The second system, the Dispute Resolution Suite, will enable the CRT to pursue further early resolution options and adjudications. The online tribunal operates in several stages. In the first instance, the facility will help users explore possible solutions. Then, parties will be required to use the tribunal's online negotiation platform, which is subject to short timelines and supported by templates for statements and arguments. If a settlement is not reached, then a tribunal case manager will be appointed to assist the parties to settle their dispute through a mediation process that will take place online or over the telephone. If parties do not settle by this mediation process, they will then be invited to agree to a third and final stage of adjudication. The adjudicator will contact the parties via the online platform, over the phone, or, when necessary, through videoconferencing, and then will make a decision that will be final and binding.

b) Financial Ombudsman Service [81]

The UK Financial Ombudsman Service (hereinafter, FOS) was established by the Financial Services and Markets Act in 2000, as the mandatory ADR body in the financial services sector. The purpose of the FOS is to resolve disputes between consumers and UK-based financial businesses with minimum formalities and at more expedited compared to the ordinary court proceedings. The FOS is a 'distance' service and each year there are usually very few face-to-face meetings with adjudicators or ombudsmen.The main approach of the FOS is constituted based on the principle of 'the sooner to act for resolving a dispute, the better the problem can be solved'. With having this outlook in the process of dispute resolution, the FOS has functioned highly successful in dealing with the referred disputes. Due to such satisfactory success rate, since April 2019, the jurisdictional scope of the FOS has been extended to also cover small and medium-sized enterprises (SMEs), allowing more businesses to benefit from using this ODR service. In Addition, the award limit threshold has been considerably expanded (by 133%) from 150,000 GBP to 350,000 GBP. [82]

For using this ODR service, the businesses need to be registered with it. A positive characteristic in FOS system is associated with the fact that, this service allows the businesses to solve the consumer complaint within three working days

[81] www.financial-ombudsman.org.uk, (Retrieved 20 November 2019).

[82] Financial Ombudsman Service Limited (2019). Annual Report and Accounts for the Year ended 31 March 2019. Presented to Parliament pursuant to paragraph 7A (3) of Schedule 17 of the Financial Services and Markets Act 2000, as amended by the Financial Services Act 2012, p. 27. Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/817100/P0002_AR_2019_Accessible_Web_09-07-19.pdf, (Retrieved 20 November 2019).

and send the result to FOS. If the dispute is not solved, then this service will start investigating into the case. The cases are solved promptly with most of them being concluded in fewer than eight weeks. The appointed adjudicators attempt to facilitate an amicable resolution procedure for both parties to be able to exchange their data and provide their evidence and necessary documents. At the end of the process, the adjudicators will write to the parties, announcing their view on what the fair and reasonable outcome should be. If both parties agree to the suggested solution (which typically happens in around 90% of cases), the dispute is resolved. However, either party may disagree and ask for the case to be referred to an ombudsman for a final and binding determination. The service has trialled new ways of working that will allow some disputes to be settled in an more informal and expedited manner, namely within hours and days. An ombudsman's determinations can be accepted or rejected by the consumer. Nonetheless, if consumer accepts the decision then it is strictly binding. The ombudsman's decision is not appealable, but it is subject to judicial review.

c) Nominet Dispute Resolution Service [83]

Nominet is a domain name registry company operating the '.UK' domain name for the entire United kingdom also the '.cymru' and '.wales' domain names for Wales. Nominet registers the referred .uk domain names on a "first-come, first-served" basis – without examining the merits of the application. It therefore established a Dispute Resolution Service (hereinafter, DRS) to provide a means of resolving .uk domain name disputes without recourse to court. To pursue a claim through the Nominet DRS, complainants should demonstrate that they have rights on an identical or similar domain name and that registration is contradicting their rights. The compliant procedure is conducted through the Online Service within five distinguished stages. As the first step, the complainant has been asked to create an account and complete an online form on the Nominet Online Service tool. [84] The system will then send a copy of the submitted complaint to the registrant of the domain name under dispute through the courier as well as by e-mail. The intended respondent will then have to send the response within a set time frame. If parties do not reach into an agreement over the disputed domain name, then they are automatically directed into the second stage which offers them a free mediation. Mediation is voluntary, thus in case the parties refuse to go through mediation or accepted it but failed to reach to a settlement, then the complainant can ask for appointing an expert – who is an independent adjudicator – to make a decision on the dispute. The expert procedure in DRS is not free and depending on the summary or full decision of the expert it costs either 200 GBP or 750 GBP.

[83] www.nominet.org.uk, (Retrieved 20 November 2019).

[84] https://secure.nominet.org.uk/auth/login.html, (Retrieved 20 November 2019).

The fourth stage refers to the possibility of appealing from the Expert decision to a panel of three experts which cost 3,000 GBP. Finally, rendering the decision by the expert panel results in the closure of the domain name case file and the final decision is published on the Nominet official website.

d) Online Schlichter [85]

The Online Schlichter (hereinafter, OS) is an online mediation service for Business-to-Consumer (B2C) e-commerce and direct selling disputes. It has been established by the Centre for Consumer Protection in Europe (ECC) in Kehl/Strasbourg, Germany since 2009. The primary goal of the OS is to increase access to justice while reducing the number of cases reaching the regular courts. The service is offered free of charge for both parties and the mediators are independent lawyers collaborating with the ECC. The adopted approach by this ODR service provider is essentially based on analyzing the case from the start and providing both parties with legal advice and evaluation of their legal position, whereby correcting any unfounded expectations about their rights. This online advice is partly automated by using textual building blocks and decision trees. This up-front advice and evaluation often helps to achieve early settlement. The mediator provides the parties with non-binding recommendations. The two-thirds of all cases are settled through accepting these recommendations by both parties. Its high settlement rate attests to the success of this technique for small claims. Nevertheless, it should be pointed out that despite the benefits of using the OS for consumers in having access to a more expedited, inexpensive and efficient out-of-court procedure, however the scope of this ODR service regarding its applicability is geographically restricted to merely seven federal states [86] over an entire Germany. This indicates that, only the consumer complaints (from other EU Member States) who are against a trader situated in one of these regions are eligible to benefit from the OS.

e) Rechtwijzer 2.0 [87]

This platform was initially developed as Rechtwijzer [88] 1.0, in 2007, by the Dutch Legal Aid Board to provide assistance to disputants with finding lawyer

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[85] www.online-schlichter.de, (Retrieved 20 November 2019). Retrieved 20 November 2019.

[86] These federal states include, Baden-Württemberg, Bavaria, Berlin, Brandenburg, Hessen, Rheinland-Pfalz or Schleswig-Holstein. It should be noted that, the referred consumer claims from Baden-Württemberg, Bavaria, Berlin, Brandenburg, Hessen, Rheinland-Pfalz or Schleswig-Holstein against a trader situated in Germany are also eligible to use the OS dispute resolution service. For more information see https://www.online-schlichter.de/vorzuege-der-schlichtung/online-schlichter-an-odr-body-for-online-trading, (Retrieved 22 November 2019).

[87] www.hiil.org/project/rechtwijzer (Retrieved 20 November 2019). Retrieved 20 November 2019).

[88] The Rechtwijzer literally refers to 'conflict resolution guide'.

and other legal supports. Since 2014, the new version of this platform was set up under Rechtwijzer 2.0 in cooperation with The Hague Institute for the Internationalisation of the Law (HiiL). [89] The service is established by the Netherlands Ministry of Justice and Security with the aim of helping the parties to resolve disputes through a process that takes them from problem diagnosis, through facilitated, Questions & Answers based framing of their case to problem solving and assisted negotiation and, finally, to various forms of ODR. As regard to providing assistance in negotiation, the process facilitates the automated legal guidance based on answers parties have given during the Questions & Answers session. The Rechtwijzer 2.0 platform began with settling matrimonial disputes, including divorce and ancillary matters, such as child custody and maintenance allowance. [90] However, later it developed its services to landlord-tenant issues and employment disputes, as well. The ADR phase is reached on failure of the parties to reach a resolution by themselves. This takes the form of online mediation or arbitration. The process takes place online on a secure and confidential platform, designed for asynchronous dialogue. The platform enables the mediator to engage in separate confidential discussions with each party, consistent with normal mediation practice. Unfortunately, in March 2017, the Rechtwijzer's partners announced that their cooperation had ended and their project was replaced by a less ambitious Justice42 – programme limited to divorce market – towards Dutch citizens only.

Justice42 took solely some of the HiiL team involved in Rechtwijzer, trying to incorporate the lessons from the Rechtwijzer. [91]

f) Resolver [92]

Resolver is another UK-based online facility that assists consumers in raising complaints with suppliers and retailers. The operators of the site have populated it with the e-mail contacts of the complaint departments of thousands of major organizations. Through a form-filling exercise and considering the provisions of standard phrases, a consumer is given online assistance in drafting the complaint. This is then e-mailed directly to the relevant complaint department. The suppliers and retailers are urged to respond to the Resolver e-mail address so that the exchange of messages can be stored on the consumer's case file that is then maintained on the site. This ODR provider currently covers a wide range of industries

[89] Cortés, P. (2017). The Law of Consumer Redress in an Evolving Digital Market: Upgrading from Alternative to Online Dispute Resolution. Cambridge University Press, pp. 53-54.

[90] Harvey, D. (2017). Collisions in the digital paradigm: Law and rule-making in the internet age. Oxford; Portland, Oregon: Hart Publishing, p. 233.

[91] https://law-tech-a2j.org/advice/goodbye-rechtwijzer-hello-justice42 (Retrieved 20 November 2019).

[92] www.resolver.co.uk, (Retrieved 20 November 2019).

from telecoms, energy, travel, finance and insurance to health and legal services. Resolver facilitates an environment for the parties to have an opportunity to discuss the occurred issues in a structured way. Interestingly, the Resolver provides the parties to use the emoticons helping consumers better express their emotions. The service holds details of the escalation procedures of the thousands of organizations and guides users from first-tier complaints handling up to the highest level. Users are notified through their e-mail to any responses from the other party and are prompted to escalate when responses are not received. The service is free of charge, both to consumers and to the organizations to whom they are using this platform seeking solutions for their disputes.

g) Traffic Penalty Tribunal [93]

The Traffic Penalty Tribunal (hereinafter, TPT) of England and Wales established a web-based portal under the Best Evidence Cloud Knowledge (BECK), to be used by appellants, respondent authorities as well as the adjudicators and administrators. The Portal enables appellants to appeal against a wide range of contraventions from car parking, bus lane, moving traffic to failing to pay other traffic related charges provided that they have been issued by the councils of England and Wales (excluding London). The BECK system enables the appellants to upload evidence and follow cases and hearings under one evidence screen and account. Similarly, each authority has a dashboard showing current cases, enabling them to submit evidence, comment, and follow progress of hearings and decisions. To use the platform, the appellants create an account and receive all notifications through emails. They can comment on evidence, request their preferred hearing type besides following progress of the case through to the decision, viewed online. The status of each case is displayed in dashboard, prompting actions. The TPT administrators, who no longer data-input, are currently more focused on customer service, for example, 'offline' appellants phon–ing for a form or help. Adjudicators can manage their own caseload, send di–rections to parties, and easily see uploaded evidence, including videos, which is also displayed to all parties. At telephone conference hearings all participants can view the same evidence, guided by the adjudicator. Using this service is generally free of charge, however if the adjudicator concludes that the party has unreasonably or frivolously made the appeal, an award for costs can be issued against the appellant.

h) Youstice [94]

Youstice is an ODR service, based in Slovakia, for handling large volumes of low value consumer complaints, relating to goods and services, regardless of whether the purchase took place online or offline. A considerable advantage of

[93] www.trafficpenaltytribunal.gov.uk, (Retrieved 20 November 2019).

[94] www.youstice.com (Retrieved 20 November 2019).

this platform is the services are offered in several languages including English, Spanish, German, Dutch, Portuguese, Slovak, Czech and French. Such multilingualism feature enables a wider range of users from among consumers or trades to benefit from using this ODR service provider. In dealing with the referred consumer claims, the Youstice deploys two types of tools. The first enables negotiation between the disputants. It provides assistance in framing arguments – parties are invited to describe their position by selecting from a series of phrases, with relevant icons for each. The site also suggests suitable solutions that again can be represented by icons. A form of structured (asynchronous) dialogue can occur within a limited area free of charge form comment. The main objective is to encourage the parties and help to facilitate reaching into an agreed settlement directly between themselves. However, by using the second tool, customers can escalate cases and seek an independent review by one of the several neutrals accredited by Youstice. Customers can file their complaints either directly at the retailers' websites or, alternatively, at websites of consumer organizations. Businesses are entitled to use the Youstice logo, provided that they reach into agreement on Youstice with consumers in at least 80 percent of the cases while implemented at least 98 percent of the agreements reached at or decisions by third-parties. Use of the facilitated negotiation platform is free of charge for consumers. Nonetheless, Youstice funds itself from the retailers who pre-register and who display the Youstice logo on their marketing.

VI. The European Online Dispute Resolution Platform

Thus far, this study has discussed the practice of ODR through the established platforms across the world, however, in this chapter the authors will focus on the European ODR platform.

Pursuant to the provisions of Regulation 524/2013 on online dispute resolution for consumer disputes – also known as "ODR Regulation" – the EU has established an ODR platform, with the aim of promoting the accessibility of ADR schemes online. This platform has been operational since February 2016, serving as a single connection point for EU-based traders, consumers, and ADR entities being strictly applicable to online transactions occurring between these parties, at both domestic and cross-border level. [95]

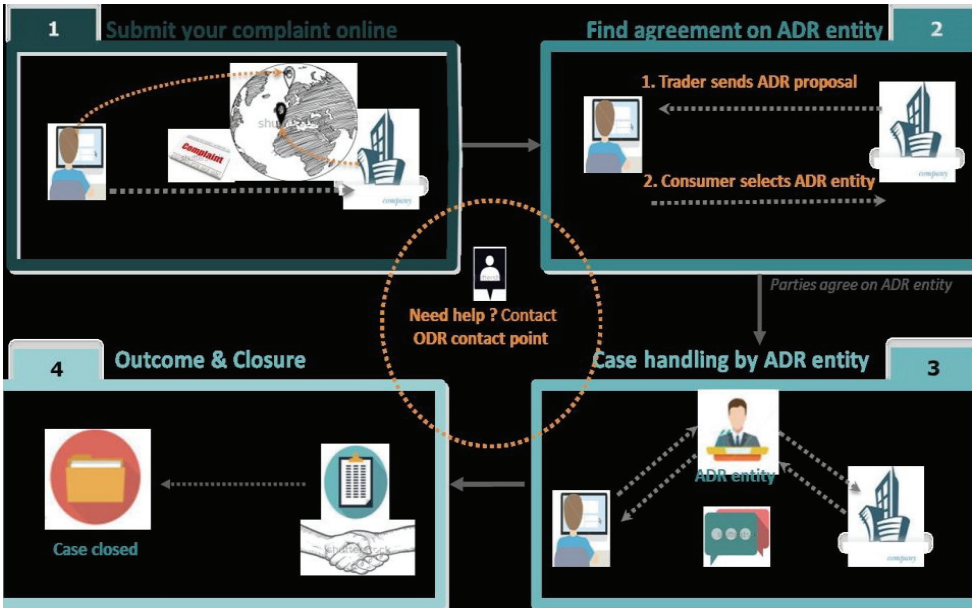
The platform has been made available to users who reside in all the EU Member States and Norway, Iceland or Lichtenstein being accessible in all EU languages including Icelandic and Norwegian.

The EU ODR platform is accessible through: www.ec.europa.eu/odr. It is also

[95] Cortés, P. (2017). The new regulatory framework for consumer dispute resolution (First ed.). Oxford: Oxford University Press, p. 5.

possible to find more information on the following website: www.ec.europa.eu/consumers/solving_consumer_disputes/non-judicial_redress/adr-odr/index_en.htm.

Figure 1. [96]



As shown in Figure 1 above, the main role is the one of the national contact points established in each Member State to provide assistance to users of ODR platform, a network [97] of online dispute resolution facilitators ("ODR facilitators' network"). This network is composed of contact points for ODR in every Member State which hosts online dispute resolution facilitators. Such facilitators should provide support to the resolution of disputes relating to complaints submitted via the ODR platform. They host at least two advisors and have access to the data of the submitted data on the occasion that anyone ask for their assistance. The role of the advisors should not be merely looked at in the capacity of assisting the disputants in case of problems. However, advisors also play a significant role in disseminating the opportunity for consumers and traders to use the achieved solution

[96] Image from the EU ODR platform.

[97] See article 7, Regulation (Eu) No 524/2013 of the European Parliament and of the Council, 21 May 2013, on online dispute resolution for consumer disputes and amending Regulation (EC) No 2006/2004 and Directive 2009/22/EC (Regulation on consumer ODR).

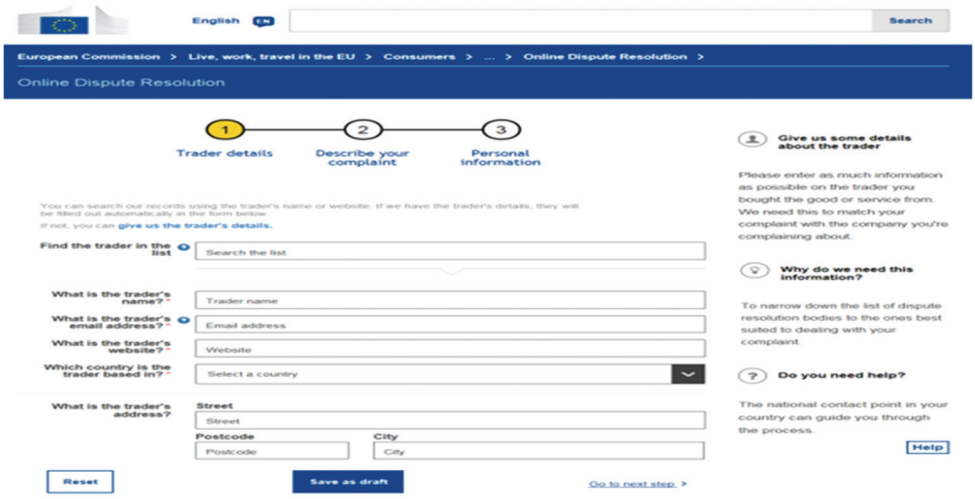
instead of going to the trial resulting in reducing the huge caseloads of the courts.

The complaint procedure of the EU ODR portal is essentially based on the four main stages.

1. *Submitting a Complaint*

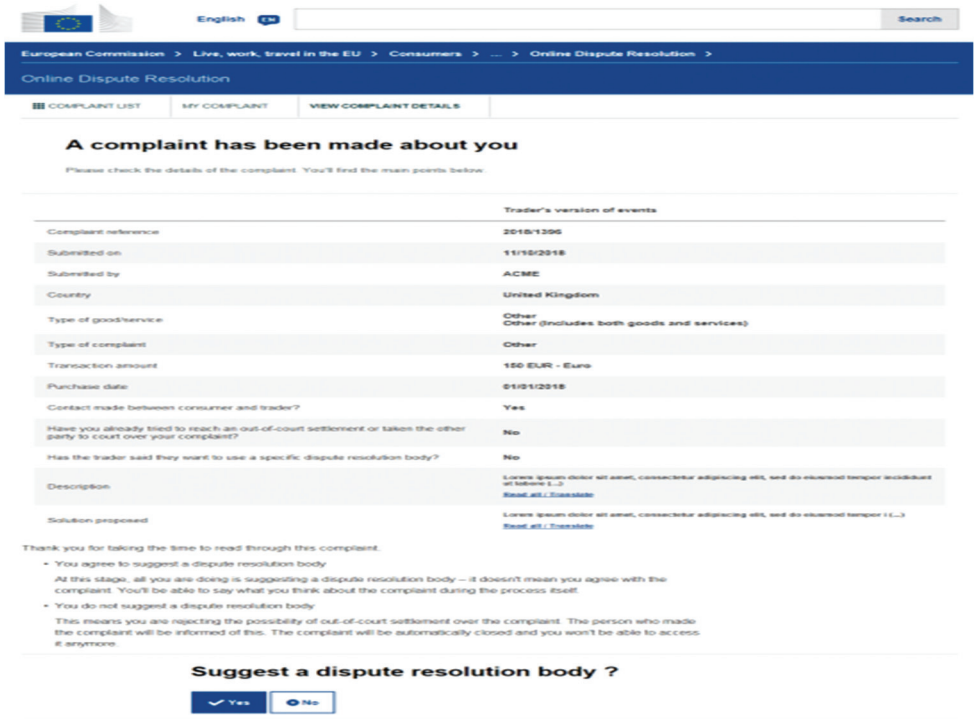
As illustrated in Figure 2, the consumer fills in the online complaint form and submit it to this platform. The other party will receive it. The recipient party will now need to agree with the other party on the dispute resolution body that will handle their dispute. They have 30 days to agree on solving the occurred dispute online through the ODR platform. If both parties agree, the complaint will be sent to the other party.

Figure 2. [98]



[98] Image from the EU ODR platform.

Figure 3. [99]



2. *Agreement on Dispute Resolution Body*

A party has 30 days to agree with the other party on the dispute resolution body that will handle their dispute. Once agreed, this site will automatically send the details of their dispute to that body. If a party cannot agree, the complaint in question will not be processed further.

[99] Image from the EU ODR platform.

Figure 4.[100]

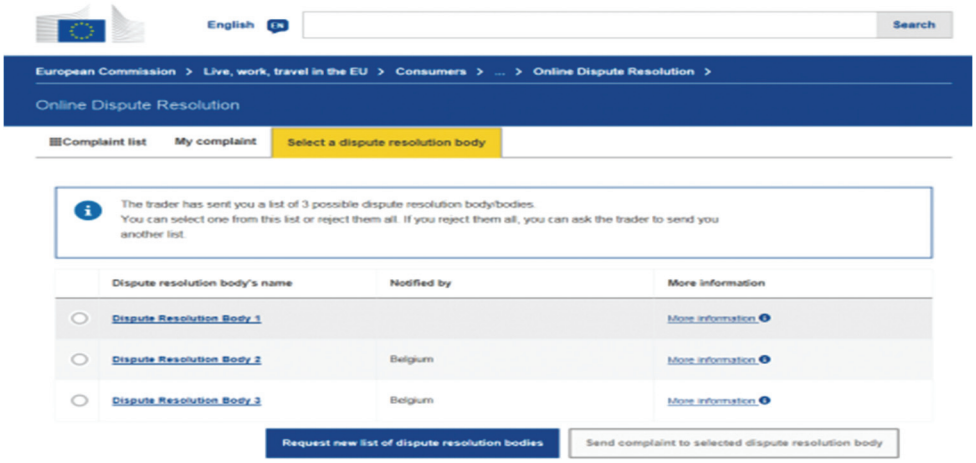
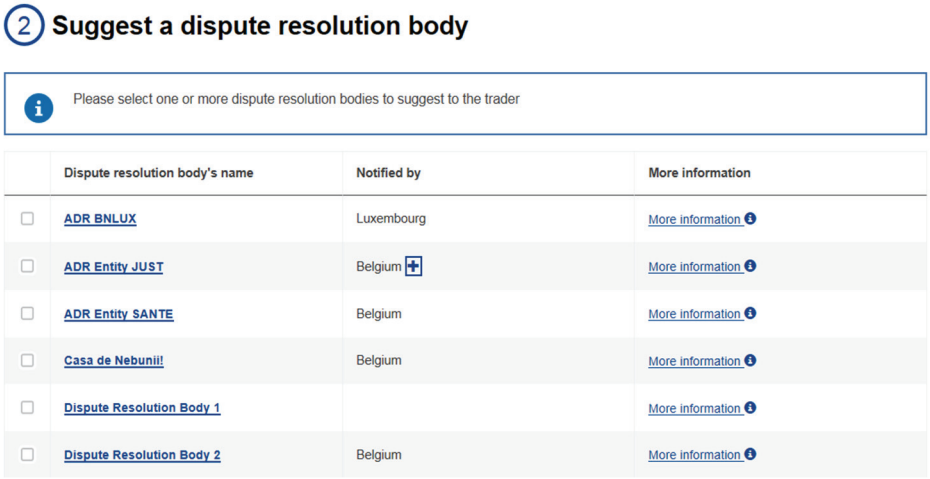


Figure 5.[101]



3. *Handling the Complaint by the Dispute Resolution Body*

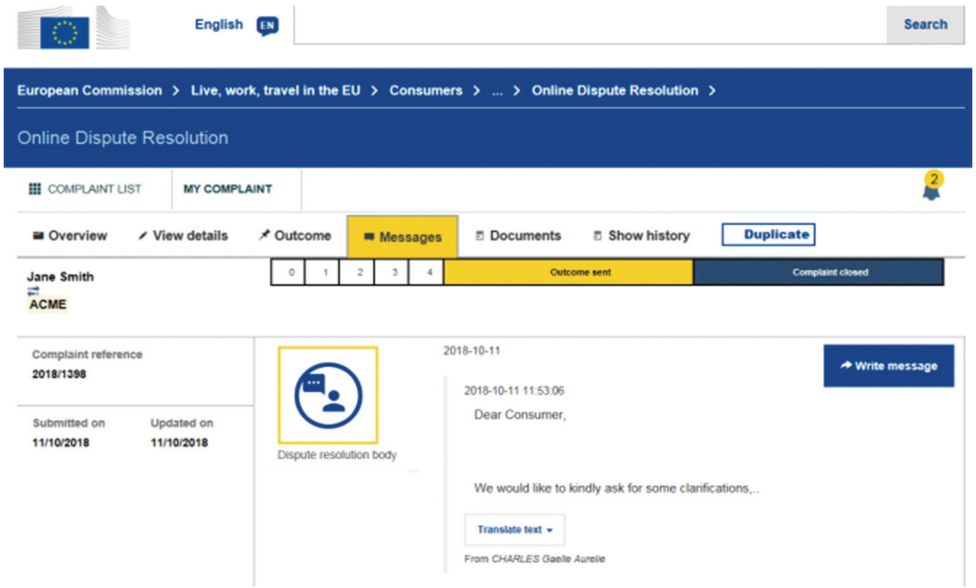
Once the dispute has been sent to a dispute resolution body (hereinafter, DRB), this neutral third party will get back to the disputants within three weeks to

[100] Image from the EU ODR platform.

[101] Image from the EU ODR platform

inform the parties whether this entity will handle the submitted compliant. However, during this time frame, the third party may ask the consumer and trader to provide them with more information or documents on the case. If the DSB agree to handle the complaint, the decision should be taken within the 90 calendar days. It is important to note that, the DRB may either use the EU ODR platform or alternatively ask the parties to use the DRB's own system for further communications. Nonetheless, either way, the ultimate outcome will be also accessible on the EU ODR website for both parties.

Figure 6. [102]

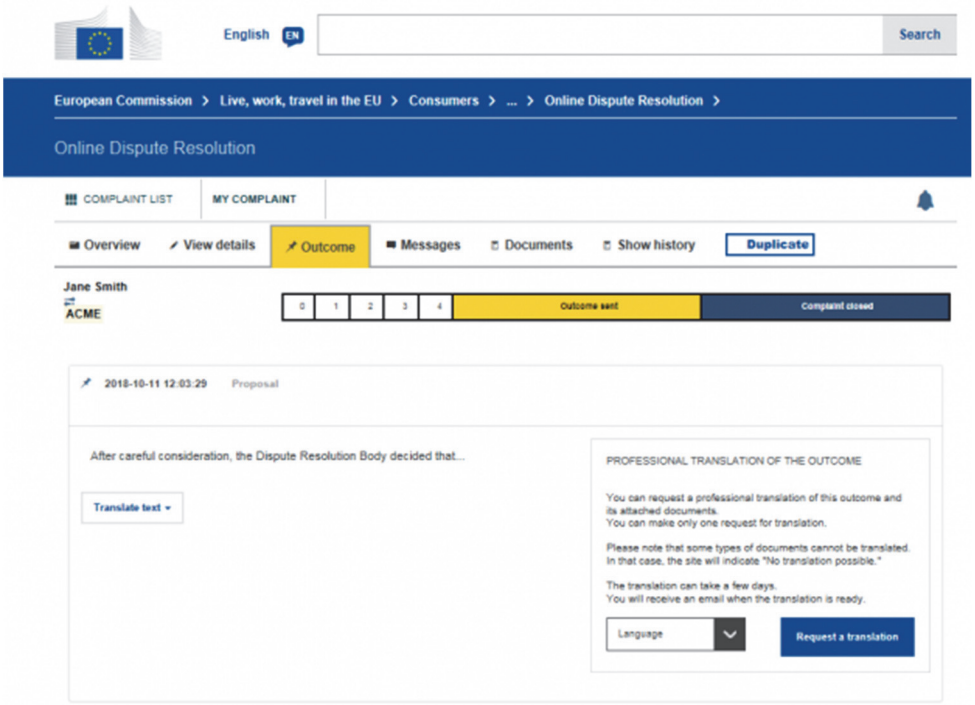


4. *Outcome of the Procedure*

Once the procedure is over, the dispute resolution body will inform the parties of the outcome. This outcome varies per dispute resolution body.

[102] Image from the EU ODR platform.

Figure 7.[103]



Conclusive Remarks

The purpose of this study was to investigate into the concept of Online Dispute Resolution and its application to civil disputes. Thus, this paper discussed the ODR as a raising phenomenon which its engagement by the law may not be straightforward given some of the properties that define it. It was argued that the ODR, as a concept, is closely linked to other alternative dispute resolution methods such as Mediation and Negotiation. It has a number of properties, however, that will likely make its engagement (especially in family disputes and e-commerce) necessary and not just alternative to the existing ordinary dispute resolution methods.

In terms of the proper approach towards ODR, the authors considered ODR systems as a place – where to settle – rather than also the process – how to settle – or both of them. This approach helps to include the benefits of developing ODR

[103] Image from the EU ODR platform.

systems such as avoiding, on the one hand, the matter of whether a specific court has jurisdiction or not over the dispute and, on the other hand, to evade expensive traveling costs.

This study also argued that E-Negotiation and E-Mediation methods are the most advanced systems, with all the differences between their correlative offline figures: negotiation and mediation. As regard to E-Negotiation, the fork roads of Automated Negotiations and Assisted Negotiations were analysed while focusing mainly on the automated ones, where the platforms already applied algorithms for solving disputes, such as the Adjusted Winner in Fair Outcomes, Inc. (www.fairoutcomes.com).

Within the last two sections of this study, the authors discussed several developed ODR platforms and the new European portal for ODR. However, one should bear in mind that, the only platform which was operating until March 2017 within a civil law system is Rechtwijzer 2.0 in the Netherlands as an assisted negotiation portal. The current available platforms are all operating within the Common law context, where there exists a different conception of resolving disputes. Thus, in terms of direction for future research, incorporating the lessons from the Rechtwijzer, further experiments could shed more light on the EU civil law context, both at a supra-national and national level.

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Appendix I. – ODR Service Providers – EU

Name	Date of creation	Date of death	Type of dispute:	The offer (what the plateform offers/any threshold?)
ADRoit3	2004	Closed-France (date of end is not clear)	International banking and finance	ADR to international Banking and Financial Services community
An Olive Branch	2015	Active – Ireland	a wide variety of disputes from commercial to employment and communications/relationships	Mediation – Arbitration – Facilitation
Chamber of Commerce of Ancona	2015	Active – Italy	Claim reinbursements	Forms are available to be directly submitted either by post or e-mail to the Chamber
CMAP France	1995	Active – France	Commercial, consumer claims, employment	Mediation, Arbitration, Amicable expertise
e-just law	2019	Active – Switzerland/France	commercial disputes	Mediation – Arbitration
e-Justice Europe	2016	Activ e- EU	Claims for online purchases of goods and services	Online platform to submit the claims online to ADR bodies in EU MSs
EUCON	2006	Active – Ukrain/Poland	Varies from commercial to labour disputes	Mediation – Arbitration
Fast Arbitre	2016	Active – France	Commercial, professional disputes	Amicable phase – Arbitration phase
Iudica	2019	Active – Germany	cryptocurrency, ICO, e-commerce	virtual arbitration court
Mediation Room (Resolver)	2002	Active – UK (Founded by Graham Ross)	shareholder, IT developments, consumer, business acquistion disputes	Mediation
Mediation-Now	2007	Active – UK	Family disputes	Neotiation, mediation, arbitration (assisting separating couples with communication regarding their children, finance, property, divorce/separation)
Money Claim Online (MCOL)	2002	Active – England and Wales	Monetary claims	A platform to claim the money online, threshold is up to 100.000 GBP

Name	Date of creation	Date of death	Type of dispute:	The offer (what the plateform offers/any threshold?)
Rechtwijzer Netherlands – De Geschillencommissie	2011	Active – Netherlands	wide variety of consumer and business disputes	dispute committee based decisions – claims with values up to 10,000 euros
RisolviOnline	2015	Active – Milan, Italy	commercial disputes	Mediation- Fcilitation – Evaluation
Square Trade	1999	Active – US/UK/Finland	Electronic devices (phones, tablets, game tools etc.)	the threshold is up to the amount of insurance contract.
UK Claims Portal	2010	Active – England and Wales	personal injury claims	A platform for claiming personal injury compensations, thereshold of 1000 to 25000 GBP
Youstice	2014	Active – Slovakia	Consumer disputes	ODR processing with neutral, low-value claims

Appendix II. – ODR Service Providers – non-EU

Name	Date of creation	Date of death	Type of dispute:	The offer (what the plateform offers/any threshold?)
Agree Online	2016	Active – Israel	Children's educational related disputes	Mediation for preparation, resolution and prevention suits in disputes
American Arbitration Association – AAA	1996	Active – US	Wide variety of disputes from commercial to labour	Mediation, arbitration
Arbiclaims	Apr-15	Active – US, LA	commercial disputes	Arbitration – Settlement
Arbitranet	2016	Active – Brazil	commercial disputes	Arbitration
ARS (ARBRESOLUTIONS)	2012	Active – US	wide variety of commercial and non-commercial disputes	Mediation – Arbitration without any value limit
Asian Domain Name DR Centre – ADNRC	2002	Active – Asia	Domain name disputes under ICANN policies	Arbitration
Brav	2006	Active – US	early stage conflict resolution	Mediation – Arbitration
Caseload Manager	1996	Active – USA	commercial and individual disputes	complete case management organisation
Conflicteam	2016	Active	financial, divorce, partnerships, payments &debts, product	Mediation, arbitration, negotiation
ConflictResolution.com	1998	Active – USA	commercial and individual disputes	Mediation, abritration, conflict strategies, dispute management, ADR consulting
Consensus Mediation	2018	Active – Canada	Family disputes	Mediation – Conflict resolution plan
Cybersettle	1996	Active – US	Monetary disputes	e-negotiation
eConciliator	2014	Active – Brasil USA Spain	claims, debts, any type of disputes	e-negociation

Name	Date of creation	Date of death	Type of dispute:	The offer (what the plateform offers/any threshold?)
HKIAC = ADNDRC	1985	Active – Asia	domain name disputes	Arbitration Mediation Domain Adjucation
ICANN Ombudsman Office	1998	Active – US	Disputes arising at ICANN website	Ombudsman through impartial mediators
ICOCR, e-Court	2016	Active – Canada	Consumer disputes	Mediation, arbitration and legal opinions
Immediation	2019	Active – Australia	Commercial disputes	Mediation,arbitration, evaluation and determination
International Institute for Conflict Prevention & Resolution (CPR)	established in 1977	Active – US	Commercial,Civil and employment disputes	Mediation, Facilitation, Arbitration, Early nutreal evaluation
It's Over Easy	2017	Active – US	Online divorce	Online divorce website to dissolve marriage through this platform
JAMS ADR	1979	Active – US	Wide variety of disputes from family and commercial disputes to construction and insurance conflicts	Mediation, Arbitration, Nutreal evaluation
Matterhorn	2014	Active – US	Civil disputesincluding small claims and family disputes, warrant prevention, traffic disputes	Providing ODR services for public and judiciary organizations
Modron	2013	Active – Australia	This platform functions as a medieum in providing human-centric dispute resolutions for professionals/entities through innovative and technological methods	ODR platform – Providing DR technology for judiciary
MyLaw BC	2019	Active-Canada, BC	Family disputes – separation plans and agreements	Prodiving an e-dialogue tool

Name	Date of creation	Date of death	Type of dispute:	The offer (what the plateform offers/any threshold?)
National Centre for Tech and DR (NCTDR)	1998	Active – US	Wide variety of disputes from commercial to social and civil disputes	Negotiation, mediation, arbitration
Net Arbitration	2005	Active – US	B2B and B2C disputes	Arbitration for small claims up to 10,000 USD
Net Neutrals	2014	Active – US	disputes referred by e-bay	To imply ODR by Reviewer feedback on dispute to settle it
Online Arbitration	2014	Active – US	Insurance reimbursement	Arbitration
Rapid Rulings	2016	Active – US	Any legal dispute	This platform provides certain legal dispute resolution through online communication for the parties.
SettlementIQ	2015	Active – US	Business disputes	ODR for insurance, debt recovery, B2B and ADR services
SettleToday	2005	Active – US	Various disputes from contracts to tenency and real propert etc.	Settlement through facilitation
SmartSettle	2012	Active – Canada	Family conflict resolution	Mediation – Facilitation – e-negotiation through the platform
The Internet Ombudsman	2014	Active	Business & public sector disputes	ombudsman evalution investigation, conflict resolution & cross cultural issues, sport ADR
The Virtual Magistrate	1996	Closed – US	N/A	Electronic arbitration
TylerTech	1966	Active – US	ODR service provider	Software, Services, Implementation, and Consulting
Virtual Court House	2001	Active – US	various areas of disputes	Mediation – arbitration – nutreal evaluation expert- case management services

Name	Date of creation	Date of death	Type of dispute:	The offer (what the plateform offers/any threshold?)
WIPO ADR Centre	1994	Active-WIPO Switzerland	International Commercial Disputes in IP matters through eADR	Mediation – arbitration/expedited arbitration – expert determination and domain names disputes

Evangelia Nezeriti *

Eventual restrictions and effective use of algorithms in civil law matters **

I. Introductory remarks

This study aims to point out the – rather underestimated – methodological restrictions on the use of algorithms emerging from the particular structure of provisions regulating civil law matters. Conversely, the much-discussed mandatory rules constitute no significant obstacle for the implementation of algorithms in the justice system.

It should be underlined that the following analysis regards algorithms (weak artificial intelligence), in other words software elaborating its decisions exclusively on the basis of specific commands, already determined in advance by the developer. Undoubtedly the technically further evolved "autonomous agents", as they are called, can be effectively used in a larger range of disputes and achieve more fascinating and varying results, since they are designed to act creatively and independently. However, it is admitted that neither the process nor the outcome of their decision making can be fully predicted, a fact that will probably lead to greater legal uncertainty and enhance the existing mistrust against justice.

II. Restrictions regarding the use of algorithms

Two main sorts of restrictions set the frame for the application of algorithms: First, various methodological restrictions, and, secondly, restrictions stemming

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** This publication consists part of the writer's homonymous oral presentation that took place in Athens in June 2019. The examples cited, based on the Greek civil legislation and deriving from Greek court practice, are limited only in the fields of family and hereditary law in compliance with CREA's targeted application sectors.

from the existing mandatory rules, principally those governing the relationship between the (ex-) spouses or the co-heirs.

1. Methodological restrictions

Not all civil law disputes are befitting to an automated arrangement. First and foremost, matters concerning the personal regime of a married couple or the exercise of parental custody are to be excluded from algorithmic procedures, inasmuch as they refer to aspects of the personality right or affect the interests of children. Therefore their regulation should be reserved to the private autonomy of the parties involved; the latter must be allowed to decide exclusively according to their personal conditions and wishes [1].

Furthermore, a computer program cannot undertake the interpretation of contracts. Finding out of the meaning of an agreement is an open process, not subject to formalization, whose outcome depends largely on the special features of the contracting parties as well as the circumstances accompanying the contract. Therefore, crystallizing rigorous interpretation parameters – compatible to automated and massive use by a smart program – is not possible. The aforementioned inability of setting concrete criteria applies *a fortiori* in case of interpreting wills, a domain where the aim of giving effect to the intentions of a particular testator prevails completely, opposing any kind of general rules or indications [2].

In addition, artificial intelligence cannot handle the task of specifying indeterminate legal concepts. It is a common practice that in drafting legal provisions legislators tend to use legal terms whose meaning is not totally preconceived (e.g. the concepts "good faith" or "principles of morality"), but is defined partly *in concreto,* in other words in accordance to the circumstances of the case under consideration, so as to ensure fairness and equity when the law is applied. It is needless to mention that algorithms would face insuperable difficulties if called to apply such "agile", beyond any standardization, rules, which nevertheless facilitate the reconciliation of conflicting interests by the judge. To the extent, subsequently, that the effective interpretation and application of the law require "creative interventions", algorithms cannot offer useful services. So a subtle and complicated task can only be performed by human beings, who are naturally gifted with flexibility and adaptability.

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[1] For example, the name and surname of the children must be chosen by their parents and not selected by a computer.

[2] For this reason, in case of wills the principles of the so-called "subjective interpretation" are to be adopted. See *Papanikolaou*, Methodology of private law and interpretation of legal acts, Athens 2000, No 474, 524; *Stathopoulos*, in Georgiades/Stathopoulos (eds), Commentary of Greek Civil Code, 2nd ed. 2016, combined articles 173 and 200 GCC (= Greek Civil Code), No 131 ff.

2. Consist mandatory rules a real obstacle for the use of algorithms?

Taking into account all the abovementioned methodological limitations, one can draw the conclusion that algorithms are suited for resolving legal problems only in cases in which there is no claim for an evaluative decision. In particular for the purpose of dividing goods, specially designed distributive platforms are likely to be proven as precious assistants, due to their unique capability of operating complicated calculations with accuracy.

On the contrary, mandatory rules do not block or discourage the integration of algorithms in the justice system. Since they embody the most fundamental principles that govern a certain legal system, they are usually formed by the legislator with clarity and precision, attributes which enable their potential digital transcription, i.e. their transformation in accurate commands, easily executable by smart programs. Besides, the hardest phase regarding *ius cogens* is undoubtedly its characterization as such, in other words the estimation whether a provision expresses a primary evaluation of a legal order not to be put aside by private agreements. But this is a process prior and irrelevant to the use of algorithms. Moreover, one must bear in mind the fact that mandatory rules are to be respected in any litigation. In this sense, they inevitably limit the discretion of every "judge", mechanized or not.

III. Examples of effective use of algorithms for the purpose of settling arguments between individuals (with emphasis on distribution of assets)

1. Solving preliminary problems

One should not underestimate the difficulties confronted by judges, mediators or arbitrators when called to ascertain whether the disputed right has actually been infringed or the disputed claim is substantiated, so as to decide subsequently on the remedy indicated. This applies in particular for rights consisting in a share of another person's estate. Confirming whether the assets already allocated by the obligor to the entitled person equalize its legal portion (e.g. its portion of the deceased's property) demands a complex sequence of calculations and comparisons. The latter can be effective and reliably performed rather by algorithms than human minds. Here are some typical examples:

a) Is one's claim to a compulsory portion been infringed?

Wills often contain various complicated distributive provisions, especially when the inherited estate is of significant value and there are quite a few heirs designated by the testator. In such cases, moreover, the deceased previously, namely during his/her lifetime, usually disposes gratuitously some of his/her as-

sets to the compulsory heirs, principally to his/her descendants. The abovementioned donations and gifts are also to be included fictively in the inheritance, so that the calculation basis for the reserved portions is broadened in favor of the obligatory heirs. As a result, when a will is combined with donations *inter vivos*, one faces serious difficulties in determining whether the goods already obtained by compulsory heirs are equivalent to or fall short of their claimable compulsory share. A smart computer program, specially designed for this scope, can accomplish efficiently and rapidly all the evaluations required, provided that it is equipped with a wide range of data needed for the precise assessment of values (e.g. prices per square meter regarding immovable property). If it turns out that the goods inherited or gifted are inadequate compared to its legal quota, then the beneficiary is entitled to overturn some or all donations that took place prior to the devolution of the inheritance. Otherwise his/her claims will be considered satisfied and no further action will be taken.

b) Can an equalization claim to accrued marital gains be established?

Defining the extent of the justified participation of a former spouse to the accrued marital gains of the other can be proven even harder than estimating the amount corresponding to compulsory shares, in particular when both the compared properties of the spouses have increased during the marriage. We should bear in mind that in such an eventuality, according to the view prevailing in Greek legal literature and jurisprudence, each spouse is entitled to a separate equalization share; the reciprocal claims of the spouses are subsequently set-off to the extent that they correspond with each other[3].

Apart from the highly challenging duty of evaluating two separate estates at two different dates, namely both at the time a marriage is concluded as well as at its ending, judges are often called to estimate the value of immaterial services offered by the claimant, such as keeping the household or upbringing children, which facilitate the undistracted exercise of professional activity on behalf of the other party and thus contribute to the increase of his/her income. One must also take into account that the statutory presumed amount of contribution to accrued marital gains can be contested by the claimant; the latter enjoys the right to prove that his/her involvement in the enrichment of the ex-spouse's property is wider[4]. All the above-described factors considered, it is no wonder that trials concerning accrued marital gains usually last for a long period of time. The intervention of an algorithm capable of operating accurate evaluations, deductions and setting-offs would considerably reduce the complexity and duration of such litigations.

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[3] See, inter alios, *Apostolos Georgiades*, Family Law, § 12 No. 30; *Lekkas*, in: Synoptic Commentary of Greek Civil Code (editor: *Ap. Georgiades*), article 1400 No 8.

[4] According to article 1400 of Greek Civil Code (henceforth citated as: GCC), the statutory presumed contribution amounts to one third of the property growth that took place during the marriage.

2. Algorithmic termination of disputes

As it has been repeatedly emphasized in this paper, algorithms are mostly suited for resolving disputes demanding complicated calculations and correlations, especially disputes related to partitioning of goods or defining the amount of periodical payments. From the wide range of civil law matters relevant to distribution of assets or judicial definition of payable amounts we will focus on: a) the dissolution of the formed partnership between co-heirs by division of inherited assets and b) the specification of the maintenance amount due to the children by each of his divorced parents.

a) Distribution of the inheritance among the co-heirs

In the event of absence of a (valid) will, provided additionally that the deceased during his/her lifetime did not stipulate an asset distribution with his/hers descendants [5], the inheritance passes under the rules of intestate succession. The inheritance may include material and immaterial assets, as well as enforceable claims against third parties. If more rightful heirs exist, the inherited assets become a joint acquisition of the co-heirs, who form a community. Every co-heir is entitled individually to request at any time the partitioning of the joint estate, thus dissolving the existing community. This is a common process because the management or disposal of jointly owned assets is subjected to significant limitations and conditions; this interdependence often leads to hostility between the co-heirs, though. For this reason, co-heirs try to make themselves exclusive owners of certain inherited goods through a consensual or judicial partition. The latter is ruled by the provisions concerning co-ownership, as well as those regulating the procedure of judicial distribution (articles 478-494 of Greek Code of Civil Procedure).

Under the circumstances described above the aid offered by an "automated distributor", skilled at achieving rational and impartial compromises, is decisive, especially since it is extremely common that the co-heirs' selections coincide; in other words they ask to obtain the very same goods, primarily the most valuable ones.

The division carried out by platforms should be based on both objective and subjective parameters. Obviously, all crucial mandatory rules regarding intestate succession are to be inserted in the program (for example, the provisions governing the degrees of intestate succession, their ranking, the shares corresponding to each degree, the right to a compulsory portion, the right to disclaim the inheritance and, naturally, the rules concerning initial joint acquisition of inherited goods by the co-heirs) [6]. In addition, the developer must enter a wide range of

[5] So an allocative agreement, effected *post mortem*, is valid solely when concluded with one's descendants (articles 1891 ff. GCC). All other types of inheritance contracts are prohibited and thus void (see article 368 GCC).

[6] It must be pointed out that admissible ways of judicial distribution are usually stipulated ex-

measuring information, which guarantee the accurate determination of the inherited estate's value.

On the other hand, personal wishes and features of the parties participating in the division should also be taken into consideration. For this purpose the co-heirs should prior to the division be asked to declare which objects of the inheritance they would preferably receive. Further particularities must also be weighed (if, for example, one of the co-heirs already possesses land adjacent to the deceased's property, it is only fair that a priority right on this specific asset is acknowledged in his/her favor).

The application of objective and subjective criteria, combined with the digital character of the algorithm's judgement (and therefore unsusceptible to any kind of pressure), secure that an equitable and unprejudiced settlement of the dispute is easily reached, without grounds for justified objections on the part of the heirs.

Finally, assets whose allocation fall into the scope of the court's equitable discretion [7] should rather be excluded from an algorithmic elaborated partition and remain reserved to weightings made *in concreto* by a human judge, arbitrator or mediator, capable of delicate adjustments.

b) Distribution of the accrued marital gains

The abovementioned remarks regarding the division of the inheritance also apply, *mutatis mutandis*, in case of assigning goods to the spouse entitled to participate in the increase of the other spouse's estate [8]. A potential distributive platform must comply with the relevant mandatory rules, contain evaluative data and take into consideration, to the extent possible, the personal preferences of the former spouses.

c) Specification of the children maintenance

Separation of belongings is not the only issue entailing perplexed calculations and thus likely to arise serious disagreements between the interested parties. Equally disturbing is the determination of periodical payments, when the sum due is not predetermined by the law but varies in accordance to the special conditions concerning the obligor and the obligee. Defining the maintenance amount owned to minor children on part of their divorced parents is a typical example thereof. For the needs of an algorithm destined to estimate an equitable maintenance, the

haustively by the law (see articles 480 ff. of Greek Code of Civil Procedure). In that sense, the partitioning process is not carried out freely, according to the interests and wishes of the parties, but is also subdued to compulsory provisions, which therefore must be applied by the algorithm in charge of the division.

[7] For example, article 1889 GCC provides that the court may assign the use of the matrimonial home exclusively to the surviving spouse of the deceased, taking account of his/her best interests.

[8] According to Greek civil law, the obligation of the enriched spouse to render part of his accrued benefits to his ex-wife/her ex-husband consists a pecuniary obligation, without prejudice of the parties' right to decide otherwise (agree, e.g., on a satisfaction of the claim *in natura*).

relevant –rather rigid– legal framework must be firstly digitized accurately.

As the legislator's main concern is to secure a decent living for the children, without risking however the maintenance of the parents, a detailed compulsory regulation, limiting considerably private autonomy, is introduced [9]. More precisely, both parents are obliged to provide maintenance for the children, as long as their remaining income is considered sufficient to cover their personal needs. If both parents fulfill this condition, their share is subsequently determined on a *pro rata* basis, taking into account their financial status (current earnings as well as property). Only if the child is considered self-sufficient (because, for example, he rents property inherited from his grandparents) are the antecedents released from the obligation to pay any allowance [10]. The maintenance, which must correspond to the child's current standard of living, comprehends all the necessities of life (alimentation, clothing, housing, etc), including the costs for education. The amount due is paid every month in advance, while summing up periodic payments is forbidden.

All the aforenamed conditions are often being contested by parents, mainly their ability or inability to offer maintenance, the current needs of the child in question, and the extent of their obligation to contribute to the child's expenses. Confirming the parents' ability to support the child (and its limits), consists a real challenge for every mediator involved in a reconciliation effort; the latter should practically engage himself/herself in an endless series of evaluations, risking meanwhile the child's prosperity. Conversely, by replacing a human by a computer program precious time is saved and possible miscalculations are avoided. The structure of such an estimative platform would be similar to the one designed to divide assets. In other words, it should contain, apart from the crucial mandatory provisions, data necessary for the appreciation of the current economic status of the divorced couple, on the one side, and the needs of the children, on the other side. At the same time both parents must state the minimum and maximum amount they are willing to spend for the sake of their children.

IV. Conclusion

The list of disputes cited above is purely suggestive. One may add other occurrences necessitating either division of goods or quantitative specification of claims [11], such as the partition of the jointly used mobile goods in the event of di-

[9] In general, and notably in Greek legislation.

[10] It should be underlined that minor children are not obliged to liquidate their personal assets in order to preserve themselves, unless their parents are judged indigent based on the criteria set by the law.

[11] Provided, of course, that the right or obligation in question fall into the dispositive power of the parties.

vorce or separation of a couple, the termination of common property regimes or even the administration of an inheritance in favor of the existing creditors. In any case, from the few examples mentioned in this short study one concludes beyond doubt that an algorithm's capacities for legal aiding are multiple and must be exploited appropriately. Mandatory provisions do not in any way hinder the automated elaboration of legal solutions, as long as they are carefully detected and subsequently coded in an executable and accurate way. Under this perspective, in the current phase of development and pilot implementation of algorithms one should rather focus on the nature of the disputes submitted to mechanized settlement, selecting those governed by rigidly formed provisions, namely provisions not containing indeterminate legal concepts or other "mobile" elements [12]. Flexible rules of this type are not destined by the legislator for immediate, general application, but require prior axiological estimations and adjustments, skills inherent mainly to human intellect. Algorithmic exercised discretion is, however, not to be excluded, provided that it takes place on the basis of measurable criteria, perceivable by artificial intelligence. Allocating assets consists, therefore, the most suitable field for the use of algorithms for the purpose of judicial or voluntary arrangement of private affairs.

One last conclusive thought must be added at the closing of this brief analysis. One must not underestimate the widespread mistrust towards judges, arbitrators and generally individuals mandated to resolve private disputes. Introducing in the judicial system electronic tools, not reacting to bribery or other means of persuasion, illustrates a perfect opportunity for reestablishing justice's lost prestige. Inevitably, of course, other complains, concerning this time massive and thus inequitable reconciliation of opposing interests, will arise. In the final analysis, the permanent disapproval of a third party's intervention (mechanized or not) in private affairs must be contemplated as an instinct reaction to the prohibition of self-redress mechanisms, which therefore is not likely to be eliminated in the future.

[12] For the concept and the function of the so-called "mobile system of legal elements" ("bewegliches System von Elementen") see *Walter Wilburg*, Entwicklung eines beweglichen Systems im bürgerlichen Recht, Graz 1950.

Irene Kalpaka *

European Union's Ethical and Legal Framework for Trustworthy Artificial Intelligence **

Abstract

Nowadays, algorithms are increasingly used to support decision-making, inter alia, in the field of civil justice. However, their use raises some questions concerning their efficacy to achieve the result sought as up to now algorithmic transparency and technical accountability are not always feasible. Therefore, some main guarantees should be established in order to avoid faults and fallible results.

European Union's existing ethical and legal framework can play a decisive role towards that aim. And as trustworthy artificial intelligence is a continuous process, a new European legislation should be enacted that would be capable to maximize the benefits of algorithms and minimize their risks.

1. Introduction

Artificial intelligence is the aim of scientific methods and techniques to reproduce the cognitive abilities of a human being by a machine [1]. It is distinguished between strong artificial intelligence and weak but only the latter is feasible nowadays as the scientific progress is not in that level in which the machines could function in a completely autonomous way.

Examples of weak artificial intelligence are the algorithms. The latter can, inter alia, be applied in the field of civil justice and its environment, for instance in a dispute resolution through mediation. They play a decisive role in the execution

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** Delimitations: The present document concerns only the automated/algorithmic decision-making systems for conflict resolution in civil law matters.

[1] There is not yet a commonly accepted definition for artificial intelligence.

of the mediation's process and contribute to its implementation, as they solve specific problems through specific range of actions.

However, their use raises some questions concerning their efficacy to achieve the result sought in each case as it is not the ingoing data that prescribe the outcome but the way that the data are analyzed by the algorithms. We can understand the input and the output but not what goes on inside, namely between these two stages [2]. In other words, which is the process behind the algorithms' decision? They elaborate the input in a way that comes out the output? If not what is the decisive factor that leads to the output? And is the latter fairly or not?

Up to now, it is not always possible to create a tool in order to understand how the system worked, what reasons are behind each decision and what actions are taken and why (algorithmic transparency and technical accountability).

However, it is feasible to define the "undecidable" problems, namely those questions for which are impossible to create an algorithm that always gives a correct answer. This is the so-called halting problem which looks for the limit of what can be computable [3].

Therefore, some main ethical and legal guarantees should be established in order to maximize the benefits of artificial intelligence and minimize its risks.

2. Ethical Framework

The European Union in order to be faithful to its cultural history, it should develop a "human-centric" approach of artificial intelligence that respects the European values and principles.

The European Commission's High Level Expert Group on Artificial Intelligence (hereinafter mentioned as AI HLEG) published ethic guidelines for trustworthy artificial intelligence [4] according to which European Union should promote the creation of a trustworthy artificial intelligence that fulfills three characteristics: It is lawful, ethical and robust.

The document of AI HLEG provides insight on how artificial intelligence's systems (hereinafter mentioned as AI systems) could become ethical and robust.

The fundamental rights as they are defined in the EU Treaties and the EU Charter of Fundamental Rights provide guidance on what technology should do in order to be ethical.

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[2] Frank Pasquale, *The Black Box Society: The Secret Algorithms that Control Money and Information*, Harvard University Press, 2016.

[3] Deven R. Desai and Joshua A. Kroll, *Trust but Verify: A Guide to Algorithms and the Law*, Harvard Journal of Law & Technology, Vol. 31.

[4] *Ethics Guidelines for Trustworthy AI*, Independent High-Level Expert Group on Artificial Intelligence, Set up by the European Commission, European Commission, Brussels, 8 April 2019.

AI HLEG sets the families of rights that are appropriate to cover that field:

- Respect for human dignity, Freedom of the individual, Respect for democracy, justice and the rule of law, Equality, non-discrimination and solidarity including the rights of persons at risk of exclusion, Citizens' rights.

From the aforementioned rights the AI HLEG derives four ethical principles:

- The principle of respect for human autonomy, The principle of prevention of harm, The principle of fairness, The principle of explicability.

Moreover, the AI HLEG points out that all these abstract rights and principles should be converted into a concrete and non-exhaustive list of requirements of equal importance for the realization of trustworthy artificial intelligence. So, AI HLEG proposes to European stakeholders to follow seven key requirements when they are developing and using AI systems:

- Human agency and oversight

To ensure that this requirement is applied in practice, a fundamental rights impact assessment prior to each AI system development should be taken. Also, humans should put in place mechanisms and measures to ensure human control and should always have the possibility to abort an operation that is problematic, for instance by the use of a stop button.

- Technical robustness and safety

That requirement concerns the cyber-security and the effort to understand and reduce the different kinds of cyber-attacks. For that reason, a series of steps should be realized in order to increase the AI systems' accuracy. Besides, humans should monitor and test those systems in order to assure that the latter meet their purposes and operate properly.

- Privacy and data governance

Namely, measures should be taken to ensure privacy, for instance via encryption, and quality of the data. Also, relevant standards for data collection, protection and governance should be followed.

- Transparency

According to this requirement, technical methods that ensure traceability should be used, namely why and how an algorithmic system is designed and developed and how it shapes the decision-making process in order to answer if that system is valid and if its outcomes are fair or produce a bias.

Moreover, the users of AI systems should always be aware that they interact with an AI system and not with another human.

- Diversity, non-discrimination and fairness

This requirement is fulfilled when the algorithms of an AI system are designed and the data are used in a way that avoids unfair bias. Moreover, such systems should take into account a wide range of individual preferences and abilities and provide a mechanism to different stakeholders in order to participate in their development and use.

- Societal and environmental wellbeing

That means that AI systems should be sustainable, environmental friendly and monitor their social impacts and effects.

- Accountability

An AI system is accountable only when justifications can be given about the occurred actions. An impact assessment tool should be used in order to measure the outcomes and to report and minimize the negative impacts of an AI system. Also, mechanisms that give the opportunity of redress if any harm would be occurred should be established for compensate users and/or third parties.

The methods to implement the requirements mentioned above can be both technical and non-technical ones and should encompass all levels of the development process of AI systems.

To sum up, AI HLEG underlines that trustworthy artificial intelligence is a continuous process that requires constant evaluation and justification of the systems. For that reason, it poses a Trustworthy AI Assessment List (Pilot Version) with a not exhaustive list of questions[5] that should be considered by those who accomplish the assessment in order to answer if their systems correspond to the seven key requirements mentioned above.

3. Legal Framework

Beside the ethical framework a legislative one appropriate for artificial intelligence should be implemented as well in order to avert or at least diminish faults and fallible results of AI systems.

AI HLEG elaborated another document[6] which complements its ethic guide-

[5] AI HLEG invites all stakeholders to pilot this Assessment List and to provide feedback, as based on that feedback the AI HLEG would proposed to the Commission a revised version of the aforementioned list in early 2020.

[6] *Policy and Investment Recommendations for Trustworthy Artificial Intelligence*, Independent High Level Expert Group on Artificial Intelligence, Set up by the European Commission, European Commission, Brussels, 26 June 2019.

lines since the latter do not refer to the first of the three components of a trustworthy artificial intelligence system ("lawful", "ethical", "robust").

AI HLEG provides insights on how regulation should be enacted in order to respond to artificial intelligence's needs.

Regulation should be based on proportionality, namely the higher the individual or social risk of an AI system the stronger the regulatory response should be.

For unacceptable impacts a precautionary principle based approach should be adopted.

Principled-based regulation is preferable instead of an analytic and descriptive one, as the technological change is rapid and unpredictable.

Also, an evaluation of all existing European Union's laws relevant [7] to AI systems should be conducted in order to ascertain the following.

To what extent and in which ways those laws are affected by AI systems, to what extent there are frameworks for enforcement and monitoring of the legislative measures concerning AI systems and to what extent existing legislation protects against risks posed by AI systems and ensures the ethical principles.

In case that the current legislation does not provide an adequate protection, then, new regulation for AI systems should be enacted from European Union, always in compliance with the principles of subsidiarity and proportionality, in order to avoid fragmentation of rules at member-states' level.

Concerning the existing legislation of European Union, Regulation 2016/679 on the protection of natural persons with regard to the processing of personal data (hereinafter mentioned as GDPR) [8] is applicable in the field of artificial intelligence too. The use of algorithms raises the question of the protection of personal data as the latter are the crucial element for the usage of an algorithm. Particularly, as the supply of data increases as the efficacy of the algorithm increases, since there is much more information to be processed and combined. For that reason, it emerges a conflict between the function of an algorithm and the "*purpose limitation*" and "*data minimization*" which are defined in article 5, par. 1 (b) and (c) of the GDPR.

However, the interested person has the right to obtain the information mentioned in GDPR, for instance in which way the data are collected and with which specific data the algorithm is provided in order to lead to a decision. The GDPR explicitly mentions in article 13, par. 2 (f), article 14, par. 2 (g) and article 15, par.

[7] For instance, legislation that concerns cyber-security, civil liability and accountability, consumer protection, competition, data protection, criminal matters and non-discrimination matters.

[8] Regulation (EU) 2016/679 of the European Parliament and of the Council of 27 April 2016 on the protection of natural persons with regard to the processing of personal data and on the free movement of such data, and repealing Directive 95/46/EC (General Data Protection Regulation), Official Journal of the European Union L 119/1, 4.5.2016.

1 (h) that the data subject should be aware of the existence of a solely automated decision-making, including profiling, and at least in those cases should be provided with meaningful information about the logic involved, the significance and the envisaged consequences of such processing for the data subject.

Also, GDPR stipulates in article 22, par. 1, that "*The data subject shall have the right not to be subject to a decision based solely on automated processing, including profiling, which produces legal effects concerning him or her or similarly affects him or her.*", therefore it enhances the right of the concerned person to access to litigation and debate in front of a judge (adversarial principle). Also, the latter should not be affected by the results conducted so far (impartiality and independence of judges) [9].

The provisions relating to processing of personal data as they are stipulated in GDPR should be applied also during the design stage of AI systems. Further, a prior risk assessment during that stage shall minimize the impact of their use on the rights of data subjects according to the precautionary principle.

In particular, according to article 35, par. 3 (a) of the GDPR, a data protection impact assessment is required especially when decisions that produce legal effects for the data subject or affect the data subject are based on automated processing of personal aspects.

4. Challenges

Concerning the ethical framework of European Union, AI HLEG's ethical guidelines are acceptable as a first step but still are not clear enough and they do not take long-term risks into consideration. Also, they do not determine which principles are not negotiable, in other words what should not be done with artificial intelligence in Europe (the so called "Red Lines"). An example of "Red Line" could be the prohibition of use of AI systems that humans can no longer understand and/or control [10].

Further, the aforementioned guidelines are not legally binding, so, there is a need for European Union's response through legal provisions that implement and complement them and ensure their implementation.

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[9] *European Ethical Charter on the Use of Artificial Intelligence in Judicial Systems and their environment*, European Commission for the Efficiency of Justice (CEPEJ), Council of Europe, Strasbourg, 3 December 2018.

[10] Thomas Metzinger, *EU guidelines Ethics washing made in Europe*, DER TAGESSPIEGEL, 08.04.2019 (English version).

For instance, legally binding common rules on transparency and common requirements for fundamental rights impact assessments should be enacted in order to answer if stakeholders are obliged to ensure transparency by design, if they can differentiate the levels of transparency required depending on the automatization of each AI system and to what extend their intellectual property rights or trade secrets will set a limit on requirements of transparency and accountability.

Concerning the requirement of explainability, namely the availability of explanations that go beyond the function of an AI system itself, it is not always possible to make explanations available concerning an algorithmic decision because of the "black box effect" that is already described in the introduction of the present document.

For that reason, European Parliament recommends the creation of a regulatory body for algorithmic decision-making that will define the criteria that can be used in order to separate the algorithmic systems that are acceptable and those that should be prohibited, for instance if transparency, explainability or accountability cannot be achieved. Also, that body will determine which would be the obligations of a provider of an algorithmic decision-making system.

More issues such as the obligation for informing the persons affected by AI systems and specific liability regimes should also be addressed. Also, a prior algorithmic impact assessment before the use of an AI system it should deployed. Therefore, in the future, a new European legislation that concerns specifically the algorithmic decision systems may be enacted in order to respond to all those new challenges [11].

Concerning the existing legal framework of European Union, some argues that GDPR does not provide a right to explanation that enhances the transparency and accountability of AI systems. Merely it grants to the data subject a right to be informed and further it does not protect data subject from discrimination [12].

As GDPR has recently applied within the member-states [13], it is necessary to monitor its application in the artificial intelligence's context, in order to conclude

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[11] *Understanding algorithmic decision-making: Opportunities and challenges*, Study, Panel for the Future of Science and Technology, European Parliamentary Research Service, Scientific Foresight Unit (STOA), European Parliament, March 2019.

[12] For instance, Sandra Wachter, Brent Mittelstadt and Luciano Floridi, *Why a Right to Explanation of Automated Decision-Making Does Not Exist in the General Data Protection Regulation*, Oxford Internet Institute, University of Oxford, International Data Privacy Law, 2017.

Also, Bryce W. Goodman, *A Step Towards Accountable Algorithms?: Algorithmic Discrimination and the European Union General Data Protection*, Oxford Internet Institute, University of Oxford, 29th Conference on Neural Information Processing Systems (NIPS 2016), Barcelona, Spain.

[13] The application of GDPR began from 25 May 2018, according to article 99, par. 2 of the GDPR.

if it is suitable or not for this context, and also to observe the interpretation of its relevant articles from the Courts.

Having as an aim to find a balance between protection and innovation and not stifle the latter, further legal research should be accomplished in order to assess in which sectors a European regulatory intervention is needed and particularly in which way. Namely, European Union has to answer if there is a need of rules as adjustments to the existing legal framework or it is necessary to create a new European legislation more suitable for artificial intelligence. Also, it has to decide if those rules would be general or sectorial and in case of new legislation, if the latter would be enacted through hard-law or soft-law approach and as state regulation, self-regulation or co-regulation.

5. Conclusion

Artificial intelligence can take life-changing decisions, especially when AI systems are used in sensitive sectors such as this one of justice. Admitted that it evolves as swiftly as we don't have yet all the answers not all the questions concerning its role, European Union should enact a legal framework in order to specify rules at least about what machines cannot do. Also, since a prerequisite for trustworthy artificial intelligence is to understand how decisions are made by AI systems, legislation ought to provide tools that allow humans to supervise decisions taken by AI systems and to challenge those decisions through judicial proceedings.

Therefore, European Union has now the opportunity to enact the kind of future that it would like to have concerning the use of AI systems and by its actions can prove to itself that is capable to avoid the "dark ages" of artificial intelligence.

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Ferruccio Auletta

A Quantitative Approach to Study the Normativity of the Jurisprudence of Courts in Countries of Civil Law Tradition

1. The measurability of the factors determining the jurisprudence and the difference between jurisprudence and precedent

In countries of civil law legal tradition, algorithms are sometimes used to *predict* the outcome of judgments. Some have even gone as far as suggesting that one day they might replace the human judge. However, a less controversial use of algorithms in civil justice is to increase the *foreseeability* of a single judgment.

Indeed, predictivity and foreseeability are cognate but distinct concepts. Predictivity is the condition of being predictive, that is to say the condition of having value for making predictions. Foreseeability means the capacity of being anticipated. Thus, they are cognate because they both «*affect the meaning and function of jurisdictional activity*» and «*concern the requirement of legal certainty, the rule of law and equality*». They are distinct concepts because predictivity goes beyond foreseeability, often to the point of becoming a self-fulfilling prophecy: «*The judge, to make decisions, uses the '*prediction*' of the algorithm, so that the prediction itself becomes the decision*» [1].

The use of algorithms to foresee judicial outcomes «*is not foreign to the very concept of jurisprudence as comprehended by the Romans, for whom '*prudence*' was a contracted version of '*pro-videntia*', derived from the Latin '*pro-video*', meaning to '*foresee*'. According to Cicero, prudence implied elements such as '*memory, intelligence and foresight*' and referred to forecasting risks; therefore, '*jurisprudence*' dealt with foreseeing what consequences would result from applying the law to specific facts*» [2].

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[1] D. DALFINO, Foro italiano, 2018, V, 385.

[2] A. FAUCHIER DELAVIGNE, A. GAJZLER, A. MARIN, The Challenges Facing Justice in the Future Judges Confronted with the Advent of Big Data Analytics, Team France: Trainer L. VUITTON, Semi-Final D, Budapest 3-6 luglio 2017, 2.

Of course, resort to algorithms to foresee the outcome of judicial proceedings presupposes the measurability of the factors underpinning the judicial ruling. That begs the question of whether measurement can be objective or whether it is inherently subjective. It also begs a vexed question of quantum physics, which is whether the very act of measuring changes what is being measured. These are just some of the philosophical and practical questions that a study of the use of algorithms to foresee judicial outcomes entails.

Measuring the factors underpinning a judicial ruling presents several challenges. The first one is access to manageable data, possibly organized as an electronic database. The digitalization of the work of courts and tribunals that took place in most developed countries during the past two decades has generated a trove of digital information that now can be mined, down to the specific components of the given ruling. The next question is: what exactly should be measured and to what end?

In common law system, the answer could be straightforward: to find what case law is on any given issue and in any given case. However, in civil law systems, the answer is more complicated, since in civil law systems there is no binding precedent, no *stare decisis* principle, but rather *jurisprudence* (*jurisprudence constante* in France, *orientamento della giurisprudenza* in Italy, or *jurisprudência dominante* in Brazil). While in common law legal systems a single decision could be binding precedent and change the law, in civil law legal systems no ruling can change the law *per se*. Typically, it takes several decisions to achieve the same result. It is the cumulative effect of several decisions, sharing the same legal rationale, that creates the *jurisprudence*. In systems of common law, the judicial precedent has its own intimate normative ability, the discovery of which by the judge is a matter of factual cognition. In Cardozo's words, «*there is a tendency toward the reproduction of kind. Every judgment has a generative power. It begets in its own image*» [3]. In the civil law tradition, the recognition of the "normative momentum", which can result in jurisprudence, requires the judge to interpret the relevant case law to find the "normative track" that s/he must follow in the pending case.

2. Examples from civil law systems

Let's consider three civil law legal systems: the French, Italian and Brazilian. Each approaches the problem of the normative nature of judicial decisions differently. France has an ancient tradition denying normative power to judicial decisions. Italy recognizes the normative value of the jurisprudence. Brazil's new Code of Civil Procedure combines the idea that some precedents are strictly binding with the idea that only jurisprudence as a whole might have normative value.

[3] The Nature of the Judicial Process, Yale University Press, 1921, 141.

2.1. France

Since the adoption of the Code Napoleon, on March 15th, 1803, France prohibits *arrêts de règlement* (regulatory judgments). Under article 5 of the Civil Code, «*It shall be prohibited for the courts to pronounce orders by general and legislative provisions on causes which are their subject matter*». The ban of regulatory judgments has been interpreted as implying that the judge cannot decide solely on the basis of a single precedent.

However, at least since the 1960's, in France, the *jurisprudence constante*, that is to say is a long series of previous decisions (as opposed to a single decision) applying a particular legal principle or rule that are highly persuasive, is equalized to a source of law. Indeed, when the Court of Cassation (*Cour de Cassation*), the last instance of jurisdiction in the French legal system, is asked to rule on a case that follows la *jurisprudence constante* the case is assigned to a panel of three judges (*formation restreinte*) instead of five, to expedite proceedings.

Clearly, the Court of Cassation, but also all lower courts, have the need to know what decisions are important, because they either affirm or divert from settled jurisprudence, and which ones do not add or detract anything. For certain purposes the Court distinguishes between: a) decisions that specify the scope of a rule; b) decisions that create new case law; c) decisions that affect or modify an old solution; d) decisions that recall principles that have been established, so that they are not lost sight of or to show the Court's commitment to them [4].

Also, when deciding which decisions should be published and where, the Court of Cassation weighs the «*normative interest of the decision*» (*l'intérêt normatif de la decision*), distinguishing between five categories of decisions:

D = Decisions for internal **d**issemination within the Court only. These are the judgments that do not add anything to the jurisprudence of the Court. They are usually called «*individual judgments*» (*arrêts d'espèce*). They are not published;

B = Decisions that the Court deems necessary to bring quickly to the attention of all judges in France. They are published in summary in the Bulletin of the Court of Cassation (*Bulletin d' information de la Cour de cassation* – **B**ICC, which is distributed biweekly to all judges);

P = Decisions that are noteworthy because, for instance, they contain novel solutions, or are an evolution of the Courts' jurisprudence, or because the Court deems it necessary publish them to remind a point of law that had not been recalled in a long time (about ten years). These are **p**ublished in full in the Bulletin;

I = Decisions that the Court deems of interest for the public. These are decisions that touch on issues of general concern or that can broadly affect the life of citizens. These are published on the website of the Court of Cassation (**i**nternet).

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[4] See A. LACABARATS, Rec. Dalloz, 2007, 889.

R = Lastly, decisions with high normative impact. These are the decisions that could change the law. They are discussed in the annual **r**eport of the Court of Cassation, and they are accompanied by an explanation of how jurisprudence of the Court has evolved [5].

Every time the Court issues a decision, it determines also the degree of publicity it should give it. It does so that everyone who needs to know (court itself, judges in the legal system or public) is alerted about what the jurisprudence of the Court is on any given issue. By doing that, it weights the normative impact of each decision (about 20,000 in total), creating the basis of an empirical measurement of the given decision's impact on the overall legal system.

Granted, this is still very different from so-called "predictive justice", which is what is currently being experimented in the Courts of Appeal of Rennes and Douai [6]. There, digitalization and artificial intelligence are used in aggregate to distill from the mass of cases specific orientations on given issues. It is a mass analysis, which is antithetical to the calculation of a single decision's normative weight the Court of Cassation does.

2.2. Italy

In Italy, as in France, the judge is not bound by precedents. Under the Italian Constitution, the judge is subject only to statutory law. However, the judge is arguably constrained by jurisprudence, that is to say, settled case law, too. A recent amendment of the Italian Code of Civil Procedure has raised the question.

Article 360-bis provides that the Court of Cassation (*Corte di Cassazione*), the country's last instance of jurisdiction, can reject an appeal on a procedural ground if the challenged judgment is based on principles of law established by the Court of Cassation, and an assessment of the grounds for the appeal does not suggest a reason to change the «*orientation*» of the Court. What is the *orientation* of the Court?

According to a 2016 *Programmatic Document* of the Prime President of the Court of Cassation , the «*orientation*» of the Court is determined by «*a decision by the United Chambers; when there is a consolidated orientation of any Chamber; when there are a few judgments of one or more Chambers, if convergent; when there is only one decision, if considered convincing*» [7].

Clearly, this is an extremely empirical estimate of the normative value of the Court's judgments, since, for example, it does not take into account *obiter dicta*, even though the Court of Cassation itself considered them normative factors and the Constitutional Court considers them capable of shaping the «*living law*».

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[5] J.F. WEBER, Comprendre un arrêt de la Cour de cassation rendu en matière civile, www.courdecassation.fr.

[6] See -for example- Predictice (www.predictice.com).

[7] April 22nd, 2016: http://www.cortedicassazione.it.

2.3. Brazil

The need to measure the "marginal normativity" of a decision arises also in other civil law countries. Brazil has faced the same problem [8]. Indeed, it seems to be a constant of the evolution of all procedural systems.

In 2015, a new Code of Civil Procedure entered into force, requiring a reliable measurement of the normative impact of any given judgment. It required the introduction of «*measures, aimed at using jurisprudence as a parameter for the elaboration of decisions*». «*The idea of enhancing the value of jurisprudence and encouraging the issuance of uniform decisions is not new since there is a repeated orientation of the courts, especially the higher ones, such as the Supreme Federal Court and the Superior Court of Justice*» [9]. However, the novelty is the formalization of the process.

The objective measurement of the normative degree of a single decision would increase foreseeability, stability and efficiency of the entire system, as well as foster the coherent development of the institutions that refer to the *uniform jurisprudence*.

Under article 926 of the new Code, «*the courts must conform their jurisprudence and keep it stable, intact and consistent*». Therefore, it has been stated that «*the new code no longer tolerates the existence of different positions on the subject matter within the same court*».

According to article 927, «*the judges and the courts shall comply with [some specific] decisions*» to which they are properly bound, such as the «*enunciados de súmula vinculante*» that are stated in case of «*jurisprudência dominante*». In this case, among others, a new lawsuit can be rejected *prima facie* because of the consolidated orientation of the courts (see art. 332).

All in all, Brazil's new approach is to combine the idea that some precedents are strictly binding with the idea that only jurisprudence as a whole might have normative value. The impetus for the reform has been the need to reduce courts caseloads, by giving them the chance to dispose of repetitive cases through one single judgment. That makes objectively determining the weight of cases paramount because it becomes a determinant of access to justice.

3. The quotient to be assigned with non-arbitrary criteria to each element concurring to the normative range of a decision

In each of the three legal systems discussed, judicial decisions impact jurisprudence, providing normative guidance to further rulings. The question is

[8] See T. ARRUDA, Le pouvoir normatif du juge – la motivation des décisions et le précédents à force obligatoire, d'après le nouveau Code de Procédure Civil (2015), Zeitschrift für Zivilprozess International, 21 (2016), 259 ss.

[9] P. LUCON, Rivista di Diritto Processuale, 2018, 1271.

whether in a system where precedent is not binding is possible to determine the "normative coefficient" of any given decision, its impact on future decisions. In other words, whether it is possible to measure the impact of any given decision on future jurisprudence, whether it contributed to open new pathways or confirm the jurisprudence.

The task is not a straightforward as it is in common law legal systems. In systems based on binding precedents, facts are determined first, and then, if they are the same of a previous decision, the rule is applied as it was applied before. Thus, as far as later cases are concerned, the rule is an exogenous factor; it is a given. Conversely, in civil law systems, precedents do not have binding force but, as it was said, jurisprudence has normative value. This means that two cases can never be considered identical, because even the slightest increase to the case law brought by the previous judgment creates a new and unique context for all subsequent decisions. The law (*i.e.* the rule plus the normative addition of the previous decision that shapes jurisprudence) applied in the first case is not the same applied in the second one. In civil law systems, law is never just applied. The very act of applying the law changes the normative context within which the next case will be decided. Determining objectively, or at least agreeing on the specific value of each element of the case, and the aggregate weight of the relevant case law, is necessary to make it possible to foresee the outcome of future cases.

In my opinion, jurisprudence (*jurisprudence constante* in France, *orientamento della giurisprudenza* in Italy, or *jurisprudência dominante* in Brazil) must be always identifiable and measurable. It must be determined *ex ante*. The identification of the driving jurisprudence cannot be left to the judge in the given case. As in physics, the observer could be «*even destructive, in the sense that it irretrievably and irreparably disturbs the observable*» [10]. The judge could create a normative reality that does not exist in the observed jurisprudence.

What are the factors of a given decision and surrounding it that should be measured? At a minimum, one should take into consideration rank of the courts that issued the decision; the content of the decision; how many times it was echoed by other courts; the social or cultural impact of the decision, etc. Also, the time factor is particularly important because the longer a given judgment stands unchallenged, the greater its weight. Other crucial factors are the density of use (*i.e.* the number of judges who refer to the prior judgment); the frequency of use (*i.e*. the number of times other judgments, in a given period, refer to it), and the reach and scope (*i.e.* the range of cases that the given decision impacts or whether it applied a general rule or a specific one). A decision that has limited reach or applies only an extremely specific rule will have little or no value to adjudicate further cases.

[10] S. IANNACCONE, March 3rd, 2019, https://www.galileonet.it/: this is because «the process of measurement and the observer who makes it are in no way separable from the measured object».

The normative impact of a decision is ultimately determined by whether it changed the *status quo*, and, looking ahead, with regard to the effects on other fields, as well as with regard to duration and intensity of its use. In any case, what needs to be assessed is whether there is a change, either in the set of legal statements or in the behavior of their makers or recipients. This defines the extent of the legal change, that is to say the normativity of the decision.

4. The measurement's technique: an analogic scale «0 1»

As to the how these factors should be measured, an analogic scale is preferable to a digital one. An analogic scale has the advantage of being able to take into account all variables mentioned. A continuous interval (0 *to* 1) accommodates a theoretically infinite number of analytical and quantification possibilities, while a digital scale (0 *or* 1) allows only for a limited number of possibilities. It could help distinguishing, for instance, between the *obiter dictum* elements of the judgment from those that pertain to its core (the *ratio decidendi*), or how many commentators welcomed or rejected it. Second, it makes it possible to use mathematical calculus (for instance, mathematics of dynamical systems or vector calculus), which leaves the door open to still unknown variable, and allows for extrapolations, interpolation *etc.*

To take as an example the system the French Court of Cassation uses to decide what decisions it should publish and where, judgments in class **D**, the so-called «*arrêts d'espèce*», will be given the value 0, since they have no impact whatsoever on the legal system. Any other judgment will be attributed a value higher than 0 up to 1. Granted, even *arrêts d'espèce* do have an impact, at least because they restate what the law is. Therefore their value should be higher than 0. However, for sake of simplicity we should attribute them the lowest possible value in the scale. At the other end of the spectrum, 1 should be attributed to judgment that, *inter alia*, have been passed in a complete legal void, in the absence of precedents and thus, created, in a sense, new law.

5. The proposal's impact on the idea that a single decision (or precedent) could be considered jurisprudence

The importance of introducing a system to weigh objectively the normative value of judicial decisions is illustrated by a recent case ruled by the Italian Court of Cassation. The Court was asked to rule on the admissibility of an appeal. Under the Italian Constitution (art. 111), appeal to the Court of Cassation is «*always*» possible when the law has been allegedly violated. Nevertheless, the Court dismissed the case because article 360-bis of the Code of Civil Procedure, as it was said earlier, allows it to set aside the appeal when the Court has already an

established jurisprudence on point and it deems there is no reason to change it. However, in this instance, the Court's jurisprudence amounted to just one single precedent. The Court argued that even a single precedent, if unambiguous, clear and convincing, can determine the «*orientation*» of the jurisprudence referred to in art. 360-bis [11]. The Court also added that since it had ruled only once on the issue, it meant that it had never felt the need to change its orientation.

There are a number of factors that arguably could have led the Court of Cassation to the conclusion that one single decision can be weighty enough to warrant considering it «*orientation*» of the Court's jurisprudence. For instance, the case concerned a temporary norm that had already lapsed, relating to enforcement proceedings pending two decades earlier (September 8th, 1998) and the Courts' judgment that established the *orientation* had been passed a decade before [12]. Since then, the Court had no chance to return to the issue again [13]. The transient nature of the law applied, the limited scope and the timing of the question considered, the fact that the Court of Cassation is the last instance of jurisdiction in the Italian legal system are all factors that could justify calling even one single decision "jurisprudence", which otherwise would appear to be simply «*absurd*» [14].

Still, the decision was remarkable and debatable because it makes confusion between precedents and jurisprudence. Yet, precedents are not jurisprudence. Jurisprudence is more than just one or more precedents. What makes jurisprudence is not just one or more precedents but also several more factors that should be objectively determined and weighted. In civil law constitutional systems, there must be a separation between those who make the law and those who apply it. If a single precedent is *jurisprudence* the distinction between law-makers and adjudicators no longer holds true. Scholars have also noted that admissibility of an appeal should be objective and technical. It cannot depend on a subjective assessment of the Court of its own jurisprudence. Assume two plaintiffs appeal before the Court, facing the same objective situation. Everything else being equal, both appeals are either admissible or are not. The fact that one appeal could be admissible but the other not contravenes the egalitarian spirit of the Constitution, which provides that access, at given conditions, to *one* Supreme Court is «*always*» permitted. As it has been noted, the Court's reasoning has distorted the concept of the Court's *orientation*, making it difficult to measure it quantitatively, since it seemed impossible to trace it to a single previous decision [15].

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[11] Cass., february 22nd, 2018, n. 4366.

[12] Cass., october 11th, 2006, n. 21733.

[13] These are factors all considered for the purpose of the publishing regime of Court's decisions in France (§ 2.1).

[14] G. PILLOT, ibid.

[15] G. PILLOT, Giusto proc. civ., 2019, 795 ss.

Yet, if the normative weigh of judicial decisions could be measured and a system to do so was agreed upon, even a single Court's decision could substantiate jurisprudential orientation as long as its measured normative weigh is heavy enough.

These are the reasons why it is now crucial, in the civil law systems, to set the quantitative parameters to weight jurisprudence and its elements that constitute it. That is necessary if efficient case management and effective judicial protection of individual rights are to be balanced.

Flavia Rolando

The improvement of the Access to justice through the integration of the ICT in the EU legal order

1. Introduction

Twelve years ago, the European Council called for the promotion of the use of information and communication technologies (ICT) in the area of justice at European level. The need to integrate ICTs into the management of justice has grown in parallel with the development of the European area of security and justice.

According to the European Commission, e-Justice can be defined as the use of ICT to improve citizens' access to justice and the effectiveness of judicial action. The European action in this area has been implemented along the lines of the e-Justice action plans and strategies, developed by the European institutions. The most recent European e-Justice strategy (2019-2023) aims to develop portals such as e-Justice, e-Law and Eur-Lex to improve access to information and, in this sense, to improve access to justice intended in a broad sense. The action plan defines how to implement the objectives of the Strategy, mainly the development of e-communications, facilitating interactions between judicial authorities as well as between citizens and practitioners, and the enhancement of interoperability between Member States' systems. There are also several possible innovations which are not currently considered in the strategy, but that are encouraged by funding research projects, also with a view to defining the prospects for their use. This is the case of the CREA project, which aims to introduce an alternative dispute resolution system using equitable algorithms.

The purpose of this work is to describe the improvement of the accessibility to justice through the ICT and to define the prospects for integrating the results of the CREA research projects into the EU legal order.

2. The EU competence in the area of judicial cooperation in civil matters

Investigate the matter about the new frontiers in the improvement of the E-justice in the EU legal order necessarily requires the definition of the competence of the European Union to regulate judicial cooperation.

According to the principle of conferral, the Union shall act only within the limits of the competences conferred upon it by the Member States in the Treaties [1]. It follows that, under this principle, the European Union can only adopt legal acts aiming at enhancing the use of the ICT complying with the limits of the competence established by the article of the Treaty on the Functioning of the European Union (hereinafter TFEU) dedicated to the judicial cooperation in civil matters [2].

The competence in the area of judicial cooperation in civil matters has been established by the Maastricht Treaty and in 1997; the Amsterdam Treaty has "comunitarised" the area, establishing that the Council shall act unanimously on a proposal from the Commission and after consulting the European Parliament [3].

At present, according to Art. 81 TFEU, the European Parliament and the Council, acting in accordance with the ordinary legislative procedure, shall adopt measures, inter alia, aimed at ensuring the mutual recognition and enforcement between Member States of judgments and of decisions in extrajudicial cases; an effective access to justice; the development of alternative methods of dispute settlement.

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[1] See art. 5 TEU.

[2] About judicial cooperation in civil matters see, inter alia, M. Andenas, *National Paradigms of Civil Enforcement: Mutual recognition or Harmonization in Europe,* in *European BusinessLaw Review*, 2006, p. 529; R. Baratta, *Art. 81 TFUE*, in A. Tizzano (edited by), Trattati dell'Unione europea, Milano, 2001, p. 241; S.M. Carbone, *Lo spazio giudiziario europeo in materia civile e commericale. Da Bruxelles I al regolamento CE 805/2004*, Torino, 2009; M. Freudenthal, *Attitudes of European Union Member States Towards the Harmonisation of Civil Procedure,* in C.H. van Rhee, A. UZELAC (eds), *Enforcement and Enforceability - Tradition and Reform*, Oxford, 2010, p. 3; C.N. Kakouris, *Do the Member States possess Judicial Procedural "Authonomy"*, in CMLR, 1997, p. 1389; X.E. Kramer, C.H. Rhee (eds), Civil Litigation in a Globalising World, L'Aja, 2012, A. Maffeo, Diritto dell'Unione europea e diritto processuale civile nazionale; verso l'adozione di norme minime comuni?, in Diritto dellì'Unione europea – Osservatorio, 31 marzo 2018; A. Maffeo, *Diritto dell'Unione europea e processo nazionale*, Napoli, 2019; O. Porchia, *Principi dell'ordinamento europeo. La cooperazione pluridirezionale*, Bologna, 2008; E. Silvestri, *Toward a European Code of Civil Procedure? Recent initiative for the Drafting of European Rules of Civil Procedure*, in academia.edu; M. Storme, *A single Civil Procedure for Europe: A Cathedral Builder's Dream*, in *Ritsumeikan Law Review*, 2005, p. 87; E. Storskrubb, *Civil Procedure and EU Law. A Policy Area Uncovered*, Oxford, 2008; G. Tarzia, *Harmonization ou unification transnationale de la procédure civile*, in *Rivista di diritto internazionale privato e processuale*, 2001, p. 869; M. Tulibacka, Europeanization of Civil Procedure: In Search of a Coherent Approach, in CMLR, 2009, P. 1527.

[3] This legal procedure has been afterwards modified by the Nice Treaty.

However, the first paragraph of this article establishes also the boundaries of this competence as it states that «The Union shall develop judicial cooperation in civil matters having cross-border implications».

Therefore, the European Union, using this legal basis, shall only adopt acts in civil matters that are related to cross-border issues. Moreover, in the adoption of measures concerning family law with cross-border implications, a special legislative procedure shall be observed [4].

Considering the topic of this contribution, between the various acts adopted to improve the judicial cooperation in civil matters, it could be interesting analyse the EU legal acts on the alternative dispute resolution (hereinafter ADR). The Directive 2008/52 on certain aspects of mediation in civil and commercial matters [5] aims to facilitate access to ADR and to promote the amicable settlement of disputes by encouraging the use of mediation. As explained by the title, the directive applies to disputes in civil and commercial matters, and it does not extend to revenue, customs or administrative matters and to the liability of an EU country for acts and omissions in the exercise of State authority (*acta iure imperii*) [6]. It is important to underline that, according to the legal basis established by Art. 81 TFEU, this Directive shall apply only in cross-border disputes, that is one in which at least one of the parties is domiciled or habitually resident in a Member State other than that of any other party. Nonetheless, as stated in whereas n. 8, even if the provisions of this Directive should apply only to mediation in cross-border disputes, nothing should prevent the Member States from applying such provisions also to internal mediation processes [7].

Therefore, it could seem strange that the Directive 2013/11 on the alternative dispute resolution for consumer disputes [8], which aims at introducing a fast and fair alternative dispute resolution procedures for consumers, has a different scope. More in depth, according to this directive Member States shall ensure that all kinds of contractual disputes that arise from the sale of goods or provision of ser-

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[4] See Art. 81, par. 3, TFEU, establishing that in such a case the Council, on a proposal from the Commission, may adopt a decision determining those aspects of family law with cross-border implications. The Council shall act unanimously after consulting the European Parliament. Moreover, the proposal of the Commission shall be notified to the national Parliaments. If a national Parliament makes known its opposition within six months of the date of such notification, the decision shall not be adopted. In the absence of opposition, the Council may adopt the decision.

[5] Directive 2008/52/EC of the European Parliament and of the Council of 21 May 2008 on certain aspects of mediation in civil and commercial matters, OJ L 136, 24.5.2008, pp. 3-8.

[6] See art. 1 of the Directive 2008/52, cited.

[7] In Italy, for instance, the mediation attempt has been established as mandatory in several civil matters.

[8] Directive 2013/11/EU of the European Parliament and of the Council of 21 May 2013 on alternative dispute resolution for consumer disputes and amending Regulation (EC) No 2006/2004 and Directive 2009/22/EC (Directive on consumer ADR), OJ L 165, 18.6.2013, pp. 63-79.

vices (both online and offline) can be submitted to an ADR entity as to obtain a simple and fast way of resolving disputes. This Directive applies to all the disputes, such as when a trader refuses to repair a product or to make a refund to which a consumer is entitled, also where consumer and seller reside in the same Member State. In this case, the scope is not limited to transnational disputes. That is because this directive, even if rules an aspect of the judicial cooperation in civil matters, has as main objective to ensure the proper functioning of the EU's single market. For this reason, the proper legal basis is Art. 114 TFEU[9], about the approximation of laws, that allows rules for internal and transnational issues.

The same is for Regulation 524/2013[10] on online dispute resolution (hereinafter ODR), that aims to create an ODR platform (website) at EU level where consumers and traders can resolve disputes that arise from online transactions. The platform allows consumers to submit their disputes online and operates in all EU official languages. This Regulation is based on art. 114 TFEU, also considering that Art. 169(1) and point (a) of Art. 169(2) TFEU provide that the Union is to contribute to the attainment of a high level of consumer protection through measures adopted pursuant to Art. 114 TFEU.

In general terms, recently there was a debate about the adoption of common minimum standards of civil procedure in the European Union. In this regard, the European Parliament has released a resolution with recommendations to the Commission[11]. The European Parliament has supported this legal act considering that the piecemeal nature of the harmonisation at Union level of procedural rules has been repeatedly criticised and the emergence of sector-specific Union civil procedure law challenges the coherence of both civil procedure systems at Member State level and the various Union instruments. Therefore, a system of Union common minimum standards in the form of principles and rules, would serve as a first step for convergence of national regulations concerning civil procedure[12].

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[9] According to Art. 114 TFEU, the European Parliament and the Council shall, acting in accordance with the ordinary legislative procedure and after consulting the Economic and Social Committee, adopt the measures for the approximation of the provisions laid down by law, regulation or administrative action in Member States which have as their object the establishment and functioning of the internal market.

[10] Regulation (EU) No 524/2013 of the European Parliament and of the Council of 21 May 2013 on online dispute resolution for consumer disputes and amending Regulation (EC) No 2006/2004 and Directive 2009/22/EC (Regulation on consumer ODR), OJ L 165, 18.6.2013, pp. 1-12. See also Commission Implementing Regulation (EU) 2015/1051 of 1 July 2015 on the modalities for the exercise of the functions of the online dispute resolution platform, on the modalities of the electronic complaint form and on the modalities of the cooperation between contact points provided for in Regulation (EU) No 524/2013 of the European Parliament and of the Council on online dispute resolution for consumer disputes, OJ L 171, 2.7.2015, pp. 1-4.

[11] European Parliament resolution of 4 July 2017 with recommendations to the Commission on common minimum standards of civil procedure in the European Union (2015/2084(INL)).

[12] See letters K and W of the cited European Parliament Resolution.

However, the European Commission in its reply [13] has stated that will determine whether and to what extent further action is required in this area and, in this case, will take Parliament's proposal into account in its future work.

This overview of the legal framework in judicial cooperation in civil matters makes a display of the boundaries of the EU competence in establishing legal acts on the judicial cooperation in civil matters and allow us to define how the EU can improve the development of the electronic justice.

3. Perspectives in the development of Electronic justice from an EU perspective

In 2007, the European Council invited the Council to promote the developing of the use of ICT in the area of civil justice at European level [14]. Following this pointer, the Council of European Union on Justice and home affairs underlined that the work in the area of E-Justice should be carried with a view to creating a user-friendly access for citizens, economic operators, practitioners of law, judicial authorities and courts [15]. According to the Council, the EU action should cover, among other objectives, the set-up of a European interface (E-Justice portal); start the preparations for the use of IT for the European payment order procedure; improve the use of videoconferencing technology for communication in cross-border proceedings.

The European Parliament too has given his contribution, inviting the Commission to complement the European area of justice, freedom and security with an area of e-Justice [16].

In response to the Council, the Commission presented its communication "Towards a European e-Justice Strategy" [17], highlighting that e-Justice represents a response to the threefold need to improve access to justice, cooperation between legal authorities and the effectiveness of the justice system itself. Therefore, the European Commission has defined the priorities for action in the period 2008-2013 and, following the indications of the other European institutions, pointed on the creation of an e-Justice portal for the public and enterprises to improve access to justice in Europe.

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[13] Suite donnée à la résolution du Parlement européen du 4 juillet 2017 contenant des recommandations à la Commission relatives à des normes minimales communes pour les procédures civiles dans l'Union européenne, 2015/2084(INL).

[14] European Council Conclusions of 21-22 June 2007, point 30.

[15] JHA Council Conclusions of 12-13 June 2007, page 43 of document 10267/07.

[16] European Parliament Resolution on e-Justice at its Plenary meeting on 18 December 2008, 2008/2125 (INI).

[17] European Commission communication "Towards a European e-Justice Strategy" of 5 June 2008 (COM(2008) 329 final.

This portal is intended to refer visitors to the other existing sites [18], to European legal institutions and is also intended to give direct access to certain European procedures such as small claims procedure [19] and payment procedure [20].

The EU action in the improvement of electronic justice has developed over time: the need to integrate ICTs into the management of justice has grown in parallel with the development of the European area of security and justice and with the development of the technologies.

The European action in this area has been implemented along the lines of the e-Justice action plans and strategies. The Council has adopted, over time, a European e-Justice Strategy and an Action Plan for the duration of four years. Therefore, these programs have been renewed as they come to the end of their terms. Actually, it has been published the e-justice Strategy and the Action plan 2019-2023.

In the 2019-2023 Strategy on e-Justice [21] the Council has recognised that Procedures carried out in a digitised manner and electronic communication have become an essential component in the efficient functioning of the judiciary in the Member States.

Nowadays, we can affirm that the European e-Justice Portal has been built up with information pages, search tools and dynamic forms. Furthermore, electronic tools now allow for digital judicial proceedings, secure communication between judicial authorities and access to certain national registers under the responsibility of the Member States or professional organisations [22]. However, it is still valid the aim at developing e-justice in order to improve access to information in the area of justice in the European Union.

In the Strategy, it has been defined three objectives of European e-Justice: the improvement of the access to information, of the e-Communication in the field of justice and of the interoperability. Considering the topic of this contribution, we will focus on the first subject.

In the view of the Council, the improving of the access to justice lies in the amelioration of the information on the rights of citizens, on EU law and on procedures.

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[18] See, for instance, Eur-lex, Pre-lex, SCADPlus, Eurovoc and IATE.

[19] Regulation (EC) No 861/2007 of the European Parliament and of the Council of 11 July 2007 establishing a European Small Claims Procedure, OJ L 199, 31.7.2007, pp. 1-22 as emended by Regulation (EU) 2015/2421 of the European Parliament and of the Council of 16 December 2015 amending Regulation (EC) No 861/2007 establishing a European Small Claims Procedure and Regulation (EC) No 1896/2006 creating a European order for payment procedure, OJ L 341, 24.12.2015, pp. 1-13.

[20] Regulation (EC) No 1896/2006 of the European Parliament and of the Council of 12 December 2006 creating a European order for payment procedure, OJ L 399, 30.12.2006, pp. 1-32.

[21] OJ 2019/C 96/04.

[22] See point 1 and point 5 of the 2019/2013 Strategy, cited.

In this purpose, the Action Plan, which is intended to deliver the vision of the Strategy, consistently develops this point in two lines: general information on Justice and access to legal information [23].

Under the first objective, the improvements are intended to make the e-justice Portal more usable and complete and reinforce its role as a one-stop-shop for European e-Justice.

Under the second objective, EU will finance the projects aiming at facilitating the access to legal data. In this purpose we know that at now EUR-Lex gives access to EU law, national law transposing EU law, case law coming from the Court of Justice of the EU as well as national case law related to EU law. Therefore, access to legal data should be facilitated by the use of identifiers for legislation and case law, which allow for easier analyses of legal data.

It is particularly interesting that, under this objective, the Council has also outlined the employment of artificial intelligence, even though in a very embryonic stage. AI is considered as one of the major developments in ICT in recent years. Nonetheless, the Council reckons that, even if its use should be further developed in coming years, at this moment its implications in the field of e-Justice need to be further defined.

Therefore, the CREA project is at the very frontier in the application of AI to justice. As will be pointed to hereinafter, the CREA Software aims to provide an alternative system of dispute resolution through equitable algorithms and this kind of solution should be considered as a tool facilitating the access to justice. In order to explain the grounds of this assertion, in the next paragraph will follow an analysis about what should be meant as access to justice.

4. The broad concept of *Access to justice* and the use of ICT for its improvement

With the sole aim to lift the major preliminary remarks and without any ambition to exhaustively tackle a so general argument, this paragraph proposes an analysis of the concept of access to justice [24].

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[23] See 2019-2023 Action Plan, cited, p. 2 ff.

[24] In the EU legal order, the Access to justice is guaranteed by the European Convention of Human Rights (Hereinafter ECHR) and by the Charter of Fundamental Right of the European Union (hereinafter CFREU). More precisely, art. 6 ECHR states the right to a fair trial and art. 13 is dedicated to the Right to an effective remedy while, in the CFREU, the chapter VI is dedicated to the Justice and its art. 47 states the right to an effective remedy and to a fair trial.

About the access to justice see, inter alia, N. Carboni, *From Quality to Access to Justice: Improving the Functioning of European Judicial Systems*, in Civil and Legal Sciences, Vol. 3, No. 4; A. Doobay, *The right to a fair trial in light of the recent ECtHR and CJEU case-law*, in ERA Forum, Vol. 14, No. 2, pp. 251-262, 2013; F. Francioni, *Access to Justice as a Human Right*, New

In particular, the access to justice should be assessed considering all its possible declensions. Therefore, it should be envisaged the real possibility to demand and obtain justice evaluating all the phases that make up the solution of a legal issue.

The first phase is made up of the awareness of own rights: the access to justice is first of all the access to information about one's right and to the information about how to exercise one's rights.

At EU level, this function is mainly performed by the EU websites [25]: portals such as e-Justice, e-Law and Eur-Lex. E-justice portal, in particular, is conceived as a one-stop-shop in the area of justice [26], while Eur-Lex is a database providing legal acts, judgments and institution's acts. As explained above, the most recent EU e-Justice strategy has announced the aim to improve access to information and, in this sense, to improve access to justice [27].

The access to justice's second phase could be considered as the material accessibility to the tools and to the procedure to obtain justice. This concept can include several issues. First of all, we have to consider the procedural aspects, espe-

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York, 2007; B. Rainey, E. Wicks, C. Ovey, The European Convention on Human Rights, Oxford, Oxford University Publishing, 2014; C. Timmermans, *Fundamental rights protection in Europe before and after accession of the European Union to the European Convention on Human Rights*, in P. van Dijk, M. van Roosmalen, et al (eds.), *Fundamental Rights and Principles*, Antwerp, Intersentia, 2013; D. Vitkauskas, G. Dikov, *Protecting the right to a fair trial under the European Convention on Human Rights*, Strasbourg, Council of Europe, 2012; A. Ward, *Commentary to art.* 47, in S. Peers, A. Ward, et al (eds.), Commentary on Charter of Fundamental Rights, Oxford, Hart Publishing, 2013; European Union Agency for Fundamental Rights, *Access to justice in Europe: an overview of challenges and opportunities*, Luxembourg, Publications Office, 2011; European Union Agency for Fundamental Rights, *Handbook on European law relating to access to justice*, Luxembourg, Publications Office, 2016; Council of Europe, *Guide to a fair trial: criminal limb*, Strasbourg, 2014.

[25] Recently, DG Justice and Consumers of the European Commission has launched a campaign aiming to make EU citizens become better aware of their key consumer rights and provide them with guidance on how to implement them, see https://europa.eu/youreuright/home_en and https://www.youtube.com/user/EUJustice/videos?disable_polymer=1. The information to the citizen is also provided through the Europe Direct desks established in several cities of the European Union.

[26] See https://e-justice.europa.eu/home.do?plang=en&action=home. As explained by the European Commission in communication "Towards a European e-Justice Strategy", cited, the portal will have the function to provide European citizens, in their language, with data on judicial systems and procedures. It is well-known that ignorance of the rules in force in other Member States is one of the major factors preventing citizens from asserting their rights outside their home country.

[27] In the 2019-2023 Strategy, cited, at point 13 the Council has affirmed that the objective to improve the access to justice includes information on the rights of citizens, which helps to raise their awareness; information on EU law, as well as national law transposing EU law; information on procedures which helps citizens to use the various tools put at their disposal for the sake of conducting such procedures, such as dynamic forms or search tools for practitioners and (judicial) authorities; information on competent authorities which helps citizens to identify competent authorities and relevant national laws, in the framework of judicial or extrajudicial proceedings; publicly available information contained in national registers and data relevant to the use of e-Justice and e-Law.

cially if the procedure can be initiated directly by the natural and legal persons without the assistance of a lawyer. In this case, the access to justice is strictly connected to the feasibility of the complaint and of the handling of the subsequent steps.

Moreover, we have to evaluate the costs of the procedure. In several cases the costs are decided by the Member States implementing the procedure. In this regard, the European Court of Justice has stated that fees and costs must be defined so that it is not in practice impossible or excessively difficult to exercise the rights conferred by European Union Law [28].

Nowadays, the ODR is the only EU procedure that can be initiated online through an EU website [29]. In this regard, the European Commission has affirmed that the ability to complete specific steps in the judicial procedure by electronic means is an important part of the quality of justice systems because the electronic submission of claims, the possibility to monitor and advance a proceeding online can ease access to justice and reduce delays and costs [30].

The access to justice' third phase should be made up by the obtaining of a fair decision [31]. In each national legal order, as in the EU legal order, this result is guaranteed by the application of the law. Nonetheless, in civil matters, considering the length of the judicial process, this result can be better achieved through an ADR. This kind of extrajudicial solution can provide a cost-effective and quick result. Furthermore, as will be see in the following paragraph, ICT can play a role in the improving of the access of justice intended as access to a fair decision [32]. In particular, AI can be applied in order to develop a new kind of decision.

Agreements resulting from mediation are more likely to be complied with vol-

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[28] See Judgment of the Court, 13 December 2012, Iwona Szyrocka, C-215/11, EU:C:2012:794. About the division of costs see also Judgment of the Court of 14 February 2019, Rebecka Jonsson, C-554/17, EU:C:2019:124.

[29] See https://ec.europa.eu/consumers/odr/main/?event=main.complaints.screeningphase.

[30] See the Communication from the Commission to the European Parliament, the Council, the European Central Bank, the European Economic and Social Committee and the Committee of the Regions *The 2018 EU Justice Scoreboard Brussels*, COM(2018) 364.

[31] As mentioned at the beginning of this paragraph and in footnote No 24, this overview on the phases of the access to justice has been carried out following a practical approach, without prejudice to the legal content of this fundamental right. The respect of all the aspects of the access to justice and, in this sense, to the fair trial and to an effective remedy, are summed up as "the application of law". Between the several declensions of the right to a fair trial it can be recalled the right to a fair and public hearing before an independent and impartial tribunal, to a legal aid, the right to be advised, defended and represented.

[32] More in detail, in the next paragraph it will be underlined the improvement to the access to justice through the use of a software developed by an algorithm able to find the best solution in the division of goods. In general terms, the ICT can also help in accessing judgments: ensuring access to judgments online increases the transparency of justice and helps citizens and businesses understand their rights.

untarily and are more likely to preserve an amicable and sustainable relationship between the parties [33]. This leads us to the effects of this kind of solution to the problem that usually belong to the fourth phase of the access to justice, that is to say the obtaining an effective execution of the judicial decision.

The execution of a judgement in another Member State is complicated by the need to start the procedure in the State of execution applying the national law.

The EU has introduced several instruments in order to facilitate the cross-border solution of disputes relates to monetary claims [34]. Through this procedure, the part can obtain a title that can be executed in another Member State without the need of an exequatur. Even if this result facilitates the circulation of the judgements, an effective easy execution trough Europe is complicated by the difficult, also for practitioners, in obtaining information on the competent jurisdiction, the procedure and the cost.

5. The implementation of CREA Project results in the EU legal order

The CREA project (Conflict Resolution through Equitative Algorithms), funded by European Union's program Horizon 2020, aims to introduce new mechanisms of dispute resolution as a helping tool in legal procedures for lawyers, mediators and judges with the objective to reach an agreement between the parties.

At the end of the project, it has been developed a software applying game-theoretical algorithmic mechanisms to the solution of certain national and cross-border civil matters concerning the division of goods between co-owners. This methodology can be applied to the allocation of goods in areas such as inheritance, divorce and co-ownership in commercial law.

The objectives and the functioning of the CREA Software have been better explained in other publications. Nonetheless, for the purpose of this contribution, it is important to underline that in this project ICT has been applied by demonstrating the efficacy of an algorithmic approach in leading the parties to a friendly solution before or during the trial. Therefore, the CREA software must be considered as an instrument based on the application of Artificial Intelligence that improves the access to justice, especially in cross-border disputes.

The CREA software brings to a solution to the dispute that is not only propor-

[33] See whereas n. 6 of the Directive 2008/52, cited. These benefits become even more pronounced in situations displaying cross-border elements.

[34] For a very general recognition of the instruments adopted by the EU for the Judicial cooperation in civil matters, see paragraph 2. In order to simplify and speed up cross-border small claims litigation in civil and commercial matters and cut costs, it has been also adopted the European Small Claims procedure. See Regulation No 861/2007 of the European Parliament and of the Council of 11 July 2007 establishing a European Small Claims Procedure, OJ L 199, 31.7.2007, pp. 1-22.

tional but also «envy-free». This way, it produces allocations in which each participant believes he or she receives the best portion of the goods being divided, based on its subjective references.

The parties express their preferences via web and the procedure leads to the best solution in a short time. Therefore, the CREA software improves the access to justice considered both as an access to material accessibility to the tools and to the procedure to obtain justice, the obtaining of a fair decision and the obtaining an effective execution of the judicial decision.

The use of the software is intuitional and easy, and it is sufficient a web connection, for this reason, this instrument allows an easy material access to the procedure.

Furthermore, in suggesting an envy-free solution, the CREA software brings the parties to the best solution. The fairness of the decision will be verified by the respect of the mandatory law.

Lastly, an agreement has higher chances of being executed quickly and of one's own accord, without the need of an execution procedure.

The CREA software will be a helpful tool in the solution of cross-border disputes as well as national ones. Evidently, in the first case, the gain in time and cost is more remarkable.

In any case, this instrument shall guarantee the complying of the solution proposed with mandatory rules of applicable law.

One of the objectives of the CREA project was the creation of a «European common ground» of available rights, different from standard legal principles in order to develop a software that uses algorithms that rapidly implement better settlements with consistent rules so that the settlement complies with the mandatory rules established in the Member States.

At now, a common ground has been created and the Law Unit has underlined the relevant question related to each field in every Member State involved. However, the algorithm implemented in the software does not include legal aspects. For this reason, at the present time the CREA software should be used under the guide of a mediator (or a lawyer, or a judge) that verifies the consistency of the settlement with the mandatory rules established by the national law. Nonetheless, when the software will be completed under this aspect, it should be used also by citizens autonomously, without prejudice to the need to submit the agreement to the judicial authority, if required by national law.

In any case, the CREA software is a useful tool for resolving disputes and it responds perfectly to the needs expressed in the European e-justice strategy and Action Plan. Therefore, it would be appropriate to include this tool, as ADR tool, on the e-justice portal.

Moreover, the new procedure could be integrated in the EU legal framework through the adoption of a legal act.

In this purpose, it should be considered art. 81 TFEU as legal basis, where it establishes that «European Parliament and the Council, acting in accordance with

the ordinary legislative procedure, shall adopt measures, particularly when necessary for the proper functioning of the internal market, aimed at ensuring: […] (e) effective access to justice; […] (g) the development of alternative methods of dispute settlement». As explained above [35], according to the first paragraph of the cited article, the development of judicial cooperation in civil matters is limited to the disputes having cross-border implications. For this reason, a legal act adopted on this legal basis, aimed at facilitating the access to ADR and at promoting the amicable settlement of disputes relates to the division of goods by encouraging the use of CREA software, will be limited to cross-border disputes. Nonetheless, as established in Directive 2008/52 [36], nothing should prevent the Member States from applying such provisions also to internal disputes.

It should be considered that this limitation could be bypassed adopting an act on the legal bases provided by art. 114 TFEU: measures for the approximation of the provisions laid down by law in the Member States. In such a case, the EU legal act could only dispose for measures which have as their object the establishment and functioning of the internal market. Therefore, such a legal act should rule division through CREA software for co-ownership of companies, considering the close relationship to the functioning of the internal market.

The adoption of a legal act requires, of course, the handling of the legal procedure by the European Institutions and this process requires times and implies an important positive impact for the parties.

We are sure about the positive impact from the use of the CREA software and also for this reason we strongly recommend the inclusion and the promotion of this tool on the e-justice portal. The use of this instrument and the feedback by the parties will allow the improvement of this instrument in order to define the best use, also in the light of the adoption of a legal act.

[35] See paragraph 2.

[36] Directive 2008/52/EC of the European Parliament and of the Council of 21 May 2008 on certain aspects of mediation in civil and commercial matters, cited.

Chapter 3

Modeling the CREA algorithm

Marco Dall'Aglio *, Daniela Di Cagno ** and Vito Fragnelli ***

Fair Division Algorithms and Experiments: A Short Review

Abstract

We concisely review established and recent results regarding procedures to allocate several objects (also referred to as items or goods) to a finite number of entities (usually denotes as agents or players). After having described how to formally represent preferences, and what defines a good allocation, we list the procedures. For each procedure we outline the assumptions, the terminology specifically employed for the occasion, and the results achieved, together with specific remarks for the examined procedure. Our analysis reveals that some procedures are mature enough to guarantee a reasonable degree of success in the legal context. Some others are very recent, but their strong theoretical grounds provide a natural perspective of applications in the legal context.

Behavioral and experimental economics provides rigorous evidence of the effectiveness of these procedures and the perception of those among the involved agents.

Keywords: Fair Division, Algorithms, Behavioral Economics

1.1. Introduction

The theory of fair division dates back to the end of the second world war. It was devised by a group of Polish mathematicians, Hugo Steinhaus, Bronisław Knaster and Stefan Banach, who used to meet in the Scottish Café in Lvov (then in Poland). Since then, many results have been obtained and many scientific papers have been written.

In this brief review, we will examine the most recent results, but we will focus

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our attention to a small area of research. The following restrictions will be enforced:

- We will be mostly interested in results regarding the problem of allocating several objects (also referred to as items, goods) to a finite number of persons, usually referred to as agents or players;
- Items will usually be considered indivisible, but also divisible items will be accepted with the idea that:
  - The procedures will return solutions where a very limited number of items will actually have to be divided among the persons involved;
  - Often, a monetary compensation is a good proxy of the actual division of an item.
- Although the majority of the scientific achievements regard the relationship between mathematical definitions, we will be interested mostly in ready-made procedures that could be, in principle, straightforwardly adapted to the legal context at hand.
- Mathematical notation will be avoided, whenever possible. [1] Our belief is that the lack of mathematical precision will be more than compensated by the opportunity to reach a larger audience.

1.2. How to represent preferences [2]

There are essentially two methods for expressing the agents' preferences over the allocations.

Ordinal preferences. A first method relies on the minimal assumption that agents are able to compare each pair of individual items. Under this hypothesis, each agent is able to rank each item from best to worst. A convenient further assumption requires that no ties are allowed.

Cardinal preferences. Each item is assigned a number. Usually these number are positive (or at least non-negative). In this case agents not only express a ranking, but also quantify the degree of preference: If an agent assigns 30 to a first item and 10 to a second item, he/she is telling us not only that he/she prefers the first item, but also that the first item is appreciated three times more than the second one.

Cardinal preferences provide more information that ordinal ones, but they are harder to assess. In specific situations, it is customary to transform an ordinal preference into a cardinal one, but the process comes at the cost of strong assumptions whose consequences are not assessed properly in most cases. This is the

[1] The description of the first procedures, however, will require some mathematical formulation.

[2] Quotes in this section are taken from Bouveret et al. (2016).

case, for instance, of the **Borda count**. Here, an agent ranking p items from best to worst, will obtain a utility level of p by receiving the most preferred item, of p-1 by obtaining the second most preferred item, and so on up to the least preferred item which will yield a utility level of only 1.

In general, "ranking [or valuating single] items is generally not enough to provide valuable information about the agents' preferences concerning different allocations. Consider for example a setting where four objects {o1, o2, o3, o4} have to be allocated to two different agents. Suppose that agent 1 ranks the objects as follows: o1, then o2, then o3 and finally o4. Does it mean that she would prefer an allocation that would give her o1 and o4 to an allocation that would give her o2 and o3? Or an allocation that would give her o1 to an allocation that would give her o2 and o4? The technical problem that lies behind this kind of questions is the problem of lifting the preference relation on individual objects to a preference relation on bundles of objects. There are two possible ways of doing it:

i. Either by automatically lifting preferences to bundles of objects using some natural assumptions;
ii. or by asking the agents to rank not only the individual objects but also the bundles of objects".

Lifting preferences from items to bundles. When **ordinal preferences** are considered, two natural, but strong, assumptions are

Monotonicity: An agent always prefers a larger set to a smaller one containing less items. This means that items are always beneficial to the agents, or they can be disposed of at no additional cost.

Responsiveness (or separability): "An agent with responsive preferences will always be able to pick unambiguously the object that she prefers among a set, this choice being independent from what she has already received, and what she will receive later on." In other words, there are no complementarities or substitutabilities among items.

When **cardinal preferences** are considered "monotonicity and responsiveness are replaced by **additivity**. The value of a bundle of items is simply given by the sum of the values of the single items composing the bundle."

"Additivity is a very strong property that forbids any kind of synergy between objects. Going back to our previous example with four objects, additivity implies that because agent 1 prefers o1 to o2, she will also prefer {o1, o3} to {o2, o3}. This makes sense if o3 is rather uncorrelated to o1 and o2: for example, if o1 is a voucher for a train ticket in France, o2 is a voucher for a night in Paris, and o3 is a camera, it seems reasonable to assume that my preference on taking the train rather than spending a night in Paris will hold, no matter whether a camera is delivered with the voucher or not. Another way to state it is to say that if in bundle {o2, o3} o2 is replaced by a better object (e.g., o1), then it makes a better bundle."

Additivity (or monotonicity and responsiveness) fails when "objects are of similar nature or are closely coupled. For example, if o3 is now a plane ticket for the same day as the train ticket, we can reasonably assume that my preferences will be reversed, because now only the night in Paris is compatible with the plane ticket (so by getting the night and the plane ticket I can enjoy both, whereas by getting the train and plane tickets I will have to drop one of the two). This is a case where additive preferences fail to represent what the agents really have in mind, because there are some dependencies between objects. These dependencies (or synergies) can be of two kinds: complementarity or substitutability. Complementarity occurs when having a group of objects is worth more than the "sum" of their individual values: the agent benefits from using them jointly. Going back to our previous example, the plane ticket and the night in Paris can be considered as complementary (if I am not living in Paris): I can use the plane ticket to fly to Paris, and then spend the night there. Substitutability occurs when objects are of very similar nature and when their use is mutually exclusive. In our example, the plane and the train tickets are exclusive, and thus, their joint value is not more than the value of one of the two."

For a review of the more sophisticated frameworks that take these issues into account we refer to Bouveret et al. (2016) Section 12.1 and Lang and Rothe (2016), Section 8.3.

1.3. Properties for allocations [3]

"In order to choose a "best" allocation, or a subset of admissible allocations, or a ranking of allocations according to their quality, we need to define criteria. We distinguish two classes of criteria: (i) those of the first class use cardinal notions, […] and (ii) those of the second class use purely ordinal notions. Since a preference relation can be induced from a utility function, all ordinal criteria are also applicable to settings with numerical preferences and, therefore, it makes more sense to start with ordinal criteria."

1.3.1. Ordinal criteria

We briefly review the most important criteria.

Pareto optimality or Efficiency. "Informally, an allocation is Pareto-efficient if it cannot be improved to another allocation which is at least as good for every agent and strictly better for at least one agent. […] The terms Pareto efficiency and Pareto optimality are used synonymously."

Envy-Freeness. "Informally, an allocation is envy-free if no agent prefers the share of another agent to her own."

[3] Quotes in this section are taken from [LR16].

"Envy-freeness and Pareto efficiency are both stability criteria: If one of them is not satisfied, then at least one agent has a strong objection against the allocation. Ideally, we should try to output allocations that satisfy both. Unfortunately, there are resource allocation problems for which there exists no allocation being both Pareto-efficient and envy-free. This can be seen on the very simple example with two objects, a and b, and two agents who both prefer a to b (note that this example shows that there may even exist no allocation being envy-free and complete, that is, assigning all objects)."

"Because it is not always possible to ensure envy-freeness while preserving Pareto efficiency, some authors have proposed to relax envy-freeness, by defining degrees of envy."

Max-Min Fair Share and Min-Max Fair Share. The max-min fair share of an agent with cardinal preferences expressed by a utility function, is defined as the maximum, over all allocations, of the utility of the worst share that [the agent] gets according to his own utility function, whereas his min-max fair share is the minimum, over all allocations, of the utility of the best share that the agent gets according to his own utility function. An allocation satisfies the max-min (respectively, min-max) fair share criterion if each agent gets a share that he values at least as much as his max-min (respectively, min-max) fair share. As shown by Bouveret and Lemaître (2014), envy-freeness implies the min-max Fair share criterion, which in turn implies the max-min fair share criterion. Even for the weakest of the criteria, that is, the max-min fair share criterion, there exist resource allocation problems for which no complete allocation satisfying the criterion exists.

1.3.2. Ordinal criteria

We briefly review the most important criteria.

Proportional fair share. "The proportional fair share of an agent is the ratio of his value for the whole set of objects by the number of agents. An allocation satisfies the proportional fair share criterion if every agent has at least his fair share." The fair share is defined as a fraction n of the total value of the goods, n being the total number of agents involved in the division.

Maximum utilitarian social welfare. Utilitarian social welfare, defined [as] the sum of the utilities all agents realize in [the actual] allocation." "Utilitarianism is the standard optimization criterion in settings where a central authority seeks to maximize overall revenue." "It does not reflect, though, how utility is distributed among the agents. For example, it might be the case that all goods are assigned to just a single agent, so this agent alone would realize the entire utility, whereas all other agents come away empty-handed. In other words, allocations with a high utilitarian social welfare can still be quite unfair. Egalitarian social welfare aims at correcting this."

Pure Egalitarian Social Welfare. "This social welfare measure maximizes the minimum of the agents' utilities; therefore, an allocation with maximum egalitarian social welfare is sometimes referred to as a max-min allocation."

Maximum Leximin Social Welfare. "While egalitarian social welfare aims at guaranteeing fairness by maximizing the utility of the least happy agent, it does so in an extreme way, to the detriment of efficiency. Leximin social welfare is a refinement of pure egalitarian social welfare: While the latter only pays attention to the utility of the least happy agent, the former allows to break ties between two allocations maximizing the utility of the least happy agent by paying attention to the second least happy agent, and in case there are still ties, then to the third least happy agent, and so on."

It is worthwhile noticing that, under very broad circumstances [4], both Pure Egalitarian and Maximum Leximin allocations satisfy.

Equity: all the agents assign the same evaluation to the part each of them received.

In this case we also speak of **equitable** allocations.

Maximum Nash Social Welfare. Social welfare by the Nash product, defined as the product of agents' utilities in the allocation "can be seen as some kind of compromise between egalitarian and utilitarian social welfare." "Even though this social welfare notion may look a bit uncommon at first glance, it does have some useful properties." For example, it is "fairer" than utilitarian social welfare in the sense that the more balanced the single agents' utilities are in an allocation, the higher is their Nash product." In fact, this measure is maximal for an allocation where all agents realize the same utility level.

1.4. The origins of fair division – First procedures

In this report, we present the most classical results, following the books by Brams and Taylor (1996) and (2000).

When one or more items have to be divided fairly among n agents, the main problem arises from the heterogeneity of their preferences, i.e. they have different evaluations of the items or of parts of them. Moreover, the preferences of an agent are her/his own private information.

Consequently, the aim of a procedure is not only to produce a fair division, but also to favor the truthfulness, i.e. an agent that misreport her/his preferences should not get an advantage from this behavior.

Another feature is that sometimes a mediator may play an important role in

[4] For instance, when all agents assign positive evaluation to all goods (and all parts of the goods in case of heterogeneity).

performing the steps of a procedure, while the decision should never be transferred to an arbitrator.

It is worthwhile to remark that the optimality properties described in the previous section have different meaning when applied to a division or to a procedure. In fact, a procedure satisfies a property when the resulting division always satisfies the same property for all the agents that correctly performed the steps of the procedure. In other words, also an unfair procedure may produce a fair division under particular situations.

The simplest example is the division of a heterogeneous cake among two agents; the classical procedure according to which one agent divides the cake in two parts and the other one chooses the preferred part, satisfies proportionality as each agent receives at least one half of the cake, and consequently envy-freeness. On the other hand, neither efficiency, nor equity are satisfied, as it is possible that another division could make at least one agent better off and it is possible that the choosing agent receives more than one half, while the other one receives exactly one half.

1.4.1. One divisible good

Let us consider a heterogeneous good (e.g. a rectangular cake) that has to be divided among n agents that do not reveal their true preferences on the different parts of the good. The abovementioned procedure "I cut, you choose", was formalized by Dubins and Spanier as follows:

Divide and choose for two agents (Dubins-Spanier, 1961)

a) A mediator moves a knife along the cake from left to right;
b) One agent calls "cut";
c) The calling agent receives the part on the left of the knife, while the other one receives the part on the right; STOP.

The calling agents considers the two parts of the cake indifferent, so s/he receives exactly one half of the cake; the other agents may receive a part of the cake that s/he prefers w.r.t. the other one.

Austin proposed a variation that guarantees equity:

Moving Knives for two agents (Austin, 1982)

a) A mediator moves a knife along the cake from left to right;
b) One agent calls "stop";
c) A second knife is placed on the left border of the cake and the calling agent moves both knives toward right;
d) In any moment, the other agent may call "stop";
e) The mediator, or the chance, decides which agent chooses one of the two parts; STOP.

The procedure preserves equity because the calling agent may move the knives in such a way that the part in between them is evaluated one half of the cake for her/him (note that the distance among the knives may vary), so that s/he obtains one half of the cake; if the second agent calls "stop" when the part in between the knives is equivalent for her/him to the remaining of the cake, then also s/he obtains one half of the cake, whatever the final decision.

It is worthwhile to remark that the previous procedures may be applied also for dividing m items if it is possible to determine two bundles that are indifferent for at least one agent.

Dubins and Spanier proposed a variation of their procedure that is suitable for more than two agents.

Divide and choose for more than two agents (Dubins-Spanier, 1961)

a) A mediator moves a knife along the cake from left to right;
b) One agent calls "cut";
c) The calling agent receives the part on the left of the knife and is considered satisfied and out of the game;
d) If there are at least two other agents, return to step a)
e) Otherwise the last agent receives the remaining part of the cake; STOP.

This procedure is proportional, as the first calling agent receives at least one n-th of the cake, leaving to the remaining agents at least one (n-1)-th of the cake, and so on. Of course, envy may show up as each agent could prefer the part assigned to one of the agents awarded after her/him.

Another procedure for more than two agents is due to Banach and Knaster and first described by Steinhaus.

Last Diminisher for more than two agents (Steinhaus, 1948)

a) A mediator, or the chance, sorts the agents;
b) Agent 1 cuts his part;
c) The following agent may further cut the part, reducing it;
d) If there are other agents, then go to step c);
e) Otherwise the last agent that operated a cut receives the part s/he cut and is out of the game;
f) The cake is reassembled;
g) If there are at least two agents, then go to step b);
h) Otherwise the remaining part is assigned to the last agent; STOP.

This procedure is proportional for the same reasoning of the previous one and may be not envy-free.

1.4.2. One divisible good

The following procedures consider the case in which m indivisible items have

to be divided among n agents, accounting for their evaluations, instead of their preferences.

Indivisible means that the value of each item is strongly reduced when it is divided (e.g. an artwork); consequently, each item is assigned to a unique agent, but monetary compensations allow for increasing the fairness.

Let $B = \{b_1, b_2, ..., b_m\}$ and $N = \{1, 2, \ldots, n\}$, $n \geq 2$, be the set of items and the set of agents, respectively. Each agent assigns to each item the value v_{ik}, $i \in N$, $k = 1, 2, ..., m$ and the value is independent from who received the other items (additivity). Let B_j be the set of items received by agent j and $V_{ij} = \sum_{k|b_k \in B_j} v_{ik}$ be the value assigned by agent i to the bundle B_j.

Knaster and Steinhaus proposed a procedure in which each agents declares her/his valuation of each item, like an auction, and the items are assigned jointly with a monetary compensation.

Sealed Bid procedure (Knaster, 1946 and Steinhaus, 1948)

a) Each agent $i \in N$ assigns the value v_{ik}, to each item b_k ($k = 1, ..., m$); let $E_i = \frac{1}{n}\sum_{k=1,...,m} v_{ik}$;
b) The item b_k is assigned to the agent i(k) whose valuation is the highest, i.e. $i(k) = \text{argmax}\ \{v_{ik}, i \in N\}$; let $v_k = v_{i(k)k}$;
c) Let $G_i = \sum_{k:i(k)=i} v_k$, i.e. the sum of the items assigned to agent $i \in N$ (clearly $\sum_{i=1,...,n} G_i = \sum_{k=1,...,m} v_k$);
d) Let $s = \sum_{i=1,...,n}(G_i - E_i)$ (surplus);
e) Let $V_i = E_i + \frac{s}{n}$, $i \in N$;
f) If $V_i - G_i > 0$, then agent i receives an equivalent monetary amount in addition to the items assigned to her/him;
g) Otherwise agent i pays a monetary amount equivalent to $G_i - V_i$; STOP.

As the surplus s is non-negative, the division is proportional.

By step e), the division satisfies equity if and only if all the agents assign the same value to the whole bundle of items, i.e. $E_i = E$, $i \in N$.

According to a note by Fink (1964), the procedure may be applied to each single item, with the same final result.

Raith (2000) proposed a variation of the previous procedure called "Adjusted Knaster" that always satisfies equity.

It is sufficient to modify step e) as

e′) Let $V_i = E_i + s\ \frac{E_i}{\sum_{j=1,...,n} E_j}$;

In this way, $\frac{V_i}{E_i} = \frac{V_j}{E_j}$ for each pair of agents $i, j \in N$.

The procedure is not envy-free; it is sufficient to consider three agents that as-

sign different values to a unique item; the item is assigned to the agent with the highest valuation, but the compensations of the two other agents are different, so that the one with the lowest valuation envy the other agent.

1.4.3. Several divisible items

Now, we devote our attention to the division of m divisible items among two agents, with no monetary compensation.

Divisible means that the value of a fraction of each item is equivalent to the same fraction of the value of the whole item for all the agents.

Again, the values of the items are additive.

The following procedures are based on the assignment of a positive value to each item by each agent such that the value of the whole bundle is 100.

Brams and Taylor proposed the following division that is envy-free (and consequently, proportional), efficient and satisfies equity, in which at most one item is divided among the two agents.

Adjusted Winner (Brams-and Taylor, 1996)

a) Each agent assigns a positive value to each item such that the value of the whole bundle is 100;
b) Each agent receives the items that s/he evaluates more (items with equal evaluations are assigned randomly);
c) If the two agents assign the same value to the bundle they received STOP; Otherwise, let I be the agent that received the bundle s/he evaluates more;
d) The items are reordered according weakly increasing values of the ratio among the evaluations of agent II and agent I (items with equal ratios are ordered randomly); let $B = \{b_1, b_2, \ldots, b_m\}$ be the reordered set of items and let x_i and y_i the evaluations of agents I and II of item $b_i \in B$, respectively; let $\{b_1, \ldots, b_r\}$ be the set of items assigned to agent I and let $\{b_{r+1}, b_2, \ldots, b_m\}$ be the set of items assigned to agent II; let $X = \sum_{h=1,\ldots,r} x_h$ and $Y = \sum_{h=r+1,\ldots,m} y_h$ the evaluations of the items received by the two agents (by step c) X > Y); let $k = r$;
e) If $X - x_k \in Y + y_k$, then item b_k is "transferred" to agent II; let $X = X - x_k$, $Y = Y + y_k$ and $k = k-1$; go to step g);
f) Otherwise, let $\in$ be such that $X - \in x_k = Y + (1-\in)y_k$ and transfer a fraction $\in$ of item b_k to agent II; STOP;
g) If X = Y, then STOP;

Otherwise, go to step f);

The following procedure, again due to Brams and Taylor, divides all the items among the two agents, in proportion to their evaluations.

The Proportional procedure (Brams and Taylor [BT96])

a) Let $B = \{b_1, b_2, \ldots, b_m\}$ be the set of items and let s_i and t_i the evaluations of

agents I and II of item $b_i \in B$, respectively, such that $\sum_{i=1,\dots,m} s_i = \sum_{i=1,\dots,m} t_i = 100$;

b) For each item $b_i \in B$, the fraction $\frac{s_i}{s_i+t_i}$ is assigned to agent I and the fraction $\frac{t_i}{s_i+t_i}$ is assigned to agent II.

The Proportional Allocation produces a division that is envy-free (and consequently, proportional) and satisfies equity. The division is efficient only if the evaluations of the two agents are equal for all the items, or conversely it is not efficient if there exist two items *k* and *h* such that $\frac{s_k}{s_h} < \frac{t_k}{t_h}$.

1.5. Most recent advances

Here we review the most relevant contribution in the last couple of decades, with a special attention to what was happened in the last few years. Sentences in quotation marks are taken from the references mentioned at the beginning of each procedure.

1.5.1. Two agents – Ordinal preferences

The AL procedure (Brams, Kilgour and Klamler, 2014 and Aziz, 2016)
Assumptions

i. Two agents (players) that must share an **even** number of items.
ii. "Each player is able to rank the items strictly, from best to worse." Moreover, "preferences satisfy responsiveness: any set becomes less preferred whenever an item is removed from it, or replaced by an item that is ranked lower according to [his/her] preferences."

Some terminology

- A novel notion of envy-freeness is needed: **Item Wise Envy-freeness (IWEF)**. This is a definition of envy-freeness that makes only item-by-item ordinal comparisons. "We say that a player does not envy the other player, if and only if there is a one-to-one correspondence from a player's items to the other player's items such that the first player prefers each of its items to the item of the second player to which it is mapped."
- This is a method that makes use of a **Contested Pile (CP)**: the set of items that cannot be assigned and must be assigned later using some another method.

The procedure (an informal description)

If the players have not yet been assigned any items, then if there is an item at the top of both players' rankings, it is put into CP, and this step is repeated until each player most prefers a different unallocated item. When this happens, AL assigns each player its preferred item. After the first assignment of items to the

players is made, new assignments are made:

i. when the players prefer different items or
ii. when they prefer the same item, provided a new assignment – of the preferred item to one player and a less preferred item to the other – does not cause envy and so is feasible.

When there is a commonly preferred item, the feasibility of assigning it to either player is assessed, one player at a time. Only if there is no such assignment is the commonly preferred item put in CP.

Results. The resulting allocation may fail to be complete – some items may remain in CP. However, the following holds:

Theorem (Brams, Kilgour and Klamler 2014). An AL allocation is a maximal EF allocation: There is no other EF allocation that allocates more items to the players.

The following result about **manipulability** holds

M1: The procedure is non-manipulable if players do not know the other player's preferences with certainty and they do not want to take the risk of worsening their outcome. Otherwise, the procedure is vulnerable to manipulation.

The UNDERCUT procedure (Brams, Kilgour and Klamler, 2012)
Assumptions.

i. Two agents (players) that must share a finite number of items.
ii. "Each player is able to rank the items strictly, from best to worse." Moreover, "preferences satisfy responsiveness: any set becomes less preferred whenever an item is removed from it, or replaced by an item that is ranked lower according to [his/her] preferences."

Some terminology:

A **minimal bundle** for a player i "is a set of items worth at least 50% to I, with the special property that any subset [of it] is worth less than 50%, as is any subset obtained by replacing some items [in the set] by less preferred items [not in the set]". A player usually has more than one minimal bundle.

The items that are not immediately assigned to either player, are put in a figurative *contested pile* for later distribution.

The procedure

1. Players A and B each independently name his or her top-ranked alternative. If they name different items, each player receives the item he or she names. If they name the same item, it goes into the contested pile.
2. This process continues until all items have been named by at least one player.
3. If the contested pile is empty, the procedure ends. Otherwise, each player identifies his or her set of minimal bundles from the contested pile and gives this information, in secret, to the referee.
4. If the sets of minimal bundles are different, each player provides to the referee,

in secret, a ranking of his or her minimal bundles. A player (say, A) is chosen at random, and A's top-ranked minimal bundle is considered. If it is not also a minimal bundle for B, then it becomes the proposal, and A is the proposer. If A's top-ranked minimal bundle is also a minimal bundle for B, then B's top-ranked minimal bundle is considered. If this set is not a minimal bundle for A, then it becomes the proposal, and B the proposer. [Otherwise], the process continues until a minimal bundle of one player is found that is not a minimal bundle of the other. Then proceed to step 6.

5. If the sets of minimal bundles are the same, and there exists a set of items [such this set and its complement (i.e., all the other items) both are minimal bundles,] then this becomes the proposal. Otherwise, a minimal bundle is chosen randomly and becomes the proposal."
6. Assume the proposer, say A, makes a proposal of taking a set S of items and leave the complement to B. Then B may respond by
   a. accepting the split which she should do if the complement is worth at least 50% to her) or
   b. undercutting A's proposal, by modifying the proposed split and take for herself a bundle strictly less preferred than A's bundle:

The procedure ends. A player's subset of X consists of all items received in steps 1 and 2, plus the player's share of the contested pile determined in step 6.

Results:

Theorem 1 in Brams, Kilgour and Klamler (2012). There is a nontrivial envy-free split of the contested pile if and only if one player has a minimal bundle that is not a minimal bundle of the other player. If so, then the Undercut Procedure implements an envy-free split.

More recently, Aziz (2015) has proposed a simplified procedure that puts all the items in the contested pile and directly proceeds with steps 3-6 of the Undercut procedure. The modified algorithm will return EF allocations unseen by the original procedure, but it may require a significantly larger effort by the players, who will have to single out the family of minimal bundles among all the items involved in the division.

The TRUMP Rule (Pruhs and Woeginger, 2012)

Assumptions:

i. Two agents a Husband and a Wife that must divide an **even** number of items.
ii. "Agents specify only a ranking of the items, from best to worse."

Some terminology:

- "An allocation is **Proportionally Fair (PF)** if all agents believe that they have received their fair share of the value according to how they value the items.". More in detail, this means that each player will receive at least half of the val-

ue of the whole bundle of goods, according to any quantitative valuation compatible with the rankings specified by the players. It can be shown that this notion is equivalent to Item Wise Envy Freeness (IWEF).

- For any number p less than or equal n., H_p and W_p denote the set of the p most desired items in the rankings of husband and wife, respectively.

The procedure:

i. Starting with p=1
   a. Consider the item in W_{2p-1} that the husband likes least and allocate it to the wife
   b. Consider the item in H_{2p-1} that the wife likes least and allocate it to the husband
ii. Repeat the above steps adding 1 to p until p reaches n.

Results: Theorem in Pruhs and Woeginger (2012). Whenever a divorce situation allows an ordinally fair allocation, the TRUMP rule succeeds in finding one.

The Singles-Doubles (SD) and Iterated Singles-Doubles (ISD) procedures (Brams, Kilgour and Klamler, 2017b)

Assumptions

- Two agents (players) that must share an **even** number of items
- "Each player is able to rank the items strictly, from best to worse." Moreover, "preferences satisfy responsiveness: any set becomes less preferred whenever an item is removed from it, or replaced by an item that is ranked lower according to [his/her] preferences."

Some terminology

- Item Wise Envy-freeness (IWEF) is considered (see the AL procedure).
- Only **balanced** allocations are considered in which half of the items are given to one player, while the other player takes the other half. If there are 2n items, each player will get n items.
- For a given allocation, the rank of the least preferred item is considered for each player, and the highest (worst) rank among the two players is recorded. An allocation is **maximin (MX)** if it minimizes such highest rank. We denote with m such highest value for the rank.
- "Assume m < 2n. An item x is called a **single** if it is a top m item for only one player. We say that x is a **double** if it is a top m item for both players."

The procedures

SD (Singles-Doubles) Algorithm

Input: The two players' rankings of an even number (2n) items.

Output: A complete MX-IWEF allocation to the two players: A and B.

i. Determine the maximin rank, m.
ii. If m = 2n, stop. There are no MX- IWEF allocations. Assign to each player its

singles. Stop if all items have now been allocated.

iii. Assign doubles using the following iterative procedure: Identify each player's most preferred unassigned double. If they are different, assign them accordingly; if they are the same, identify the player who can be assigned its second-most preferred unassigned double while still guaranteeing EF, and assign items accordingly, breaking ties at random. Repeat until all doubles are assigned.

ISD (Iterated Singles-Doubles) Algorithm
Input: The two players' rankings of an even number (2n) items.
Output: A complete MX-IWEF allocation to the two players: A and B.

i. Assign to each player its singles. Stop if all items have been allocated.
ii. If the players' least-preferred unassigned items are different, consider the ranking restricted to the unassigned items, determine the new values of m, identify the new singles, and repeat step i to assign them. Repeat this step until either all items are assigned, or the least-preferred unassigned items are the same.
iii. Assign doubles using the following iterative procedure: Identify each player's most preferred unassigned double. If they are different, assign them accordingly; if they are the same, identify the player who can be assigned its second-most preferred unassigned double while still guaranteeing EF, and assign items accordingly, breaking ties at random. Repeat until all doubles are assigned.

Results: Theorem (Brams, Kilgour and Klamler, 2017b) Assume the two players rank an even number of items. Then an MX-IWEF exists if and only if for no odd number k, the set of top k items for both players is never identical.

Manipulability: "Although not strategy-proof, SD seems relatively invulnerable to strategizing whenever players do not have complete information about an opponent's preferences. The manipulator's task might be further complicated if the opponent is aware that the manipulator might try to capitalize on its knowledge and adopts countermeasures (e.g., through deception) to prevent such exploitation."

1.5.2. Any number of players – Ordinal preferences

The SA procedure (Brams, Kilgour and Klamler, 2017a)
Assumptions

i. There are $n \geq 2$ players and $m = kn$ distinct items to be allocated, where $k \geq 1$ and integral.
ii. Each player has a positive utility for each item and ranks the items strictly.
iii. The utility of a set of items for a player is the sum of the utilities of the items that comprise it.
iv. The notion of Item Wise Envy-freeness (IWEF) is adapted to the context of more than two player. We say that a player, say A, does not envy another player, say B, if and only if there is an injection (a 1-1 mapping) from A's

items onto B's items such that A prefers each of its items to the item of B to which it is mapped.

Procedure

i. On the first round, descend the ranks of the players, one rank at a time, stopping at the first rank at which each player can be given a different item (at or before this rank). This is the stopping point for that round; the rank reached is its depth, which must be the same for each player. Assign one item to each player in all possible ways that are at or above this depth (there may be only one), which may give rise to one or more SA allocations.
ii. On subsequent rounds, continue the descent, increasing the depth of the stopping point on each round. At each stopping point, assign items not yet allocated in all possible ways until all items are allocated.
iii. At the completion of the descent, if SA gives more than one possible allocation, choose one that is PO and, if possible, IWEF.

Results

i. SA rules (i) and (ii) produce at least one allocation that is PO.
ii. When there are two players, if an EF allocation exists, then SA will give at least one allocation that is IWEF and PO.

In terms of manipulability, the usual result M1 holds.

PICKING SEQUENCES (Brams and Taylor, 2000 and Bouveret and Lang, 2011)

Assumptions

i. Any number of agents (players), say p, that must share any number of items, say n.
ii. Each player is able to rank the items strictly, from best to worse.
iii. agents have additive utilities;
iv. a scoring function maps the rank of an object in a preference relation to its utility value – the agents may have different rankings, but this scoring function is the same for all agents;
v. the arbitrator does not know the agents' preferences but only has a probability distribution on the possible profiles.

The procedure. "The […] arbitrator defines a sequence of p agents. Every time an agent is designated, she picks one object out of those that remain. For instance, if n = 3 and p = 5, the sequence 12332 means that agent 1 picks an object first; then 2 picks an object; then 3 picks two objects; and 2 takes the last object."

The results. "Such a protocol has very appealing properties: first, it is very simple to implement and to explain and secondly, it frees the central authority from the burden of eliciting the agents' preferences". "This protocol has been discussed to some extent by Brams and Taylor (2000), who focus on two particular sequences, namely, strict alternation, where two agents pick objects in alternation,

and balanced alternation (for two agents) consisting of sequences of the form 1221, 12212112, and so on. One can feel intuitively that these kinds of sequences are quite fair, in the sense that alternating the agents in the sequence increases the probability of obtaining a fair allocation in the end (for example, the sequence 1221 is more likely to make both agents happy than 1122, where agent 2 is very likely to be disappointed)". The problem of finding the best (fairest) sequence has been investigated by Bouveret and Lang (2011), who proposed a formalization" [with the goal of finding] "the best sequence […] that maximize the expected […] collective utility." Later Kalinowski et al. (2013) have shown that "the strict alternation policy is optimal with respect to the utilitarian social welfare" thus proving "the intuitive idea that under mild assumptions, a sequence like 12121212... maximizes the overall utility of the society".

THE DESCENDING DEMAND PROCEDURE (DDP) (Herreiner and Puppe, 2000)

Assumptions

Any number of agents who can rank all subsets of items. Preferences satisfy monotonicity: adding items to the allotment of an agent can only bring him/her benefit.

Some terminology

An allocation that maximizes the rank in the preference ordering of the bundle obtained by the worst-off agent is called **Rank Maximin Optimal (RMO)**.

The procedure

An ordering of the agents is fixed beforehand: one by one, they name their preferred bundle, then their next preferred bundle, and so on. The procedure stops as soon as a feasible complete allocation can be obtained, by combining only bundles mentioned so far in the procedure. There may be several such allocations, in which case the Pareto-optimal ones are selected.

Results

Theorem (Herreiner and Puppe, 2002) DDP produces a PO and RMO allocation.

The allocation however, may fail to be EF.

1.5.3. Two players – Cardinal preferences

Extension of the Adjusted Winner procedure to indivisible items. Dall'Aglio and Mosca (2007) consider the problem of allocating a finite number of indivisible items to two players with additive utilities. The algorithm proceeds by allocating a large subset of objects directly without having to rank them. and makes repeated use of an extension of the Adjusted Winner. With techniques from integer programming, the procedure finds new candidate solutions, and suggests which items should be assigned to the players, making a repeated use of the

Adjusted Winner procedure to bar proposals, or to measure the goodness of the surviving suggestions.

1.5.4. Any number of agents – Cardinal preferences

Extension of AW to more than two agents (Olvera-López and Sánchez-Sánchez, 2014, Dall'Aglio, Di Luca and Milone, 2017)

"The Adjusted Winner (AW) procedure works for two players and returns an allocation with many appealing properties: it is (strongly) Pareto-optimal, it is equitable, and it is envy-free. Moving from a two-player setting to a three-player one is not an easy extension. Dall'Aglio and Hill (2003) presented a series of examples, with three or more players, where the three properties ensured by the AW procedure could not coexist. This difficulty was also noted by Brams et al. (2013). Since Pareto optimality is an essential requirement, it is necessary to choose between equitability and envy-freeness. Olvera-López and Sánchez-Sánchez (2014), adopted a linear programming approach to find the [Pure Egalitarian] allocation, which, under the hypothesis of Mutual Absolutely Continuous (MAC) utilities is also Pareto-optimal and equitable for a finite number of players. This is done by transforming the fair division problem into an optimization problem over a bipartite graph, with the nodes on one side representing players, and the nodes on the other side denoting goods. In Dall'Aglio, Di Luca and Milone (2017) for the case of three agents, an alternative graph-based approach, with the graph originating from the tight relationship between two geometric objects, the Individual Pieces Set (IPS) and the Radon-Nikodym Set (RNS) specifically introduced to deal with problems in fair division. This graph is obtained by placing the objects in the RNS and by considering the objects and the intersections between lines joining these objects to the vertices of the RNS as nodes, and these lines as edges. While the generality of the previous approach [is not achieved], this approach is more intuitive. In fact, once the objects are plotted on the RNS, every Pareto-optimal allocation can be visualized immediately. The optimal allocation is sought from among all these allocations by moving from one node of the graph to an adjacent one, until a local optimum is found. This is the global optimal allocation for the problem." [5]

The Nash Product Maximizer (NPM) for Divisible Items (Bogomolnaia, Moulin, Sandomirskiy, and Yanovskaya, 2017)

Assumptions

- Any number of agents dividing any number of divisible goods.
- Cardinal utility. Each player assigns a value to an item. Utilities are linear, i.e.
  - Utility is additive: the value of a bundle is the sum of the values of the sin-

[5] Quote from Dall'Aglio et al. (2017).

gle items.
  - Goods are divisible and homogeneous: receiving a fraction between 0 and 1 of an item yields the same fraction of the utility for the whole item.
- Utility does not need to be normalized for all the agents. The sum of the utility for all the goods may be different for different agents: what counts is the relative weight that each agent assigns to the assorted items.

Some terminology:

- A **Competitive Equilibrium with Equal Income (CEEI)** equilibrium allocation is an allocation where each good is assigned a public price and agents are endowed with an identical budget. Agents can buy the goods in their entirety or only a fraction of them at the posted price. If at the end of the process, all the goods are sold in their entirety with no duplications, the prices and the resulting allocation form the equilibrium.

The procedure: The Maximum Nash Social Welfare Function is computed by means of convex programming techniques

The results (see also Moulin, 2003):

i. The optimal allocation is Proportional, Pareto Optimal and Envy-free.
ii. The optimal allocation is also a CEEI Equilibrium allocation.

Manipulability. The Optimal allocation satisfies several axioms that show its robustness with respect to manipulability, especially when compared to the Pure Egalitarian solution [6]: In particular:

- **Resource Monotonicity (RM).** "As we add new goods to the pot (or increase the quantity of some goods) the welfare of all beneficiaries should improve at least weakly. This is a compelling and popular solidarity property in the common property regime. Originally introduced for the fair division of private goods, it was then applied to a broad range of resource allocation problems with production and/or indivisibilities. Its incentive aspect is that, if RM fails, some agents have an incentive to sabotage the process by destroying some goods or failing to discover them."
- **Independence of Lost Bids (ILB)** "means that nothing changes when we lower a losing bid: it remains losing and the allocation selected by the rule does not change. The ILB axiom implies a weak incentive property: misreporting on an item which I do not consume anyway (whether I misreport or not) does not pay, and does not affect anyone else either".
- Other axioms confirm the robustness of this solution in terms of manipulability.

In fact, the Pure Egalitarian rule does not satisfy any of the above axioms.

An important extension to the allocation of bads and mixed goods and

[6] Under the hypothesis that all the agents assign a positive value to every item.

bads. The same framework is used in (2017) and (2019) to analyze the important problem where not all goods bring benefits to the agents. When an item gives negative utility to an agent it is usually referred to as a "bad". Since both positive, zero and negative utilities are considered, in place of the MNSW solution, which may be meaningless in this situation, the CEEI solution is sought. It turns out that:

- **When all items are bads for all players**. While for goods, the Competitive division fares better […] and makes it harder to strategically misreport preferences, […] for bads, the Competitive rule, unlike the Egalitarian one, is multivalued, harder to compute, and admits no continuous selection.
- **When goods and bads are mixed**, and there are items that are goods, or bads, for every agent, or goods for some agents and bads for others, there can be two distinct types of problems.
  - "If goods overwhelm bads the Competitive rule behaves just like an all goods problem: it picks a maximizes the product of utilities, yields a unique utility profile, is resource monotonic and continuous."
  - If instead "bads overwhelm goods we are back to the potentially messy situation of an all bads problems with a host of different competitive divisions and no continuous selection from this set."

The Nash Product Maximizer (NMP) for Indivisible Items (Caragiannis et al., 2016)

Assumptions

Same as above, with indivisible items. Cardinal and additive preferences are considered. Also in this case, utilities need not to be normalized.

Some terminology:

- When indivisible items are considered, envy-free allocations may fail to exist. We consider the following weaker notion. An allocation is **Envy-free Up to One Good (EF1)** if an agent envies another agent, but the envy can be eliminated by removing a single good from the bundle of latter agent.

The procedure

The Maximum Nash Social Welfare Function is computed by means of integer programming techniques (see section 5 in Caragiannis et al., 2016, for details)

The results

- The MNSW allocation is PO and EF1.
- The MNSW is close to the Max-Min Fair Share allocation. In detail, the MNSW allocation value for each player is always greater than a constant fraction of the Max-Min Fair Share value, with the constant fraction being dependent only on the number of agents.

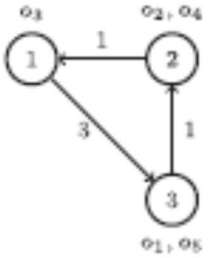
The Envy Cycle Procedure (Lipton et al., 2004)

Assumptions

Any number of agents. Each agent assigns a utility to any bundle of items. The utility is monotone: receiving more goods can only benefit the agent.

Some terminology:

- **The marginal utility** of a good given an agent and a bundle is the amount of additional utility that this object yields when taken together with the bundle. In an additive setting, where the value of a bundle of items is simply given by the sum of the value of the single items, this is thus simply the highest utility (value) that an agent assigns to a good. We denote as α **the maximum marginal utility.**
- The **envy graph** is a graphical tool in which agents are represented by circles. If, for a given allocation of the goods, an agent envies another agent, this is represented by an arrow from the envying agent to the envied agent. For instance, the following picture shows a situation where agent 1 envies agent 3 (and the amount of envy is 3), agent 2 envies agent 1 (amount 1) and agent 3 envies agent 2 (amount 1). The arrows form an envy cycle. If each agent gives his/her bundle to the agent who is envying him/her the envy cycle is broken and the envy is eliminated or reduced.



- When indivisible items are considered, envy-free allocations may fail to exist. One may seek allocations that reduce the maximum amount of envy, i.e., the maximum number on the arrows of the above graph. A first estimate of the best that can be done in this respect was formulated in Dall'Aglio and Hill (2003).

The procedure

i. Goods are allocated one by one.
   a. First allocate one good arbitrarily.
ii. Now consider the end of round k, and suppose k items have been allocated, and that envy is bounded by α.
iii. At round k + 1 we build the envy graph.
   b. If an envy cycle appears, we rotate the bundles as previously described.
   c. At some point, there must be an agent that no one envies.
iv. We then allocate object k+1 to this agent. Envy is thus at most α.

Results: Theorem (Lipton et al., 2004) It is always possible to find an allocation whose envy is bounded by the maximal marginal utility of the problem.

1.6. Further topics

1.6.1. Allocation of Money and Indivisible Goods [7]

Money is the divisible good par excellence. We briefly describe two papers (Alkan et al., 1991, Bevia, 1998) where money is used to level the disparities that may arise when indivisible goods are allocated.

Assumptions

- Any number of agents share any number of indivisible goods and an amount of money. Money is divisible and mitigates the inequalities that could arise by the distribution of the indivisible goods.
- Any agent will equate any subset of items to an amount of money. Preferences are therefore cardinal.
- Each agent will receive a subset of the items and an amount of money, with the constraint that the sum of the amounts given to the agents equals the amount initially available.

Some terminology:

- In principle, an agent may receive a negative amount of money, meaning that she has to pay some money to compensate for the goods received.
- An allocation of goods and money "is **consistent** if whenever an allocation is a good recommendation […], the restriction of this allocation to any subgroup of agents is also a good recommendation […] for the problem of allocating the resources received by this subgroup".
- An allocation is **Group Envy-Free (GEF)** if "no group of agents is able to make all its members better off, and at least one of them strictly better off, if they were given the resources attributed to any other group of the same size".

Results:

- An allocation that is Pareto-Optimal and Envy-Free (POEF) always exists (Proposition 2.2 in Bevia, 1998).
- If only solutions where each agent receives money (and does not have to pay), a congruous amount of money is required for the POEF solution to exist (Proposition 2.3 in Bevia, 1998, where the exact quantity is specified).
- Usually, there are many POEF solutions. Among these, the one in which the objects are as evenly distributed as possible (in terms of number of items per agent) stands out because it is consistent (Proposition 3.1 in Bevia, 1998).
- If all agents are entitled to receive at most one item, any POEF solution is also GEF and it is a CEEI equilibrium (see Alkan et al., 1991, for details]).

Further remarks: In the same group of methods, we may also consider variations of the Adjusted Winner procedure, with its extensions, and the Nash Product Maximizer for divisible items. In fact, whenever the solution indicates to split an

[7] Quotes from Bouveret et al. (2016).

item among several agents, we may consider giving the whole item to one of the agents. This agent will compensate the other agent entitled to the good with monetary side payments.

1.6.2. Allocation solutions via optimization problem

When preferences are cardinal, an optimal solution can be obtained as a solution of an optimization programming.

- "The problem of computing [Maximum Leximin Solutions] has been studied in depth by Bouveret and Lemaître (2014), who describe five specific algorithms using constraint programming and compare them both theoretically and experimentally. For further work on resource allocation by a leximin approach, we refer to the survey by Luss (1999)."
- "The practical computation of [Pure Egalitarian] allocations via mixed integer linear programming was addressed by Lesca and Perny [LP10]. Additionally, they go beyond the max-min fair share criterion and propose a general family of criteria, based on ordered weighted average operators that span from maximum egalitarian to maximum utilitarian social welfare."

1.7. The role of behavioral and experimental economics

Behavioral and experimental economics have recently shown the relevant role that fairness, inequity aversion, envy and trust in affecting individual decision making. When individual preferences are influenced by such characteristics the outcome of individual decision for example in sharing decision could be very different from those suggested by the traditional maximization of self-interests.

Many experiments have shown in fact that in many games the prediction made by the theory miss to account for several others aspects, the so called "others' regarding preferences" that impede the attainment of the theoretic solution and tend to different equilibria.

Fischbacher et al. (2001), as an example, found that in a voluntary contribution to a public good game a certain proportion of the population behave as conditional cooperators: A conditional cooperator increases (decreases) her independent contribution if the other's independent contribution is larger (smaller) and does not adapt if it is equal to the own one, whereas the theory suggests just exploiter subjects, or free-rider, never contributes, either independently or when adjusting.

Also, Berg et al. (1995) found experimentally are available to share with the others a given amount in a simple trust game showing the existence of a large proportion of experimental subjects that trust the others and therefore receive reciprocity feedbacks, also if the theory states for profit maximizer subjects no trust and no reciprocity.

In many experiment of division of a given endowment (from the seminal Ultimatum game to its modifications (the impunity game, the yes/no game, the dictator game) people offer positive amounts and is accepted by the others only if those amounts are not lower than a given threshold. This introduces the theme of envy and reciprocation in sharing situations.

The combined outcome of those results is that people trust, care about the others and do not want to be undervalued or overcome.

This is particularly relevant when you consider the high number of situations in real life where subjects are involved in a division process of a good, an endowment, a firm, as example in a legacy or a divorce or a bankruptcy and so on.

The question that follows is if the proposal and the acceptance behavior of individuals involved in such a situation could be affected by the way in which the proposal of division is presented to the participants to the bargaining process and if the presence of an automata that applies a given well known and legally accepted algorithm could help in reducing conflicts, inequity aversion, envy fear and therefore increases the overall welfare of the participants involved and their feeling of satisfaction.

To the aim to check on this issue we have designed an experimental study that allows of evaluating how potential division choices differ when receiving legal or technical advice from a robo-advisor (AI) compared to a (human) consultant.

For this purpose, we implement a setting where either a computerized algorithm (AI) or a consultant (human) gives advice to a subject involved in a sharing bargaining, after which the investor makes his decisions.

We suggest the use of computer algorithms to automatically handle at least several tasks related to the bargaining decision process since they have the main advantage of cutting costs significantly compared to more "traditional" judicial processes. This reduces also the entry barriers that induce people lacking of income and/or juridical literacy to make adequate request of division and increasing trust reduce consequently the length and the cost of the litigation process. Therefore, applying such kind of approach to some specific and well identified problems of division could result in wealth enhancing.

With this experiment we aim to answer the following research questions:

i. Whether there is a difference in the amount of trust that the type of advisor induces in the bargainers?
   a. Are the participants decisions closer to the advice they were given based on the type of advisor?
   b. Do subjects gain more in the division based on the type of advisor that has advised them?
ii. Does attitude towards risk change with the specific type of advisor? i.e. measuring the direction of deviation from the advice given.
   iii. Do risk preference and trust towards the two types of advisors change

based on a variety of external variables (ex. level of legal and digital literacy of the subject, gender, intertemporal preferences, and so on)?

Experimental subjects will be furnished of a given outcome that they know they have to share with another /or many other participants. After having answered the questionnaire, participants are given advice on which one of the 6 possible divisions will be would best suit them according to the answers provided. Then they take their decision.

Participants conclude the experiment with some (un)incentivized questionnaires:

a. The risk preference questionnaire to elicit risk attitude (Holt and Laury, 2002, or Hey and Orme, 1994) and the intertemporal choice test by Coller and Williams (1999) to elicit intertemporal preferences; the lotteries are incentivized and the final payment (for one random lottery) is selected at the end of the experiment;)
b. trust game and inequity aversion tests.

Note on treatment design differences: AI sessions can be done as in the traditional manner (all participants entering the lab at the same time), however for the human consultant sessions we suggest smaller 5 to 10 participant sessions (every 30 minutes) so as to not have to wait too long in order to get advice from the expert.

We are also interested in understanding if the different advisors affect in a different way subjects who tend to make intuitive choices respect more reflexive ones or depend from their level of education.

Whereas exist psychological studies that analyze the impact of the use of computers/robots as counterparts in different kind of situational and technological settings (see as an example Nass and Moon, 2000) showing that it avoids gender and ethnicity stereotypes, politeness, reciprocity, reciprocal self-disclosure and self-serving bias), as far as we know direct metanalytic comparison between the human-human and the human-algorithm contexts have not been yet conducted in an experimental setting.

There exists a restricted bunch of experimental literature that compare the different impact on the individual economic decision making when facing automata versus human beings in several different situations. Even if slightly indirectly all of them seem find evidence that supports the idea that introducing robots allows to switch versus “better” equilibria not only in games with multiple equilibria (where the introduction of artificial subjects induce greater coordination) but also in bargaining and sharing interactions (where the introduction of programmed artificial subjects reduce strategic behavior). More in details all of them share the results that the presence of non-human opponents increases the convergence towards the theoretical equilibria, reduces participants aggressive behaviors, in-

creases their trust and reduces envy increasing the overall earnings respect when participants face human opponents. Moreover, it has been shown that this kind of repeated interaction helps learning to behave rationally and on its own behalf.

More in details; also many experimental papers focus on the different bidding behavior when human versus human or human versus computers are involved: we can refer to the seminal paper of Cason (1995) regarding the optimal behavior in EPA's emission trading auction where he found that buyers facing human opponents compete more aggressively than the risk neutral predictions whereas bids do not differ significantly from the theoretical predictions when buyers face computerized Nash "robots". Also, Walker, Vernon and Cox (1987) found the same behavior in bidding attributing the overbidding against human opponents to strategic responses to a subset of aggressive high bidding subjects. Moreover, they found that nearly one-quarter of subjects who played against human opponents always followed recommendations, whereas in the robot treatment subjects almost always followed recommendations.

Other experimental studies (i.e. Cason and Friedman, 1997) focused on the impact of the existence of computerized behavior of several subjects in markets on the convergence towards the theoretical market equilibrium. In a simple experiment designed to examine the price formation process in the Single Call Market they found more support for the Bayesian Nash equilibrium when subjects compete against Nash "robots" opponents, also when the latter play randomly (that is applying a complete non-strategic "zero intelligence" trading behavior); in particular zero intelligence trading behavior leads to average efficiency of about 33%.

Gode and Sunder (1993) have shown that simple strategies lead to nearly fully efficient outcomes in the continuous double auction, often exceeding the efficiency of markets with human traders.

More recently Cason and Sharma (2007) showed through a laboratory experiment wherein subjects play a howk-dove game that when participants play against robots that always follow recommendations it is possible to implement the theoretical equilibrium. This is due to the fact that introducing robot opponents substantially reduces the possible influence of social preferences such as subjects care about the distribution between themselves and another person.

Houser and Kurzban (2002) similarly use computerized opponents to limit the influence of social preferences. If subjects care about the distribution of earnings of the subject's opponent as in well known models such as Fehr and Schmidt (1999), Bolton and Ockenfelds (2000) and Zizzo (2004) then these differences between human versus automatized opponent matter.

Nishio et al. (2012) examined the factors with which robots are recognized as social being running an experiment on the Ultimatum game, a procedure commonly used for examining the attitude toward others in a sharing context. Aim of the experiment was to compare through this game how people behave

when facing different types of artificial agents with different appearances. They interestingly found that the number of fair proposals to agents playing with the computer were higher than those with humans and that the number of rejection of unfair proposal are lower. This means that some part of behavioral unexplained components of individual choices are eliminated by the presence of the machine.

1.8. Conclusion

Fair division theory has now entered its seventh decade of life. This seems an extremely long span of time. Born as a mathematical entertainment, the topic has acquired the status of independent field of research only in the late 90s, when the first book on the subject by Brams and Taylor (1996) was published. Since then, we have witnessed an increasing number of publications on the subject. Also, several scientific communities have increased the knowledge set on the topic, bringing their vantage points and their toolboxes. First the mathematicians, then the political scientists and the economists. More recently, computer scientists have had their say, adding new procedures and testing their computational complexity. With this project, we hope to bring law scholars and law practitioners into the community of developers and users of the field.

In this review we have examined many procedures. A few of them are very old, but many more have been developed only in the last ten years or so. Not all the procedures that we have listed and reviewed find an immediate application in the legal context. For instance, while the allocation of money and goods with the one-object-per person case described in Alkan, Demange and Gale (1991) is perfectly fit for scheduling jobs in a work environment, it cannot reasonably be enforced to facilitate the allocation of an inheritance among a group of heirs. “If, for one of these heirs, a certain set of objects has special sentimental value, why should he not obtain more than one object if he is able, in turn, to compensate the rest of the heirs?”

Our first analysis reveals that some procedures are mature enough to guarantee a reasonable degree of success in the legal context. We refer for instance to the celebrated Adjusted Winner procedure. Some others are very recent, but their strong theoretical grounds look promising enough to continue their exploration in the forthcoming months with a perspective of applications in the legal context. We refer, in particular, to

- The AL procedure, the Undercut procedure, the Trump rule, the SD and ISD procedures for two agents with ordinal preferences and indivisible goods.
- The Nash Product Maximizer Rule for any number of agents with cardinal preferences and divisible or indivisible goods.

- The consistent allocation for any number of agents with cardinal preferences, indivisible goods and money.

Behavioral and experimental economics will provide rigorous evidence of the effectiveness of these procedures and the perception of those among the involved agents.

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Marco Dall'Aglio *

Fair Division and the law: How real cases help shape allocation procedures in the legal settings across European countries

Abstract

Existing fair division algorithms are tested under many legal cases analyzed by the legal teams of the CREA project in three sectors: Inheritances, Divorces and Company Law. Each example is commented from the algorithm designers' viewpoint. A list of the features that should be present in a general-purpose procedure for allocating goods in a judicial setting are given, together with other additional features that could be developed as side projects if resources allow. A guideline for designing procedures based on the existing specimens but tailored for the specific needs of aiding legal specialists and citizens is outlined.

Keywords: *Fair Division, Algorithms, Family Law, Company Law*

1.1. Introduction

In Dall'Aglio, Di Cagno and Fragnelli (2020), the most recent advances on fair division procedures and algorithms were examined. The work pointed out that some procedures are mature enough to guarantee a reasonable degree of success in the legal context. We refer for instance to the celebrated Adjusted Winner procedure. Some others are very recent, but their strong theoretical grounds look promising enough to invest on their exploration with a perspective of applications in the legal context. We refer to the following classes of procedures:

I. Procedures in which contenders (also denoted as players or agents) express a ranking of the contested items and these items are indivisible. These procedures include the AL procedure (Brams, Kilgour and Klamler, 2014), the Un-

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dercut procedure ((Brams, Kilgour and Klamler, 2012), the Trump rule (Pruhs and Woeginger, 2012), the SD and ISD procedures for two agents with ordinal preferences and indivisible goods (Brams, Kilgour and Klamler, 2017). This is the simplest setting since contenders are only required to state what is the most valuable item for them, then the second best, and so on.

II. Procedures in which contenders allocate points to the items that can be divisible or indivisible. These procedures include the Adjusted Winner (Brams and Taylor, 1996 and 2000), with some of its recent extensions, and the Nash Product Maximizer Rule for any number of agents (Nash, 1950 and Bogomolnaia et al., 2017).

III. Procedures in which contenders allocate indivisible items and money among themselves. This is obtained by asking each contender to provide a personal monetary evaluation of each item (or, if contenders are willing to do so, of items' bundles).

In Dall'Aglio, Di Cagno and Fragnelli (2020) these methods are investigated in detail to deliver the best instruments that help the legal specialists and the parts involved in any (amicable or formal) dispute.

1.2. Specific Cases Brought by the Project Legal Team

In January 2018, a discussion involving all teams of the CREA project has begun. It regarded which sectors in the legal environment should witness the application of fair division tools. The discussion led to the emergence of three areas in which fair division algorithms provide a useful and intuitive toolset for the automatic solution of disputes or, at least, they provide a valuable aid for the disputants to settle issues inside and outside courts:

A. Inheritance (issues involving division of assets).
B. Divorce (issues involving division of assets).
C. Company Law (issues involving value of the shares of the companies).

We then received the input of several partner units in the project, who give a list of exemplary case that help us focus on the specific needs and features for the algorithms needed to fulfill the project. In what follows we will list those cases, divided by area of application and we will comment about what these examples can bring to the development of the project: Do they fit into the framework of a fair division problem? Do they suggest new features that an algorithm should encompass? How general is the instance that the example suggests? How consistent should the effort be for the general algorithm to encompass the special case set forth by the example?

We analyze the three classes separately.

1.3. Inheritance divisions

The division of a deceased person's wealth is often regulated by his/her will. It is rather common, however, that the departed left no will, or the will was incomplete. Furthermore, a detailed will may be disputed by one or more parties effectively or potentially involved in the inheritance. In all these cases, an algorithm could help find an amicable agreement before proceeding to the court, or it could help the judges in their decision.

Typically, an inheritance problem involves several parties (usually more than two) with different entitlements. A list of the goods forming the patrimony should be made prior to the division, together with the share to which each party is entitled.

1.3.1. Specific cases

A list of the cases proposed by the legal units of the CREA project helps to define the features that an algorithmic aid should provide.

Example A.1

The inheritance was based on the legal provisions as there was no will. The first instance court decided that all heirs will inherit in the legal set proportions. One of the heirs disagreed as he was claiming that he should be entitled to a bigger share of the inheritance as he was the one, staying at home, taking care of the whole family and nourish the deceased and his wife until their death.

The Court decided that deceased family members and their kids, who lived together with the deceased and helped him with their work, money or assisted him in any other way, have the right to demand that a portion of the deceased's property be excluded from the inheritance. The value of the excluded inheritance shall be defined due to their contribution to the increase or preservation of the value of the deceased's property.

The assets can be excluded from the inheritance only if and to the extent that the value of the property is greater that the value of the assets before the family members started to live together. The courts have to establish the difference in value between the property with the contribution of family members and without their help. Than the value of the contribution of the family member (in money, work, help, nourishing) has to be determined. In calculating the preservation / increase of value, taking into account the circumstances of the individual case, it is necessary to assess whether the property may be devalued due to the descent of the use (or whether the property would be greater if the offspring had not been used by it), and not whether the descendant using things enriched.

The portion of deceased inheritance in immovable property (House) is not paid out in money, but it increases the inheritance share (portion) of the family member who helped/lived with the deceased.

Remarks: This example introduces an important notion: typically, the shares of entitlement are fixed, but a Court or the Law may impose the accrue of a party's shares, according to the party's effort, in terms of capital and/or work that increased the value of the disputed patrimony. Conversely, a careless conduct by an heir in his/her past interaction with the person now deceased, may decrease the patrimony's worth and this should result in a reduction of his/her share in the patrimony division.

A Dispute Resolution System (DRS, onward) should additionally provide an algorithm that inputs the (positive) efforts and/or the (negative) harms by the parties and suggests a modification of the parties' shares before entering the legacy division.

We also notice that the estimation of efforts lies outside the traditional realm of fair division and social choice theory in general. Therefore, it will require the acquisition of expertise through study and specialists' consulting.

Example A.2

It's a similar case to the case No. 1. The special question or problem was the fact that the immovable property was a Farm. In Slovenia, farms of a certain (defined by Law) size are called protected Farms, which means, the land of the Farm cannot be further divided or sold off. This means that there are also special rules in inheriting protected Farms. Under the Law only ONE can inherit the Land and he has to pay off the other heirs. Special rules on the value of payment are established by Law.
General rules do not apply, neither to the question of defining heirs, nor in defining the split of inheritance between the heirs.

Remarks: In principle, this is a very simple problem that should not require a proper algorithm. Once the Land that has to remain under a single owner is properly evaluated, then proper compensatory payments should take place according to the Law. In practice, a DRS may help suggest a solution by pointing out which goods of the assets in the patrimony (different from the land) may be traded for cash, or which goods may be considered as side payments.

Example A.3

If the spouse should inherit together with the second order heirs (if spouses don't have kids, then the legal heirs are the remaining spouse and the parents of the deceased – if they are dead, then their heirs) the court may, at the request of the spouse, decid, that the spouse may inherit all or part of the inheritance of second order heirs if the inheritance is of small size and if the spouse would lack the means to have a decent life if he would not inherit the whole inheritance.

When deciding on this, the court shall take into account all the circumstances of the case, in particular the financial situation and the ability of the spouse to get in-

come, the financial situation and the gainful ability of other heirs and the value of the estate.

Remarks: This case suggests another interesting feature that a global DRS should feature. If required by Law or by the court, a procedure could offer an alternative solution that does not provide a division fully proportional to the parties' shares of entitlement, but it constraints the division to the financial situation and the gainful ability of every heir and the value of the estate. A simpler method would require the forced assignment of some items that are easily liquidated or that guarantee a future income to the worst-off parties. A more sophisticated procedure could instead begin by distributing the legacy only to the parties that currently fail to reach a minimum standard of living. Only after those basic needs are fulfilled, the residual patrimony may be divide according to the standard rules and procedures. It is important to notice that what defines an insufficient income level or gainful ability cannot be endogenously determined by the algorithm, but it should be sharply defined by the Law or by the Court issuing the decision.

Example A.4

The spouse was 82 years old, had a pension in the amount of 416 € and gets additional social help in the amount of 230 €. The other heir has a house and a flat. He is renting the flat. The court decided that the spouse is entitled to inherit the whole inheritance.

Remarks: This case is very similar to Example A.3 and raises identical remarks.

Example A.5

The spouse was challenging the court's decision on the wrongful division of inheritance. The spouse got 2/3 and her son 1/3. She appealed and claimed that the son was disrespectful, did not take care of her and his father and the he should only get a tractor, which he predominantly used. She also claimed to be invalid and being depended on social benefits.

The Appellant Court decided that the rules which enable a spouse in need to inherit the whole inheritance are used only in cases, were other heirs are second order heirs. In this case, the other heir was the son of the deceased, so those rules don't apply.

Remarks: The decision on whether a welfare-based criterion should only be applied to a certain order of kinship lies entirely upon the Court or the Law. Algorithmic procedures cannot replace the role of the human legislator, but they can provide a concrete and often unreplaceable aid. Here (and everywhere else in the listed examples), the Legal context should precede the design of algorithms.

Example A.6

A and B lived together since 2001 with their 16-year-old daughter and 12-year-old son but they were never married. The house was property of two owners: A (decedent) and C (A's step-mother). They inherited the house after the death of A's father, and C's spouse (May 2008), each one half of the house. They didn't think it was necessary to set up condominium. C lived on the ground floor (100m2, valued at 120 000 euros) and the rest of the family lived on the first floor (120 m2, valued at 140 000 euros). Together they used the garden (50m2, ¼ of the whole plot) with a small pool and a fully furnished barbecue with expensive appliances that were sometimes used by C's son but paid by A. They often fought over that part of the property. During the marriage A and B bought a small cottage in the mountains worth 50 000 euros and vineyards worth 15 000 euros. In the land register the sole owner was A. A was in debt after his firm went bankrupt (30 000 euros).

Succession proceedings after the death of A started in April 2017, two months after his death.

B claims half of the inheritance as marital property. Her intention is to sell the house as she always had a bad relationship with C. B also claims household items, especially the ones in the garden which are mostly the reason for the fight. The court decided that children have together 2/6 of the house, and that B has 1/3, that children have together 1/3 of the rest of the properties and B has 2/3.

C, being an old retired judge, thought that something was wrong, and she sued both B and her children seeking another determination of co-ownership share, partitioning co-ownership, and establishing condominium. She wanted to make sure that the garden will belong only to her. She argued that A and B were never married.

Remarks: This is an interesting case, where the designated algorithms could help settle the case: A real estate asset equipped with pertinences and several heirs with different shares of entitlement and different preferences. There is, however, one important point that should be made clear. In accordance with the previous cases, the Court or the Law in general should define the set of rules. In the present situation, the Court (or the Law) should decide whether marriage makes a difference. It should also decide which share of entitlement should be associated to every hair, and it should determine whether those shares can be modified according to precise rules. In principle and if the Law allows it, the Court could simulate different outputs deriving from different distributions of shares to come up with a decision about each heir's share. Only then, the true process of dispute resolution should begin.

Example A.7

After the death of N.N. five of his cousins became his successors (each receiving 1/5 of the estate). N.N. was an owner of a property worth 860 000 Euros which consists of a house, a big garden and a small store with electric appliances placed near

the street (the property has an area of 625 m2). He also started renovating his house (the ground floor) in order to open a small café with a big terrace in his garden. N.N. was living alone; he never married but his passion was collecting rare furniture antiques. The first flat in the house was crowded with old antique furniture worth around 50 000 Euros which he wanted to use for the renovated ground floor as café decorations.

Cousin 1 wants to use his cousin's (N.N.) idea of creating this special café with his furniture just like N.N. has imagined and planned. He doesn't care how it will legally be taken care of. He already has experience in café management and he strongly believes that such a café in that location would be a mayor success.

Cousins 2 and 3 (brothers) are both interested in the part of the property where the garden is situated (around 200 m2) because it is an area suitable for development, and by creating a small building plot they would be able to together build a house for their families.

Cousins 4 and 5 (also brothers that were not very close to N.N.) want everything to be sold as soon as possible because they have enormous gambling debts.

Remarks: This example too offers new insights on the inheritance allocation problem. First, we notice that the heirs' goals may be not only different but totally conflicting. From what we understand on the case, Cousin 1's idea of destining part of the garden as a terrace for a café is totally conflicting with Cousins 2 and 3's intention of using the whole garden as a development plot for themselves. A simple idea would be to split the garden into two parts: the terrace area and the remaining part. Cousin 1 will express a high preference for the terrace area and the ground floor. Cousins 2 and 3 will express their appreciation for the two plots. Most likely, one of the two factions will prevail if it is willing to compensate the "losing" counterpart with side payments. Another interesting feature of the example is offered by the presence of heirs who only care about liquidity. A simple option would be for them to focus on easily sellable goods, such as the art collection. A more sophisticated but realistic option would contemplate those heirs to explicitly state their intention to accept only those solutions in which they are compensated with money – either through side payments or through the partial or total liquidation of the asset.

Example A.8

During his life, X was the owner of a land plot in Zadar with a building and garden (180m2) with three flats: one on the ground floor (90m2, 180 000 Euros), one on the first floor (60m2, 120 000 Euros) and one on the second floor with a wonderful view of the shore and beach (60m2, 130 000 Euros). All flats were condominiums and were rented out. He also owned another land plot in Zagreb with a building with three flats; one on the ground floor (55m2, where his son A had a mechanic's workshop, 77000 euros not including equipment), one on the first floor (55m2, where X lived, 80 000 Euros) and one on the second floor (45m2, but needs full renovation, 45000 Eu-

ros). This second building was not condominium. After death of person X he is succeeded by his sons, A, B and C.

A is most interested in the ground floor because he operates a mechanic's workshop which is crucial for his livelihood. He wouldn't mind getting another apartment either in Zagreb or in Zadar.

B already had a house so he was interested in the house in Zadar. He wants two flats, the one on the first floor but especially the one on the second floor (this is his mayor priority).

C has a tourist agency and he wants all flats in Zadar.

Remarks: With three heirs and six flats we have an example where fair division tools can operate at their best without any further assumption. We only need to "translate" the three heirs' preferences into numbers – either absolute or monetary values.

Example A.9 (Cass. civ., Sez. II, 5 settembre 2016, n. 17576)

One brother, D.G., and his sister, D.T., inherited, in communion with each other, goods both from the father and the aunt. D.G. wants to dissolve the communion and turns to the court to proceed with the dissolution. The sister, D.T., adheres to the application for division, but asks that the properties donated to life by the father to his brother are also collated.

D.G., in turn, asks to proceed with the collation of the property donated by his father to his sister. At the time of carrying out the divisional operation, the Court finds itself in front of three distinct communions (represented by the property fallen into the succession of the father, the aunt and those acquired in the ordinary communion by the two brothers).

The Judge decides to unify the masses and to proceed to a single division as if the three masses of goods constituted a unique communion.

Following an appeal by DG, the sentence is reformed by the Court of Appeal approving three separate division projects, one for each mass, estimating the donated assets, according to both the collation and the withdrawals, with regard to value of the same when the succession is opened.

D.G. is not satisfied and, using the Court of Cassation, denies that the Court of Appeal has adopted a divisional criterion different from that adopted by the court of first instance, in violation of the internal judgment formed on the point, as in the appeal deed no objections were raised regarding the choice made by the Court. The applicant also considers that such compliance should be considered equivalent to the provision of consent in writing, a condition of legitimacy for the joint division.

The Supr. Court rejects the appeal and states that, correctly to what has been decided by the Court of Appeal, in the case of division of common goods coming from different titles, autonomous division projects must be prepared and if the masses are to be reunited for the purpose of a single division requires the express consent of all the parties.

However, he believes that, in the case in which the heirs have equal shares with respect to each mass, as in this case, the judge can still approve a unitary division pro-

ject, even without express consent, because, in this case, the result remains the same regardless of the method of division used and any damage may result to the parties requesting the division.

The choice of the divisional criterion – supports Supr. Court – falls within the regular exercise of the decision-making power of the judge at the time of the formation of the shares.

The Supr. Court states that the assets, which non-donors can take from the inheritance mass following the collection by imputation made by the joint donors, must be estimated for the value they had at the time of the opening of the succession and not at the time of the division.

The intervention of Supr. Court seems interesting and certainly to be investigated from the point of view of the division in the case of "multiple masses", since it seems to have introduced a further principle of law in this matter consisting of the attribution of relevance to the element of "identity of shares" as criterion of legitimacy for the judge to proceed to a single division even in the presence of multiple masses deriving from different titles.

Remarks: This example points out two interesting features of inheritance problems: To begin with, we notice that a heritage comprises assets still owned by the deceased person, as well as assets already donated to the heirs. A procedure for the fair allocation of the whole patrimony should suggest an allocation that takes into account what has already been donated and compensating those heirs that have not enjoyed any donation with more assets and, if necessary, with side payments. A proper DRS should therefore allow the formulation of a constrained problem in which assets already donated do not change owners, but the other heirs receive fair compensation.

Moving to the second issue, as algorithm designers we remark that, beyond the Courts' decisions at any level, automated dispute resolution systems can take into account three different division or one division of the assets' union. It is important to notice that the latter has a considerable advantage. In fact, any efficient division of a patrimony between two parties can be obtained by splitting at most one item (or by considering side payments that replicate the same outcome). Consider to this aim the features of the Adjusted Winner procedure, which singles out an equitable and efficient division. Consequently, in the division of the assets deriving from the union of the three masses, no more than one item will be split. Conversely, in the three separate divisions up to three items for which splits, or equivalent side payments, will have to be considered.

Example A.10 (Cass. civ., Sez. II, 4 maggio 2016, n. 17576)

Think again, how the lack of the quality of coherence, at the time of the opening of the succession, being for example the subject, at the time, linked to the de cuius by a relationship of affiliation and not of adoption, does not prevent the others co-heirs, in the

exercise of their negotiating autonomy, to negotiate the dissolution of the hereditary communion, attributing a share of the same to that, having thus given place not to a real division, but to a plurilateral contract, however binding and effective between the contractors.

Remarks: The attribution of the assets' share to a person affiliated to the deceased person, but not officially recognized as an adopted child, can be considered as an input in the DRS, once the appropriate share of entitlement is agreed between the group of co-heirs and the affiliated.

Example A.11 (Cass. civ., Sez. II, ord. 15 gennaio 2018, n. 726)

In the judicial division, standard of the drawing by lot, in the case of equality of shares, by art. 729 of the Civil Code, to guarantee the transparency of divisional operations against any possible favoritism, is not absolute, but only tendential, and, therefore, can be derogated on the basis of discretionary assessments, which can not only meet objective reasons linked to the functional condition and of assets, but also to subjective factors of appreciable and proven opportunity.

Remarks: Drawing by lots may assure transparency, but it usually results in totally unsatisfactory and inefficient solutions. The purpose of the DRS that we are building goes exactly in the direction of proposing satisfactory solutions for all parties with transparent procedures that take into account the preferences of the parties involved and avoids biases and manipulations from all sides.

Example A.12

A, a professional butcher, dies without writing a will. His wife B, in his own right and as tutor of his daughter C, sues his sons, D and E, asking for the dissolution of hereditary communion and the division of property.
The hereditary axis, net of debts, consists of:
1. a residential building used as a family home for a value of 250,000 euros;
2. a property for residential use worth € 180,000;
3. a property used as a business for a value of € 150,000;
4. a building used as a slaughterhouse, worth € 150,000;
5. a land for agricultural use worth € 50,000;
6. movable assets for a value of € 20,000.
The wife points out that the business premises had remained in good state of conservation until the death of A and that it had deteriorated, losing 1/5 of its value, due to poor maintenance, during the period in which it had been in the enjoyment of his son D, who had continued his father's business, together with his brother E, since A's death.
Conversely, his son E, who had continued the management of the slaughterhouse, had made improvements to the property, increasing its value by 1/10.

Remarks: This case is similar to Example A.1. Also in this case, if the Court or the Law prescribes so, the actual division of the patrimony should be preceded by another procedure for the assessments of the efforts or the harms that may increase or decrease the relative share of each heir. The continuation value of the activity should also be taken into account.

Example A.13

A was born from an incestuous relationship between B, entrepreneur, divorced from E and father of F and G, and his half-sister C. B dies and A, before becoming of age, asks for recognition of paternity, but the judge denies authorization required by art. 251 of the Italian Civil Code. The status of B, therefore, is that of an unrecognizable child, entitled, instead of a hereditary share, to a life allowance equal to the "income of the portion of inheritance to which he would be entitled, if the filiation had been declared or recognized" (Article 580 of the Civil Code). In this sense, according to the prevailing opinion, the share due to the child must be calculated on the relictum.
The hereditary axis consists of a residential building used as a family home for a value of 3,000,000 euros;

- *a residential building used as holiday home worth € 700,000;*
- *shareholdings in a real estate corporation for a value of € 3,000,000;*
- *valuable movable assets for a value of € 1,500,000;*
- *money equal to € 450,000.*

In this hypothesis, the application of equitative algorithms could support the identification of the ideal hereditary portion of the unrecognizable child and the quantification of the suitable corresponding annuity.

Remarks: This case bears close similarity to Example A.7. A's apportionment in this case will consist of a monetary amount. In the previous example this was an option expressed by some heirs, and as such, it could be obtained only with a general agreement among heirs. In the present case, instead, the solution is imposed by Law. This monetary amount will then be transformed in an equivalent annuity. In this case, the great amount of money will certainly help in finding a satisfactory arrangement.

1.4. Divorce divisions

In principles, the division of assets which were enjoyed in communion by a pair of partners who have decided to end their relationship is the simplest example of allocation problem: there are two partners who have equal shares of entitlements (unless otherwise prescribed by Court or Law). In practice, however, this idyllic setting is marred by several factors: most often, the resentful attitude of the two ex-partner that act more in their counterpart's spite rather than guided by their

own self-interest, does not help in finding a satisfactory overall arrangement. More importantly, though, the custody of minor children is a very delicate issue that certainly affects the economic outcome but cannot be left to an exclusive algorithmic treatment. Other issues and hints originate from the analysis of the example brought by the project's legal team.

1.4.1. Specific cases

A list of the cases proposed by the legal units of the CREA project helps to define the features that an algorithmic aid should provide.

Example B.1

> *Despite the fact of a valid marriage new joint property of spouses stops accumulating at the time when spouses stop living together (when their joint life together stops), even though they are still married.*
>
> *A flat bought in the time of marriage does NOT represent joint property of spouses if the money used to pay the flat was a part of the personal assets of one of the spouses – assets already existed at the time of concluding the marriage or acquired in a manner that does not count as work (inheritance ...).*
>
> *For determining if the flat is a part of joint property of spouse or special property of one of the spouse the criteria, who was repaying the loan is not important. It only matters out of which money the loan was repaid.*

Remarks: The example points out, prior to any algorithmic processing, it is important to define explicitly which part of the common facilities that the two ex-partners were using is actually joint property and which other parts are special property of either of the two ex-spouses.

Example B.2

> *In the dispute as to the proportion of each spouse on joint assets, the court takes into account not only the income of each spouse, but also other circumstances, such as the assistance given by the spouse to another spouse, the protection and upbringing of children, the pursuit of domestic work, the care for the preservation of property and any other form of work and participation in the administration, maintenance and expansion of joint assets.*

Remarks: In a fashion similar to example A.1 and if explicitly required by the Court or Law, a preliminary routine could be used to measure the efforts/harms that the two ex-spouses brought to their joint property and modify their shares of entitlement accordingly. The second part of the example shows that this example is particularly suitable for a solution obtained through a DRS. In fact, the ex-spouse who is interested in the objects intended for the pursuit of his craft or pro-

fession will attribute a high value to those items when asked about his preferences.

Example B.3

Determining shares on joint assets is not a matter of calculating operation, but it's a matter of a comprehensive assessment of any contributions of partners at the time of marriage.

The applicant covered the costs of the defendant in connection with her studies (staying) abroad from his personal assets. His property was therefore not brought into the common household, but was a gift from a defendant. That circumstance therefore does not justify a higher proportion of the plaintiff on the joint assets.

It is not important who of the partners has hired a loan and who repaid it, is not essential to decide on the shares on the joint property if the loan was not repaid from the special property of the spouses.

The scholarship, the family pension and the social security help represent the special property of the spouse.

Repayment of liabilities in relation to joint assets represents the issue of debts of joint assets, the payment of which is settled in accordance with the rules of the law of obligations and does not alter the contribution of each partner to the creation of joint assets.

The defendant did not assert that the inquiries concerning the level of personal incomes in the relevant state bodies or holders of public authorizations attempted to carry out the self, and failed. For such a circumstance, the defendant, on the basis of Article 226 of the ZPP, would undo her other subordinate duties.

Remarks: As in the previous examples, a preliminary assessment of what constitutes the joint property to be divided should be performed.

Example B.4

Marc and Anna were married since 2011. Before the marriage, in March 2009, Marc bought a flat/apartment in the centre of Zagreb. At that moment, the prices of apartments and houses were very high. He paid 50 000 Euros in cash but for the rest of the amount (100 000 Euros) he took out a 10-year loan. The contract was concluded under special terms since Marc was a bank manager so the interest rate was only 3,54%. His monthly payment was (and still is) 990 Euros. Also, there is a hypothec encumbering the apartment. After the marriage, Anna moved in. She bought some expensive pieces of furniture: a designer leather sofa for 8 persons (valued at 5400 Euros), a vintage styled dining cabinet (valued at 3000 Euros) for her special Rosenthal and Versace collection of dinnerware (valued at 4500 Euros).

Marc was unemployed for two years (from May 2013 to May 2015) so the burden of repaying the loan was on Anna. In April in 2016 she bought a car with the help of another loan of 23 000 Euros (annual interest rate of 7,3% for a two-year loan, with a monthly repayment of 1030 Euros). Anna was repaying her loan regularly. They both

used the car until 2017 when Marc inherited his father's car (worth 5000 Euros). In March 2018, they decided to dissolve the marriage.

Marc made clear that the apartment in Zagreb is his, since it was bought before the marriage. The value of this apartment is 15% less as compared to the moment the original sale was made. He is aware that Anna did repay some part of his loan but he thinks that this doesn't matter because she lived in his apartment for free. Also, he claims that his father's car is his own.

Anna wants the car that she chose and bought and she thinks that she is the sole owner of the car, but she also wants an equal share in the apartment and a refund for the amount she had paid for two years' worth of spousal support. She wants all movable things she bought for the apartments (furniture and dinnerware).

Remarks: As in the previous examples, a preliminary assessment of what constitutes the joint property to be divided should be performed. In a proper DRS, the parties involved have all the possibilities to express their preferences.

Example B.5

The Claimant and the Respondent lived together from 2009 to 2015 (marriage was not concluded) in the apartment belonging to the Claimant as personal property, managed everyday life, had a child. In 2012 the Company was established; the idea to start construction business was the Claimant's, the Claimant and the Respondent decided on positions within the Company: The Respondent was the founder of the Company with the capital of Euro 1 448.10, the Respondent became the executive of the Company, the Company's domicile was registered in the apartment that belongs to the Claimant as personal property. Since 2012 the Claimant worked in the Company, managed its bookkeeping, lent the money to the Company. The Company acquired land plots which were bought from the Claimant's father where 4 apartments were build, the income received from the sale of these apartments were the main income of the Company.

The courts established these circumstances as sufficient to declare that the Claimant and the Respondent concluded a factual (unwritten) agreement on partnership (joint venture). The Courts indicated that it is to be analysed if the Claimant has the right to ½ of the capital of the Company and subsequently, to ½ profit of the Company. For that the factual investment of both parties are to be analysed.

The courts also decided on the division of the real estate that was bought by the Respondent as personal property, financing it with the loan from the Company. The courts established that the particular real estate is not to be divided between the parties since the Respondent took obligations resulting from the loan agreement as personal obligations (not joint).

Remarks: This example introduces the issue of non-marital partnership. Whether they should be assimilated to traditional marriages is a decision entirely upon the Court or the Law. As it is the decision about what constitutes the joint property to be fairly divided.

Example B.6

The courts established that the Claimant and the father of the Respondent concluded a factual (unwritten) agreement on partnership (joint venture) since they both lived together and managed everyday life without concluding marriage.

The Courts refused to divide the apartment in which the Claimant and the father of the Respondent lived, declaring that the particular apartment was bought as personal property of the Claimant using funds that were her own, the father of the Respondent did not pitch in financially and started living with the Claimant after the apartment was bought and repaired. The court also refused to divide the summer cottage using similar grounds.

The courts established that the Claimant has the ½ right to a part of 50% of the real estate used for commercial purposes and rent. The conclusion was made after establishing that the Claimant was directly involved in acquiring the assets (including financing part of it). Therefore, the courts decided that the Claimant has the right to ½ market value of the disputed real estate (instead of ½ of the price that was paid for the real estate back in 2004). The Claimant therefore awarded monetary compensation of the ½ part of the shared real estate and ½ of the rent received from renting the particular real estate. The Respondent was awarded with the sole ownership of the disputed real estate and was obliged to compensate to the Claimant ½ of it.

Despite that another apartment was registered as personal property of the father of the Respondent, the courts declared that the Claimant has the right to ½ of the particular real estate since at the time it was acquired the factual partnership between the Claimant and the father of the Respondent was already in place and the Claimant pitched in financially, also, furnished the apartment.

The court refused to award to the Respondent ½ of the funds acquired pursuant to agreement of investment insurance. The court decided to award to the Claimant all expenses related to the funeral of the Respondent's father. The courts held that the Claimant was awarded the ½ of the assets as not the successor of the Respondent's father but was awarded part of the asset that she pitched in as a partner. The Claimant was not the wife of the Respondent's father; therefore, the funeral expenses are to be financed by the Respondent as the son the deceased.

Remarks: Once again, the recurring themes of non-marital partnership and that of the exact definition of the joint property show up in this example. A new problem emerges: that of the multiple ties that bind people. In this case, the Claimant was simultaneously the (de-facto) spouse and the financial partner of the deceased. It is up to the Court or the Law to define which tie linked the two persons involved with the item in dispute, and in which proportion the item has to be shared.

Example B.7

The Claimant and the Respondent lived in marriage before filing for divorce and division of assets.

The courts established that even though particular real estate is registered as the personal property of the Respondent, these assets are considered to be joint property and are to be divided between the Claimant and the Respondent.

The courts identified all dividable assets, out of which the real estate and one of the vehicles was awarded to the Respondent obliging him to compensate the Claimant (the Claimant lived in Norway during the divorce). The second vehicle and the land plot were declared to be personal property of the Respondent.

The courts established that there were reasons to deviate from the principle of equal shares of the dividable assets during the divorce and established that the Respondent are to be awarded 2/3 of the assets while the Claimant – 1/3, therefore, the compensation to the Claimant is to be calculated respectively.

Remarks: Once again, the recurring themes of non-marital partnership and that of the exact definition of the joint property show up in this example. A new problem emerges: that of the multiple ties that bind people. In this case, the Claimant was simultaneously the (de-facto) spouse and the financial partner of the deceased. It is up to the Court or the Law to define which tie linked the two persons involved with the item in dispute, and in which proportion the item has to be shared.

Example B.8

Divorce entails the dissolution of the matrimonial regime and thus requires a pecuniary organization of the relations between the former spouses. Divorce carries patrimonial effects between the spouses and the liquidation of the matrimonial regime. Liquidation is an accounting operation that fixes the rights of each spouse over all the property of the communion. The sharing of common assets and liabilities is divided between the spouses.

• In the divorce by mutual consent, the spouses present to the judge an agreement regulating the liquidation of their matrimonial regime.

• In other cases of divorce, the law encourages the spouses to anticipate by agreement the liquidation of their matrimonial regime thus favoring an agreement between them.

If this has not been the case, during the divorce proceedings the spouses may enter into liquidation agreements. To prepare the liquidation of the matrimonial regime, the family judge appoints a qualified professional or a notary to make an inventory of the assets of the spouses and to develop a liquidation project.

This qualified professional who draws up the inventory and distributes property between spouses can be replaced by the algorithms stated in the project.

For the sharing of common assets: the principle is equal sharing, as assets must be divided in half between the spouses. This rule is not mandatory, the spouses can derogate by marriage contract or within a settlement agreement. the spouses can, when they agree on an amicable sharing, divide their assets as they see fit, the equality of the division is an equality in value. The evaluation is done when the goods are shared.

Article 1477 of the French Civil Code sanctions the fraud of a spouse whose purpose is to break the equality of sharing in his favor, which constitutes a concealment of communion.

For the sharing of common liabilities: (difference between obligation to debt and contribution to debt).

Obligation to the debt: Each spouse can be sued for the totality of the common debts existing on the day of the dissolution, as soon as they entered into communion.

The creditor may exercise his right to sue on all the assets of the spouse. As for the spouse of the debtor spouse, he is liable for half of the debt.

Contribution to the debt: If one of the spouse has contracted a debt, there may be a reward for the other spouse, to compensate his loss. For instance, if one of the spouse paid for the other's debts he may have a recourse against him. Other debts of the communion are due for half by each spouse.

Remarks: The most important lesson to be taken by this example is that an asset to be divided between ex-spouses may include liabilities (denoted as "bads" in the economic literature) as well as valuable items ("goods"). An effective DRS should take both kinds of assets into account. This topic affects all the three areas that we are considering, and should be included in the general procedure to be adapted to the specific problem at hand. Finally, one small remark concerns the role of the algorithms, that are meant to replace human mediators only if the parts involved in the dispute agree to do so. Typically, the DRS will help the appointed mediator in his/her hard task.

Example B.9 (Cass. civ., Sez. III, 14 marzo 2013, n. 6575)

The nature of communion without shares of the legal communion of the spouses involves that expropriation, for personal debts of one of the spouses, of a good (or of more goods) in communion, have as object the good in its entirety and not for half, with the dissolution of the legal communion limited to the well endured at the time of its sale or assignment and right of spouse not debtor to half the sum gross from the sale of the asset same or the value of this, in case of assignment.

The legal communion between the spouses constitutes, in the italian jurisprudential interpretation absolutely prevailing (since the Court Cost. 10 march 1988, n. 311) and despite dissensions in part of the doctrine, a communion without quotes, in which the spouses are jointly holders of a right concerning all goods of it and with respect to which it is not admitted the participation of strangers (among the latest: Cass. 24 july 2012, n. 12923; Cass., ord. 25 october 2011, n. 22082; Cass. 7 march 2006, n. 4890), being a directed communion, unlike ordinary communion, not already to the protection of individual property, but rather than that of the family (among others: Cass. 9 October 2007, n. 21098; Cass. 12 january 2011, n. 517); it can melt in the suns cases provided for by law and is unavailable from part of the spouses, who, inter alia, they cannot choose which goods to return to and which not, but only completely change the patrimonial regime, with solemn acts opposable to third parties only with the annota-

tion formal in the margins of the marriage act; the quote is therefore not a structural element of property: and, in relations with third parties, each of spouses, while he has no right to dispose of its quote, it may however be available of the whole common good. This setting prevents, first place, the reconstruction of the legal communion as a universality; secondly, it precludes the applicability of the discipline of expropriation of shares (referred to in Article 599 Code of Civil Procedure), and that against the third non-debtor: of the one, because the good belongs to another subject solidly for the whole, that it could not however act separately for the dissolution of the communion only to that asset; of the other, because it is exceptional and therefore insensitive to application analogy the subjection to an executive procedure of an individual who is not a debtor . The only reconstructive option that would satisfy the only requirements of legal communion it would be the exclusion of the foreclosure itself of the assets belonging to different credits from family ones: but it is a reconstructive option which unduly justifies the reasons of the creditors of individual spouses for non-family claims, the latter, however, although married, not cease to answer for their debts with everyone the assets belonging to their assets, of which art. 2740 c.c.; in addition, the destination of the goods in legal communion with the needs of the family it does not absolutely determine its impossibility to meet the claims of individual spouses, only envisaging a subsidiarity regime (art. 189 c.c.; regime which, then, is understood correctly do not even lead to the burden, for the proceeding creditor, to experiment in advance and with negative result the executive action on the personal property of the obligated spouse, such as also to investigate the existence of them: on the other hand, it is preferable to refer to each one spouses - and therefore also to the non-debtor - a real burden to oppose or to plead the existence of personal property of the debtor, to be attacked beforehand); finally, the removal of assets in legal communion expropriability for personal credits of one of them ends up depriving the same singles spouses of every useful possibility of access to the credit and, paradoxically, with the negative burden on the management of family assets, for the suffocation in the root of fullness of the participation of each of the individuals spouses to legal traffic. It must therefore end by affirming the following principle of law, pursuant to art. 363 c.p.c., paragraph 3 (to which the applicant is the same should have been rejected from the beginning his opposition), with the obvious specification that for sale or assignment means the moment when, as a result of these, depending on the peculiarities of the individual expropriations, the concrete transfer of ownership of the well endured: the nature of communion without shares of the legal communion of the spouses implies that expropriation, for credits personalities of one of the spouses, of a good (or more goods) in communion, have to object the good in its entirety and not for half, with the dissolution of the legal communion limited to the garnishee at his act sale or assignment and right of the spouse not debtor at half of the gross sum derived from the sale of the asset or value of this, in case of assignment.

Remarks: This example points out that creditors of one of the spouses cannot claim on the whole asset owned by the couple, but only on the half that is equivalent to the debtor share. A DRS could be used to simulate the couple's divorce and to extract the assets that the debtor may use to extinguish the debt.

Example B.10 (Cass. civ., Sez. III, 14 marzo 2013, n. 6575)

A, wife of B, asks for the statement of termination of the civil effects of the marriage, three years having passed since the judgment of personal separation.

The goods in common are:

1. an apartment, used as a family home, worth 1,500,000 euros;

2. an apartment in a seaside resort worth 1,250,000 euros;

3. a prestigious building, inherited by the couple through testamentary disposition, worth 1,750,000 euros;

4. valuable furniture (works of art) contained in the aforementioned buildings for a value of 550,000 euros;

5. two cars with a value of 60,000 and 50,000 euros, respectively;

6. a vintage car, worth 170,000 euros;

7. company equity investments for a value of 750,000 euros;

8. a sum of money equal to 1,500,000 euros.

The spouses exercise both professional financial activities in the risk capital market and are involved in several types of entrepreneurial activities. For this reason, both have an interest in retaining company holdings.

The wife also asks for the sub-4 assets for herself as part of her entrepreneurial activity involves the buying and selling of works of art. For its part B requires the assignment of works of art and vintage cars, as collectors.

The wife also requires the exclusive custody of the daughter of 7 years and a check for her maintenance amounting to 2,500 euros per month. Although not requiring a maintenance allowance for himself, she demands to keep the surname of the husband (Article 5, Law No. 898 of the Italian Civil Code), as this would allow her to continue more profitably her business, being many of the financial contacts she has made from her relationship with her husband.

Remarks: This example clearly shows the importance of child custody in a divorce. Clearly no algorithm can replace the Court in deciding who will take care of the children. An algorithmic procedure (but not a fair division one) can be helpful in computing the maintenance check. Finally, when data is input into a DRS to find a mutually satisfactory solution, the ex-spouse who will have to provide the maintenance check will express his/her preferences for those items that can guarantee a future income (such as the apartments in this example), or that can be easily liquidated.

Example B.11

Pursuant to art. 709 ter of the Italian Civil Code, the parent who proves to be in default of the procedures for the assignment of the child can be obliged by the judge, to compensate the other parent. The nature of this compensation, as underlined by most of the doctrine, is that of civil sanction and for this reason should be proportional to the gravity of the violation and ***adequate*** *to the patrimony of the defaulter.*

For this reason, the parent A, head physician in dentistry, custodial of B, unilaterally changes the residence without parent C's agreement, will suffer a more severe sanction than the parent D, construction worker, who modifies, in turn, without parent E's consent, the residence of their son F.

Remarks: An algorithm can be used to define a sanction proportional to the defaulter's income. We note, however, that the example cannot be treated as a fair division problem, since it deals with a fault in the application of the provisions issued by the Court after a divorce, and not with the provisions themselves.

1.5. Company Law Divisions

The division of company laws is not an easy task to model as there are several situations that require a detailed consideration. The ownership may be divided into many co-owners, or with just a few of them with very different percentages and different willingness about whether or not to quit the business. Once it has been decided for a division, it is not always immediate to redistribute the asset side of the company since a redistribution has to consider the capital in all the forms it has been transferred from every co-owner and the effort and time the put in the business by them. Moreover, stakeholders have to be considered with their different percentages of credits. It may be the case that a company faces a bankruptcy procedure or that one or more creditors propose an agreement to avoid bankruptcy. In these cases, the algorithmic division has to follow certain criteria set up by law. The issue and hints originate from the analysis of the example brought by the project's legal team.

1.5.1. Specific cases

Example C.1

The law allows a dominant shareholder to squeeze out minority shareholders. If one shareholder has 90% or more of all shares he can demand that all other share be sold to him. He has to pay a suitable price for the shares. The law does not give any concrete definition of the term suitable price. It's a legal standard that has to be "filled" by the courts in every single case, using the tools and techniques of modern company price evaluation.

In this case one share was valued at a price of 180 €. Minority shareholders complaint and demanded a price of 35858 € per share.

The Supreme court concludes that it is true that the valuation of the present value of shares subject to a squeeze out (minority shareholders forces sales of shares to a dominant shareholder) should also reflect the future returns of the company (hypothetical and uncertain). So, the auditor should not use only the static balance sheet meth-

od. However, the auditor operates with ex ante data and values the future cash flow and the discount rate from an ex ante stand point. Therefore, the complainant is using a methodologically misleading approach as he attempts to correct the accuracy of the valuator's findings with the data of the assets of the company and the price at the ex post time (including general information about the financial crisis and its consequences on the share price). This means the price of the shares must be determined by the knowledge and fact at the time of the squeeze out and not at the time of the court dispute.

Remarks: The Court should properly define the rules under which a proper negotiation could be carried out. The counterparts should than make their technical valuations and carry the negotiation following binding rules. Though not a proper fair division problem, a suitable DRS may support such decisions providing an equitable algorithm procedure to compensate the minority shareholders for the squeeze out.

Example C.2

A, B and C concluded a partnership contract in 2006, agreeing to contribute their work and/or property to achieve a common objective – a small carpentry factory and a store for selling goods. They had different stakes/contributions which would determine their shares as joint owners. A was a carpenter with experience especially in kitchens and bedrooms. He contributed equipment (valued at 35 000 euros) and of course with his "know-how" and experience. B had business premises large enough for the factory and for the store, and this was his contribution. C contributed in cash 30 000 euros. After the financial crisis, the business began to deteriorate so person B proposed to change the purpose of their business to stocking and selling electronic appliances which would be directly imported from China. B still thinks that he is the only one who can decide about the purpose of the business premises. A was disappointed because they didn't need him or his work anymore. C only cares about profit. The content of their common asset (joint ownership) changed during the decade. They bought new machinery but they also had a special website for selling furniture with the possibility of on-line interior design as an additional service. To set up this website they had to spend 4500 euros and they pay 1200 euros monthly for software licenses and website maintenance fees.

They decided to dissolve the joint ownership and the first step that court had to make was determining their shares. The court decided that A has 3/9, B has 5/9 and C 1/9 of the business. By determination of their shares joint-ownership was transformed into co-ownership.

At the dissolution of co-ownership (in May 2016) the assets consist of all of the above mentioned but also includes new machinery (valued at 20 000 Euros, store items valued at 30 000 euros, and a profit of 15 000 Euros). In the process of partitioning co-ownership, A wants all machinery, but also a part of the property where the factory was located because he wants to continue running the same business by him-

self. B wants a part of the profits to start with his idea and all business premises. He is also interested in the website because he wants to sell online. C is interested in money only and proposes to sell the business as a whole, but he might consider the possibility to continue working with A with slight changes he would additionally propose.

Remarks: This is a case where the Court already assessed the share of entitlement for each co-owner of the company: 3/9 for A, 5/9 for B and 1/9 for C. There are enough goods and enough differences in the counterparts' preferences to make this an easily solvable case through an adequately flexible DRS.

This example also introduces the notion of multiple scenarios: The company liquidation might imply the complete shutdown of the premises, with the proceeds resulting from their selling distributed to the partners according to their quotas. Other outcomes may alternatively take place. For instance, we understand that the major breakup occurs between B and the other partners. A and C may therefore join their forces with the idea to continue their collaboration. They may therefore participate in the DRS as a single player with quota 4/9, the sum of the two co-owners quotas. A predictable outcome could be a fair division of the assets that will allow A and C on one side, and B on the other, to continue their entrepreneurial activities.

Example C.3 – Liquidation amiable

The objective of the liquidation is, after settlement of liabilities on assets, to convert the company's monetary elements, so that sharing can be performed. It also consists in determining the share that each partner must take in charge of the liabilities that cannot be settled on the asset. Liquidation is essential to achieve the sharing. Complex set of operations following the dissolution of the company, it pursues three objectives:

1. To discharge the social liabilities, the social creditors being paid thanks to the patrimony of the dissolved society

2. To refund, if applicable, the contributions made by the partners

3. To establish a net active mass that could be distributed, by way of sharing between the partners.

Remarks: Liquidation of a company is often seen as the simplest way to achieve the sharing. In practice, it is not an easy task deal with because liquidation has to account for the great complexity of the business that the company is involved in. Rarely, the liquidator finds an entrepreneur or a pool of entrepreneurs interested in taking over the entire company. More often, parts of the company may be sold, opening to a threefold implication: the redistribution of the liquidated assets among the stakeholders and shareholders, the redistribution of the unliquidated assets, the expected amount of time to liquidate the entire company. In all cases, a trade off arises between the sold off due to a need of liquidity and

the willingness to wait for a better offer. DRS may provide useful procedures in helping the judge to guarantee equitable and efficient division once the Court decides case by case who the shareholders and stakeholders are and what percentage of the total value they are entitled to. In particular, a DRS based on fair division, may help in all those cases where not all assets were traded in for cash, but there are still goods, as well as liabilities ("bads") to distribute.

Example C.4 – Fiducie (trust)

The trust is the transaction by which one or more constituents transfer property, rights or security, or a set of property, rights or security interests, present or future, to one or more trustees who, holding them separate from their assets, act for a specific purpose for the benefit of one or more beneficiaries, if there are different beneficiaries, there is a division of assets. The main consequence of the trust is to transfer to the trustee, which is necessarily a financial institution, a business of insurance or a lawyer, the property initially owned by the grantor.

• Fiducie assignment: The trust is used for the transfer of property, precisely the property of the settlor to that of the beneficiary, after management of the property put in trust by the trustee. This assignment must necessarily be made à titre onéreux, i.e. in exchange for a price. The validity of the trust agreement assumes that the beneficiary of the trust, when not the settlor himself, has a consideration equivalent in value to the value of the property transferred to him by the trustee under the terms of the trust agreement.

• Trust management: The trustee undertakes, if necessary for remuneration, to manage the property sent to him on behalf of the settlor and to surrender it to him on a specified date.

• Security trust: The constituent and the beneficiary are united on the same head, the trustee being a credit institution lending money. The settlor, the initial owner of a property, transfers ownership of the property to the trustee as security for the loan. If the loan is not repaid at maturity, the trustee irrevocably retains ownership of the property.

Remarks: A fair division DRS may be employed whenever the asset has more owners and only a part of them must undergo the trust. The procedure will then determine which goods of the larger asset will actually be managed by the trustee(s).

Example C.5

Company A, that has business in the real estate, faces a bankruptcy liquidation procedure.

Its assets are composed of:

- *Real estate properties, to the value of 1.250.000 euros;*
- *Securities, to the value of 250.000 euros;*
- *Credits, to the value of 200.000 euros.*

Its liabilities are composed of three mortgage secured credits for a total credit val-

ue of 300.00 euros. Ten unsecured credits for a total value of 3.000.000 euros, of which one subordinated for a total credit value of 100.000 euros, three resulting from credit provisioning for a total credit value of 1.000.000 euros and five resulting from employment for a total credit value of 150.000 euros.

Two out of the five unsecured and unsubordinated creditors propose the following (bankruptcy) agreements:

One of the two company, B Spa, is in liquidity needs, and propose to divide the liabilities in four classes of creditors to be paid with the following percentages:

- *Mortgage creditors (who cannot receive less than what the liquidation would pay them, art.124, c.3 l. fall), payment at 80%;*
- *Credit resulting from provisioning and employment, payment at 75%;*
- *Other creditors, payment at 45%;*
- *Subordinated creditors, payment at 15%.*

The other company, C Spa, is also in liquidity needs but can wait longer than company B. Therefore, it proposes an agreement with deferred payment of a greater percentage of the credit than the previous agreement and three classes of creditors:

- *Secured creditors, payment at 95% in 5 years;*
- *Credits resulting from employment, payment at 85% in 4 years;*
- *Other creditors, payment at 65% in 4 years.*

The company debtor, A, proposes an agreement itself, in order to maximize stakeholders and shareholders' profits in the short term. Moreover, every year the company's real estate properties appreciate 5% whereas securities depreciate 1%.

In the agreement proposed by company B, creditors of class III play the main role as the breakdown of the votes shows:

- *Class 1: 9%;*
- *Class 2: 35%;*
- *Class 3: 53%;*
- *Class 4: 3%.*

Still the same class of creditors is crucial in the agreement proposed by company C, as the breakdown of the votes shows:

- *Class 1: 9%;*
- *Class 2: 4,5%;*
- *Class 3: 86,5%.*

The application of equitable algorithms is intended to determine the equilibrium among the percentage of share, the deferral in the payment and the appreciation/depreciation of the company's assets.

Remarks: This example is instructive and suggests the introduction of procedures that were not discussed so far. Note that the only agreement that will be signed will be the one voted by the majority of creditors of company A (art. 128 l. fall). Moreover, a creditor who disagrees with the majority may appeal the decision. This setting solicits several questions:

a. What is the appraisal rate of a given plan (i.e., the percentage of creditors who should approve the plane according to their self-interest)? What is the discon-

tent rate (the percentage of creditors who could appeal with a reasonable rate of success)?

b. Given two different liquidation plans? What is the most likely to be voted because it should receive the approval of most creditors? What is the less likely to be appealed (because it creates less discontent in the minority who opposed it)?
c. Does there exist an optimal plan in terms of approval and low discontent?

A sketchy analysis suggests that Company B's plan will be chosen provided that the majority of creditors prefers a 65% payment in 4 years rather than an immediate payment of 15%. Also in terms of discontent, Company B's plan should prevail.

Clearly, technical matters such as the percentage of share, the deferral in the payment and the appreciation/depreciation of the company's assets should be evaluated together with independent financial consultants. However, it is worth to recall that some of the percentages are determined by Law.

Example C6

to avoid bankruptcy (ex. art 161 c.4 l.fall.). Such agreement is composed of two elements:

1) Availability for creditors of the company's assets (real estate properties, other company's shares, credits) for the total value of 35.000.000 euros;

2) Future cash flows (by the continuity of the business) to the value of 7.100.000 euros each year for a total value of 28.400.000 euros in four years.

Financial resources deriving from 1) aim to totally satisfy:

- *Secured preferential creditors for 4.000.000 euros;*
- *Secured creditors for 17.000.000 euros;*
- *Mortgage-backed secured creditors for 7.000.000 euros.*

Financial resources deriving from 2) aim to satisfy six classes of unsecured creditors:

1) Ordinary credits for 50% of their value, that is 6.000.000 euros;

2) Contested claims for 40% of their value, that is 400.000 euros;

3) Credits due to employees, pension institutions and Treasury for 100% of their value, that is 4.000.000 euros;

4) Strategic unsecured creditors (banks, etc.) for 100% of their value, that is 9.000.000 euros;

5) Other unsecured creditors for 50% of their value, that is 8.000.000 euros;

6) Subordinated creditors who will be paid with shares up to their total credit of 1.000.000. euros.

At time of approving the agreement the judge lowers the percentages of the secured creditors after the majority of creditors voted for a less optimistic forecast of future cash flows.

Algorithmic procedures are intended to provide the most convenient allocation that keeps into account the need to continue the business and together with probabilistic valuations of the activity. Issues regarding the breakdown of votes, in a manner similar to what was done in the previous example can be carried through.

Remarks: Once again, a technical consultant should be appointed to forecast future scenarios of the firms' business and to provide a valuation of future cash flows which is a common practice in business valuation. To evaluate the plan in terms of approval and discontent rates, the consistency of each class of creditors should be assessed.

Example C.7

Company A is a construction company in financial needs and proposes an agreement to keep continuing its business.

Such agreement is based on a business plan that aims at generating future cash flows, enough to satisfy all the company's obligations. Specifically, the business plan proposes to renovate and then sell some real estate properties (apartments) at a price that is twice as much as their actual value of 5.000.000 euros.

The plan proposes a breakdown of the company's creditors as follows:

Class 1) Mortgage creditors for a total value of 2.000.000;

Class 2) Other secured creditors for a total value of 4.000.000;

Class 3) Unsecured creditors for a total value of 1.500.000.

The plan provides a full payment of the three classes of creditors in 5 years, with creditors of class 1) being paid in the first two years and creditors of class 2) being paid in the following two years.

Costs for the apartments renovation are accounted in the business plan as new debt for about 1.500.000 euros.

The three classes of creditors must vote for accepting or not the agreement and B, a creditor of class 2) for 3.000.000 euros, wants to understand whether voting as creditors of class 2) and 3) or voting the opposite, as creditors of class 1).

The algorithm may help creditor B deciding how to vote.

Remarks: The decision whether to accept the agreement will depend on the market price of the apartments, all other creditors' subjective criteria being equal. Again, a technical expertise is called for to evaluate the real market prices of the apartments.

From the breakdown of the classes of creditors, class 2) has the majority of votes, and creditor B's vote is decisive for any outcome (in technical terms this creditor is called *pivotal*). Therefore, this creditor may vote in disagreement with other creditors in his class, if he foresees a greater advantage. The algorithmic procedures may reveal what are the consequences of B's vote.

Example C.8

Company A, that has a business in food distribution, faces a bankruptcy liquidation procedure.

The company assets are:

- *A real estate shed for a value of 1.700.000 euros;*
- *Industrial equipment for a value of 700.000 euros;*
- *A lorry and four refrigerated vans for a value of 200.000 euros;*
- *Cash for a value of 250.000 euros.*

B, the only secured creditor, proposes an agreement that allows to recover all the credits in 5 years and pays the preferential creditors cash immediately.

The agreement proposes a breakdown of the creditors as follows:

- *Class 1) One secured creditor (for a value of 2.000.000 euros) fully paid with the allocation of the only building;*
- *Class 2) Secured creditors (for a value of 2.000.000 euros) paid 100%;*
- *Class 3) Unsecured creditors with contracts since at least 2 years (for a value of 400.000 euros) paid 60%,*
- *Class 4) Unsecured creditors with contracts since at least 4 years (for a value of 500.000 euros) paid 40%,*
- *Class 5) Unsecured creditors with contracts since at least 6 years (for a value of 100.000 euros) paid 10%;*
- *Class 6) Subordinated creditors (for a value of 50.000 euros) not paid.*

The agreement is approved by creditors from class 1), 2), 3), 4). Creditors of class 5) and 6) make opposition when approving the plan.

These creditors have to show the reasons why the plan is not convenient, ex art. 129 l.fall. However, the judge may approve the plan anyway whether he/she believes that the opponents' credits may not be satisfied by other proposals.

Remarks: As noted before, algorithmic procedures may help the judge in finding the best economic allocation among different proposals in terms of approval and discontent removal.

In this case, the algorithm could also help creditors of classes 5) and 6) in their claim that fairer liquidation plans are possible.

Example C.9

A and B decide to enter a joint venture in the agricultural sector. At the time of the establishment of the company A transfers:

- *Stocks of animals and vegetables (cattle, plants, seeds) and two agricultural machinery (a tractor and a thresher machine);*
- *Cash for 50.000 euros.*

A also pays for the building of the shed and stable. B transfers:

- *A plot of 30 hectares of land together with a wrecked house;*
- *Cash for 20.000 euros.*

After a not rewarding business year C joins the company and transfers:

- *Cash for 100.000 euros;*
- *His expertise as agronomist.*

After the inclusion of C, the company double its business and buys new plots of land, animals and enters the wine market, the most rewarding among the business it is involved in.

After 30 years, A, B and C decide to quit the company that has asset side as follows:

- *A ploy of 60 hectares of land, 1/3 for the wine production, 1/5 for the cattle business and the rest dedicated to other production;*
- *The wrecked house is a farm house;*
- *A patent for a new machinery aimed at the wine production;*
- *Three sheds (one for each kind of business) one stable and a basement for food and wine tasting;*
- *Five agricultural machinery;*
- *Two vehicles;*
- *Products for a value of 400.000 euros;*
- *15 cattle and 10 sheep;*
- *Gazing rights;*
- *Cash for a value of 650.000 euros.*

A, B and C express their preferences:

A wants the animals to keep with the dairy production. C wants all that is related with the wine production: lands and machinery. B, the oldest among the three, wants cash and the wrecked house.

Remarks: We return to a context were a proper fair division DRS will help the three partners to find a mutually satisfying agreement. The large amount of cash should make the agreement as smooth as possible. It is very important to determine the parties' shares based on the resources and efforts spent in the enterprise.

Example C.10

The communion can be distinguished in voluntary (i.e. dependent on the will of the participants in the communion, more people, for example, buy together the same good and become, for this reason, co-owners), incidental (not dependent on the will of the participants: more people for example, they receive the same good as an inheritance and find themselves, independently of their will, to be co-owners) or forced (which cannot be avoided: so it is in the condominium of the buildings, so in the case of the forced communion of the wall that the neighboring owner can impose on the other owner who has not respected the legal distances).

The institute of the division is indeed a complex sector; think, for example, in the substantive field, how, in the matter of dissolution of the communion, the registered creditors and the assignees from a participant, while having the right to intervene in the division, pursuant to art. 1113, paragraph 1, of the Italian Civil Code, are not parties in that judgment, to which only the holders of the relationship of communion must participate, being able instead, the registered creditors and the assignees, to intervene in it in order to supervise the proper conduct of the divisional procedure or to propose opposition to the division not yet performed following judgment which they have not participated, without having any device power, as not sharing (and it follows that the

failure to evoke registered creditors and assignees, in the dissolution proceedings, implies that the division has no effect on them, as expressly provided for by Article 1113, paragraph 3 of the Civil Code.

Remarks: The example outlines a case where creditors and assignees of a participant in a communion to be dissolved (such as co-owners of a company that is about to file for bankruptcy) can intervene in the process of division, but cannot appeal the same decision. A fair division DRS can certainly consider the perspective of such external agents in a very natural way. The creditor or assignee can formulate a preference profile, just like the actual participants. That profile will then will be weighted as a share (proportional to the participant's contracted debt) of the participant share of entitlement. For instance, if a participant owns ¼ of a company and the contracted debt with a unique creditor is estimated at 1/3 of the participant's share value, the creditor's preference profile will be weighted as ¼ *1/3 = 1/12, while the participant will be able to count on the residual share ¼ - 1/12 = 1/6.

Example C.11

- *Case A – (Corte d'Appello Roma, Sez. III, 20 settembre 2006, n. 3905)*
  The concept of "convenient divisibility" of the property referred to in articles 720 and 1114 of the Civil Code. It postulates, from a structural point of view, that the division of the asset can be carried out through the determination of concrete quotes capable of autonomous and free enjoyment and, from an economic-functional point of view, that the division allows maintenance, even if proportionately reduced functionality that had the whole and does not entail a significant depreciation of the value of the individual shares proportionally proportional to the value of the whole, taking into account the normal destination and use of the asset itself.
- *Case B – (Cass. civ., Sez. II, 27 novembre 2017, n. 28230)*
  The non-divisibility of a property, by adding an exception to the right of each participant in communion to achieve the goods in kind, can be considered legitimately viable only when the recurrence of its assumptions, constituted by the unrealizability of the splitting of the property, or from its feasibility under penalty of considerable depreciation, or from the impossibility of forming in concrete portions susceptible of autonomous and free enjoyment, taking into account the usual destination and the previous use of the asset itself.

Remarks: These cases point out the delicate issue of the divisibility of goods. Agreeably, certain items in the patrimony lend themselves easily to being divided, while for others the simple act of division devalues its worth. In the former category we may include, for instance, plots of land, as long as each resulting piece is large enough for its exploitation. In the latter we list, for instance, motor vehicles for personal use (if the owners live in two distant cities). We also note that when-

ever all goods of an asset con be divided according to any proportion of shares among the parts involved, a "satisfactory" division can be achieved, while the same is not necessarily true if all items are indivisible (think of an asset composed of a single car to be allocated in its entirety to one of two claimants living in distant cities). With these premises, we note that a flexible fair division DRS is able to handle divisible and indivisible goods in a variety of ways.

Example C.12

A. *The criteria for settling the damage in the contractual and extra-contractual terms are dictated by the Civil Code (articles 1223-1226-1227 c.c. regarding liability for breach of contract and 2056 ss. c.c. for non-contractual liability) and are arranged by the judges.*

The liquidated damage clause is a particular clause of the contract, expression of the agreement with which, on a lump-sum and preventive basis, the amount of compensation for the damage caused by the non-fulfillment of the obligation or by the delay in the fulfillment is determined.

The penalty clause could dictate criteria to be applied with the algorithm to be implemented with the use of private autonomy.

B. *The evaluation pursuant to art. 1226 c.c. it consists in the possibility given to the judge to appeal, even ex officio, to criteria of equality to make up for the impossibility of proof of the indemnifiable damage in its precise amount. For such evaluation, it is sufficient that the judge gives the indication of reasonable, even if summary, reasons for the logical process on the basis of which he has adopted it, thus remaining in the legitimacy, the exercise of this discretionary power is inconceivable. About the equitable settlement in jurisprudence (ex plurimis,* ***Cass., Sez. Un., 13 settembre 2005, n. 18128, Cass. civ., Sez. II, 1° luglio 2009, n. 15468****) has clarified that:*
   a. *the recourse to the equitable assessment of the damage pursuant to art. 1226 c.c. assumes that there is no evidence of its precise amount and that the demonstration of it is impossible or at least very difficult in relation to the peculiarity of the damaging fact or the subjective conditions of the injured, so that the judge can't proceed when the findings of the case offer elements for a precise quantification.*
   b. *Article 1226 of the Civil Code, as the use of the equitable criterion for the liquidation of pecuniary damage provided by the rule always presupposes that the economic prejudice of which the party claims compensation is certain in its existence and is allowed to the judge only in the presence of an impossibility or an objective difficulty for the interested party to prove the exact amount of the damage.*
   c. *the exercise of the discretionary power to liquidate the damage on an equitable basis, conferred on the judge by the art. 1226 and 2056 of the Civil Code, expression of the more general power referred to in art. 115 of the Italian Civil Code, gives rise not to a fair trial but to a legal judgment characterized by c.d.*

corrective or supplementary judicial fairness, which, therefore, on the one hand is subject to the condition which is objectively impossible, or particularly difficult for the interested party, to prove the damage in its precise amount, on the other hand it does not include the assessment of the which liquidation is involved, assuming that the party's burden of proving the existence and materiality of the damage has already been fulfilled, or exempts the party from providing the evidence and factual information which it can reasonably have, so that the fairness can be appreciated. both as far as possible, brought back to its function to fill only the unsurpassable gaps in the process of determining the pecuniary equivalent of damage.

In addition, the party is called upon to make those deductions that could give concreteness to the specific claim of quantification of one of the components of the liquidation of damages, for which the previous general condemnation has taken place: this deductive activity has the purpose of providing a starting point for the judge for the consequent liquidation pursuant to art. 1226 c.c.: correctly the request is rejected if such deductions are omitted, since the application of the jurisprudential principle, according to which the damage is due to the deprivation of enjoyment or the unavailability of an asset, may be sufficient for the decision limited to the an debeatur, but not exempting the damaged party for the purposes of the quantum, the burden of proof or at least specifically deductive, to provide evidence to the court for the settlement.

C. ***Trib. Milano 28 dicembre 2016***

In the judgment of liquidation of the loss-of-chance damage, both the aspect is relevant of the proximity of the factual situation to the achievement of the desired result, both the profile of the greater or lesser suitability to guarantee this result. Under the first aspect, the value of the loss it will depend on the sufficiency of the behavior held or missed by the manager to determine the desired result (ie the need, on the contrary, of the intervention of further events, to be evaluated case by case as to the probability or only to the possibility of their occurrence); under the second aspect, it will detect the concrete suitability of the situation to determine the desired result, ie the probability enjoyment of the result, even in percentage terms.

At the same time, jurisprudence has clarified that precisely because the illicit fact has not been the cause of the death in itself, but only of the death in that date and not subsequently, the compensation of the damage patrimonial and non-patrimonial in favor of the entitled persons will be able to invest that anticipation of death, and therefore have as a reference term the period of time between the date in which the event (exitus) actually occurred and the one in which it would presumably be verified if the tort (failure to slow down the disease or accelerator of pre-existing pathogenic factors) was not been committed. The principle must necessarily be adapted to the case where the damage consisted in a loss of probability, having to modulate the criterion of quantification of compensation by reason of the greater or lesser fitness, even percentage, of the chances to produce the desired result. Otherwise to what happens in the case of non-pecuniary damage due to injury to physical integrity (where he/she rescues, for invalidity from 1 to 9 percentage points, the normative dictate art. 139 Cod. Ass., And for greater injuries the evaluation grid of the notes in the Milan ta-

bles), the loss of chances of healing or survival does not find liquidation point of damage, neither constant, solid and convinced jurisprudential findings. This type of damage can be resolved by recourse to a rigid liquidation scheme; the Court must therefore necessarily identify a criterion that guides the exercise of the equity that is called to apply in the liquidation operation of the non-pecuniary damage from loss of chance. In the absence of normative forecasts, it can be referred to as the first guideline criterion, which can compete usefully to identify balanced values, as elaborated by this Court through the compilation of the Tables for the settlement of non-pecuniary damage.

Within them – for the surviving relatives, today's actresses – can take on, as an evaluation point of departure the damage from parental loss, adequately reconfigured in compliance with the peculiarity of the wound.

The relative figure concerning the liquidation measure, in line with the principles expressed above, should be considered considered and, therefore, adequately remodeled in consideration of the clinical event, of the conditions family and social circumstances of the injured person and of any other index present in the concrete situation of the injured party, appreciable according to the perimetration of the specific content of the possibilities (healing or survival) lost, to be evaluated in every relevant profile and relevant to the reflections on its integrity psycho-biological, conditioning and prejudice in carrying out its areddítual activities, ad any further moral aspect that contributes to describing non-pecuniary damage, and, necessarily, based on the findings and also the presumptive allegations offered by the party (it should be remembered, in fact, that non-pecuniary damage even when determined by injury of inviolable rights of the person, constitutes damage consequently, which must be attached and proven, with the effect that the injured party must in any case attach the appropriate elements to provide, in the concrete case in point, the linked series of facts that make it possible to presume and identify the various profiles of damage) (Cass. Sez. un. 11 November 2008, n. 26973).

The Court considers that in this case they cannot therefore to find the application of the grids of values identified by the notes in the Milanese tables; the components offered by the tabular evaluation can however offer a shared parameter to proceed to one equitable settlement of damage from loss of chance of survival which, taking into account age and gender general conditions of health of the patient, allows to proceed to a reasoned abatement of values tabulated information, to be carried out directly proportional to the statistical percentages of favorable outcome (to be understood in the present case in terms of five-year survival, considered in medical-legal terms a long-term survival) that statistical medical science is able to report for the case in question.

The general health risk conditions presented by the patient, a tobacco addict with a pathology chronic respiratory system, even if moderate, impose a further prudential abatement, having regard predictable more limited life expectancy, even in the absence of oncological pathology, compared to statistical average.

The incompleteness of the necessary elements of knowledge can be attributed to it exclusive to the negligent conduct of the defendant institution and, for it, its health care; the shortage evidenced cannot therefore be placed on the charge of actresses, who could not in any way access to evidence tools able to fill the highlighted gap; for this, in application of the principle of the cd. of proximity of the proof, the right to

compensation of the injured party cannot be limited in the quantum deriving from a possible flattening on the statistical minima resulting from the application of the statistical data related to the least favorable prognosis.

It is therefore necessary to proceed to the identification of an average statistical value to be identified within the scissors consisting of the minimum percentage attributed to the survival predictions for stage III A (15%) and the statistical value of maximum long-term survival, ie five years, in the case of stage II A (50%) neoplasia, mean value that is determined in 32.5% chance of survival a five year.

The overall subjective conditions of M.T., particularly concerning the fact of being a man of 66 years old, a great smoker, a bearer at the time of COPD, as far as a slight degree is concerned, they impose the adoption of a parameter close to the minimum values of the reference tables based on the verified those health risks related to the conditions indicated (and in particular to smoking, behaving, as it is known, an exponential increase compared to the statistical average, to contract potentially pathologies mortals), for which a prudential assessment of the survival of the subject with respect to statistical mean even in the absence of neoplastic pathology.

The application of the indicated criteria leads to the determination of the following amounts:

- *As to the damage borne by M.T. for the loss of long-term survival chances, stimulated reasonable, at a prudential valuation, the sum of € 40,000.00, adopting as a basis for calculation the minimum table value identified for the parental loss of spouse / child, € 163.990, taken into consideration to the subjective characteristics of the patient (general health conditions, life habits and age of subject at the time of the facts, -66 years), pulled down to € 53,296.75 due to the statistical percentage, 32.5%, of favorable possibilities recognized in medical legal terms; further reduced until recognized as a result of the limited survival time span identified by the medical expert in five years;*
- *as to the damage from parental loss (deriving from the loss of chances to enjoy the years of survival of one's own joint that a timely and correct therapy to combat the disease would have able to guarantee) in favor of the surviving wife and the daughter, today's actresses, the congruent estimate corresponding sum of € 40,000.00 each, having regard to the duration over time and the intensity of the emotional bonding as attachments, and not disputed, in acts.*

The amounts thus liquidated must be understood as inclusive, having taken into account, aspects not only contingents as related to acute suffering resulting from the harmful conduct, but also the broader prejudice that has resulted in all aspects of the patient's life and relatives in relation to the broader affective relational context of the injured subject, in this by re-entering each vulnus directly deriving from the injurious event in question, in a unitary assessment of the non-pecuniary damage deriving from it from the loss of survival chances.

The damage deriving from the failure to timely enjoy the pecuniary equivalent must also be recognized that, in the absence of various evidence, it is considered to compensate by adopting that parameter of the legal interests to be calculated, according to the teaching of the United Sections of the Supreme Court (n. 1712/95), on the sum gradually revalued by the production of the damage event to date, time of the liquidation.

As to the degree of responsibility to be attributed to each of the negligent conduct examined, yes considers that – given the lack of specification by the experts of specific and autonomous percentages of damage, considering the impossibility to distinguish, in scientific terms, an appreciable one different gravity of the causal contribution of each defendant, considering that the loss of chances of survival appeared as the result of a series of successive clinical consequences without solution continuity – the responsibility itself must be attributed to each of the defendants at the same level.

As for the hospital body, suffice it to observe that the serial nature of the omissions of the errors committed against M.T. in the course of repeated visits of the patient to the Institute of Multiple professionals carrying out their activities, is indicative, on the one hand, of a clear culpa in eligendo of the professionalism calls to operate inside; on the other, a clear violation of control duties (to be exercised primarily through the adoption of operational protocols and verification procedures) of the work of its employees and collaborators.

Remarks: These examples deal with the design of clauses in contracts that describe "agreement[s] with which, on a lump-sum and preventive basis, the amount of compensation for the damage caused by the non-fulfillment of the obligation [(the subject matter in the contract)] or by the delay in the fulfillment is determined". An example is given, where the sum to compensate the damage arising from the misconduct of a medical team is determined. The resolution of these issues requires advanced statistical, actuarial and medical tools such as survival tables under various medical conditions, tables for the settlement of non-pecuniary damages, invalidity percentage estimates. The project's goal is to augment the use of fair division procedures to provide a quantitative help in solving all disputes involving the assignment of assets and liabilities deriving from the dissolution of a conduction arrangement (whether caused by death or divorce or bankruptcy). According to these goals and due to the reduced size of the team and to the short duration of the project, we were not able to assign a high priority to the treatment of the kind of problems described in this example.

1.6. Conclusions

1.6.1. Lessons from the Examples

In the light of the examples brought to us by the project's legal team, a general-purpose fair division procedure suitable for the legal sectors singled out in the present project, should have the following features:

i. It should avoid random outcomes – as pointed out in example A.11.
ii. It should be able to deal with agents having different shares of entitlement – both as a result of a different involvement (as is the case of close and distant relatives in a succession) as well as of a different effort in the maintenance of

the communion regime prior to the event ending that regime. This was pointed out in examples A.1, A.12, B.2.

iii. It should be able to consider allocations where items and/or money are preventively assigned to one of the agents. This could take place in a succession as a result of the deceased's will, or by a previous donation (see examples A.3, A.4, A.5, A.9).

iv. It should take into account that certain items may have to remain indivisible, and, therefore, these have to be assigned to one of the agents in their entirety – possibly with a monetary side payment to compensate for the excess of value that this designation might imply. We refer to Examples A.2 and C.11 for further details.

v. It should consider the need imposed by the Law or by the circumstances to certain agents for liquid assets. Also, the simple inclination by certain agents for monetary or easily tradeable goods should be considered. These situations were pointed out in examples A.2, A.7, A.13.

vi. It should encompass liabilities ("bads", in the economic jargon), as well as assets ("goods") – as noted in the example B.8.

There are other features which cannot be included in a general-purpose procedure, that should, however, be developed as side projects:

a. A preliminary routine should measure the contribution or the dissipation that an agent has brought to the asset which was in communion prior to the dissolving event. The same routine should also convert this effort or harm in to the corresponding increase or decrease in the share of entitlement. We refer to examples A.1 and A.2.
b. A similar routine should also modify the shares so to provide allocations proportional to the economic need of the agent involved in the division – as pointed out in examples A.3 and A.4.
c. A procedure should consider those situations where the interested agents are linked by more than one relationship: This was the case in example B.6 in which economic and affective relationships are mingled.
d. As pointed out in example C.3, a preliminary routine should also provide alternative and credible scenarios where agents can find a common ground and perform the division.
e. The intervention of creditors over participants in the division, as pointed out in the example C.10, should be considered. Here, a simple procedure could compute the share of entitlement that those creditors can claim.
f. Examples C.5 through C.8 illustrate cases where companies go through bankruptcy and liquidation plans are devised to satisfy the creditors. More than a side project, this is a complete system to be set up in order to achieve a satisfactory liquidation of the company, while minimizing the creditors' discontent.

The cases we examined also consider delicate issues, such as child custody in a divorce (see example B.10). These matters cannot be treated as one of the items composing the asset. For their sensitivity, we recommend the exclusion of such issues from an algorithmic treatment. Only the economic consequences, such as for instance the quantification of an alimony to help in the custody could be considered.

Also, the other questions may be faced with the aid of algorithms. Among these, we consider the assessment of lump-sum penalties for defaulting a contract described in example C.12, or the definition of the just price for the squeeze out of a minority shareholder (seen in example C.1). While certainly relevant, the solution of these problems requires the development of skills and techniques that are quite distant from the body of the project.

1.6.2. Further Considerations

The examples listed in the current report motivate us to reconsider the algorithms reviewed in Dall'Aglio, Di Cagno and Fragnelli (2020), to see whether they can be adapted to the juridical context. In the conclusion of that paper we listed three broad classes of algorithm. We note that:

- Procedures in class I (in which contenders express a ranking of the contested items and these items are indivisible) are certainly the easiest one to implement. Most of these procedures work for two agents with equal share of entitlements. For this reason, this procedure may be applied in divorces, or in all those contexts where there are two contenders. We note, however, that since these procedures deal with indivisible items only, they may fail to return a satisfying solution. Also, their adaptation to the required features in the current context is not a trivial one. For instance, handling unequal shares may prove challenging.
- Procedures in class II (in which contenders allocate points to the items that can be divisible or indivisible). These procedures, while less intuitive than those in which only a ranking of items is required, typically return solutions that satisfy many optimality properties. In the case of indivisible goods, they return solutions that are quasi-optimal. These procedures lend themselves easily to the adaptation required: Some of them can be implemented for any number of agents and for any share of entitlement attributed to each agent. Also, recent results (Bogomolnaia et al., 2020) shows that one of them, The Nash product maximizer rule, is capable of handling goods and bads simultaneously. When goods prevail over the bads, the proposed solution shares many appealing features and guarantees an efficient and envy-free solution. These procedures are also flexible enough to consider the forced attribution of items to players as well as the indivisibility of certain items.
- Procedures in class III (in which contenders allocate indivisible items and

money among themselves). Also in this case, the proposed solutions satisfy many optimality conditions (though not as neat as the methods listed in Class II), and the presence of money usually smooths out the disparity induced by the presence of indivisible goods.

The development of a general-purpose procedure capable of handling as many features as possible among those listed in the previous subsection should give precedence to the procedures in classes II and III, with two additional remarks:

- While procedures in class II share a larger number of optimal features, procedures in class III are particularly appealing because agents are required to provide a personal monetary estimation of each item involved in the division – since items usually have a market value, this is certainly more natural than assigning a score in a 0-100 range to each item. The possibility of including monetary valuations in Class II procedures should be carried on.
- Procedures in class III typically assume (see Bevia, 1998) that an unlimited amount of money is available for the allocation. This is often unreasonable, since in a succession or a company liquidation, only a limited amount of cash is available, and agents are unwilling to use their own money to solve arising disputes. A more realistic assumption should consider a limited availability of money for each party.

Simultaneously, the feasibility of the side projects listed as a. through e. in the previous subsection should be pursued. Some of these are simple additional routines. Consider for instance the computation of the creditors' share over the participants in the division. Others, as for instance case f., require the development of a more complex model in which creditors are able to express their preferences, and the feasibility of their proposal are evaluated by suitable algorithms.

The side projects that will pass this feasibility analysis will then be properly implemented.

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Marco Dall'Aglio * and Vito Fragnelli **

On the Manipulability of the Division of Two Items Among Two Agents Using a Bargaining Approach

Abstract

We investigate the possibility for an agent of manipulating her/his declarations on the evaluations of the items in order to increase her/his total utility, when the division is stated as a bargaining problem. Our analysis shows that the Nash solution has a slight advantage with respect to the egalitarian solution in terms of manipulation prevention.

Keywords: *Fair Division, Manipulation, Bargaining, Nash Solution, Egalitarian Solution*

1.1. Introduction

Fair division theory is a well-recognized problem at the border between mathematics and the social sciences, with an established scientific literature and a long list of applications ranging from allocation of CPU time to border settlements in land disputes.

The most important results in this area typically rely on an assumption that is very hard to get rid off: When prompted, all agents are assumed to reveal their true preferences over the disputed items. On the contrary, twisting the personal preferences may reveal advantageous for an agent, especially when that agent has some information over the other agents' preferences.

In what follows, we conduct an analysis of these gains in a very simple setting. First, we consider two agents, I and II, and two items, a and b; second, we suppose agent I prefers item a, while agent II prefers item b; third, as the egalitarian solution requires to normalize the declarations of the agents, in order to avoid completely inefficient solutions when the evaluations of one agent are very large

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w.r.t. the evaluations of another one, we consider the normalized declarations also for the Nash bargaining solution. We recall that a similar hypothesis is made also for the Adjusted Winner solution and the Proportional Allocation (see Brams and Taylor, 1996).

The note is organized as follows. Section 2 surveys the manipulability proposed in Brams and Taylor (1996). In Section 3, we present the model and the main hypothesis; Sections 4 and 5 are devoted to the Egalitarian solution and its manipulation; Sections 6 and 7 consider the Nash solution and its manipulation; Section 8 concludes.

1.2. An important precedent

The possibility of altering the declared value of an item in order to improve the gain of an agent, was tackled in Brams and Taylor (1996) referring to two procedures, namely the Proportional Allocation and the Adjusted Winner. Both procedures require the two agents to declare their valuation of the items, assigning a total of one hundred points to them (with the additional hypothesis of assigning only integer numbers of points). The Proportional Allocation procedure divides each item among the two agents proportionally to her/his valuation of the item; the Adjusted Winner procedure first assigns to each agent the items s/he evaluates more than the other one, then some items are transferred from the agent that received more points to the agents that received less points in such a way that both receive the same amount of points, possibly dividing a unique item. Note that result of the Adjusted Winner procedure coincides with the Egalitarian solution.

The Adjusted Winner procedure has the advantage of dividing at most one item, but results more easy to manipulate; on the other hand, the Proportional Allocation procedure is not Pareto efficient, as it is possible to increase the values assigned to both agents.

According to our results for the Egalitarian solution, the optimal manipulation for the Adjusted Winner procedure is to assign just one point more than the other agent (the minimal variation that an integer point allocation allows) to the preferred item.

The manipulation of the Proportional Allocation procedure leads at most to a very small advantage, when one agent knows the declaration of the other agent, but may result in a reduction of the utility of the manipulating agent.

In view of this, the proposal by Brams and Taylor is a so-called combined procedure. The idea is to use the Adjusted Winner procedure that is efficient, allowing the agents of asking for the Proportional Allocation procedure when the result of the Adjusted Winner procedure seems affected by a manipulation, possibly revising their declared values; this should be a very good way to reduce the incentive for manipulating the Adjusted Winner procedure.

1.3. The Model

We model the declarations of the two agents I and II on the two items α and β as the table given in the following:

Agent/Item	α	β
I	A	1-A
II	1-B	B

A and B may be interpreted as the percentage of value assigned by agents I and II, respectively, to the two items α and β; consequently, agent I assigns 1-A to item β and agent II assigns 1-B to item α. The preference of agent I for item a corresponds to the inequality

$$\frac{A}{1-B} > \frac{1-A}{B} \Rightarrow A + B > 1$$

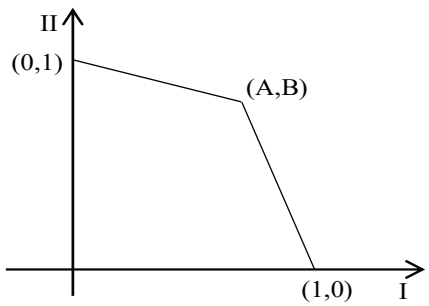
So, we state the following hypothesis.

Hypothesis 1

We suppose that A + B > 1.[1]

It is possible to represent the above situation on a Cartesian diagram, where the horizontal axis corresponds to the utility of agent I and the vertical axis to the utility of agent II.

The quadrilateral area represents the feasible region, i.e. the possible amounts of utility assigned to the two agents, and the boundary correspond to the Pareto efficient solutions, i.e. those divisions of the two items among the agents for which there does not exist the possibility of increasing the utility of an agent without decreasing the utility of the other one.



[1] The same hypothesis may be obtained starting from the preference of agent II for item β, that is $\frac{B}{1-A} > \frac{1-B}{A}$.

The three extreme points corresponds to the following situations:

- (1,0) - agent I receives the two items;
- (A,B) - agent I receives item α;
- (0,1) - agent I receives no item.

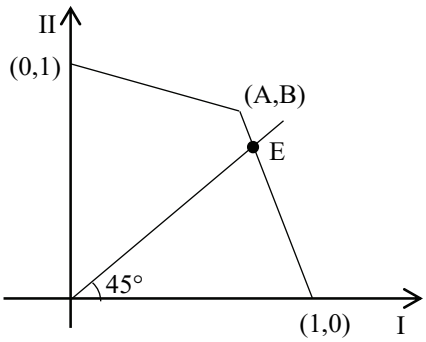
Any point on the edge (1,0) (A,B) indicates that agent I receives item α and a fraction of item β and any point on the edge (A,B) (0,1) indicates that agent I receives a fraction of item α.

Obviously, agent II receives the remaining part.

In the following sections, we apply two well-known solutions for a bargaining problem with two agents, namely the Egalitarian solution, due to Kalai (1977) and the Nash (1950) solution that is the one proposed in the pivotal paper of 1950.

1.4. The Egalitarian Solution

The egalitarian solution E (see Kalai, 1977) assigns to the two agents the same fraction of utility, and it corresponds to the intersection of the boundary of the above quadrilateral with the bisector of the first quarter.



In the figure above, E assigns to agent I the whole item α and a fraction of item β so that the utility for the two agents is the same. This is due to the evaluation B of item β for agent Π that is higher than the evaluation A of item α for agent I. We remark that, being the maximal utility of the two agents equal to 1, then the Egalitarian solution coincides with the solution proposed by Kalai and Smorodinsky (1975).

In general, when Hypothesis 1 holds, we have the following cases.

A = B

The feasible region is symmetric and the solution corresponds to the point (A,B), i.e. agent I receives item α and agent II receives item β.

A > B

In this case, agent I receives a fraction of item α and agent II receives item β plus the remaining fraction of item α. More precisely, the fraction of item α assigned to agent I is $\frac{1}{1+A-B}$; in this way, the utility of agent I is $\frac{A}{1+A-B}$ and the utility of agent Π is $\left(1-\frac{1}{1+A-B}\right)$(1-B) + B = $\frac{A}{1+A-B}$.

A < B

In this case, agent I receives item α plus a fraction of item β and agent II receives the remaining fraction of item β. More precisely, the fraction of item β assigned to agent I is $\frac{B-A}{1-A+B}$; in this way, the utility of agent I is $A+\left(1-\frac{B-A}{1-A+B}\right)$(1-A) = $\frac{B}{1-A+B}$. and the utility of agent Π is $\frac{B}{1-A+B}$.

1.5. Manipulation of the Egalitarian Solution

In this section, we analyze the possibility that one agent makes a false declaration of the evaluation of an item in order to obtain a larger utility. The situation is symmetric for the two agents, so we consider that agent I declares an evaluation A* for item α, but her/his true evaluation is still A; we suppose that Hypothesis 1 holds.

A = B

In this case, if agent I declares A* > A, s/he obtains a fraction of item α, so the manipulation is profitable only when A* < A and A* > 1-B due to Hypothesis 1. In this case, agent I receives item α plus a fraction $\frac{B-A^*}{1-A^*+B}$ of item β, that is decreasing in A*, so the optimal choice is to declare A* = 1-B + ε; with the optimal manipulation agent I receives item α plus a fraction $\frac{2B-1-\varepsilon}{2B-\varepsilon}$ of item β and her/his utility is $\frac{A+2B-1-\varepsilon}{2B-\varepsilon}$. On the other hand, agent II receives a fraction $\frac{1}{2B-\varepsilon}$ of item β and her/his utility is $\frac{B}{2B-\varepsilon}$.

A > B

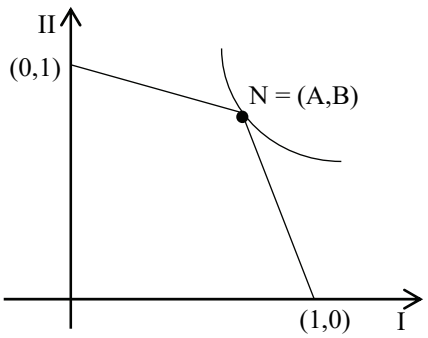
In this case, the fraction of item a assigned to agent I is $\frac{1}{1+A^*-B}$, i.e. it is decreasing in A*, so the optimal choice is to declare A* = B + e; with the optimal manipulation agent I receives a fraction $\frac{1}{1+\varepsilon}$ of item a and her/his utility is $\frac{A}{1+\varepsilon}$. On the other hand, agent II receives item β plus the fraction $\frac{\varepsilon}{1+\varepsilon}$ of item α and her/his utility is $\frac{B+\varepsilon}{1+\varepsilon}$.

A < B

In this case, the fraction of item b assigned to agent I is $\frac{B-A^*}{1-A^*+B}$, i.e. it is decreasing in A*, so the optimal choice is to declare A* = 1-B + e; with the optimal manipulation agent I receives item a plus a fraction $\frac{2B-1-\varepsilon}{2B-\varepsilon}$ of item b and her/his utility is $\frac{A+2B-1-\varepsilon}{2B-\varepsilon}$. On the other hand, agent II receives a fraction $\frac{1}{2B-\varepsilon}$ of item b and her/his utility is $\frac{B}{2B-\varepsilon}$.

1.6. The Nash Solution

The Nash solution N (see Nash, 1950) assigns to the two agents an amount of utility $U_I(N)$ and $U_{II}(N)$, respectively, such that the product $U_I(N)\,U_{II}(N)$ is maximal over the points of the feasible region.



We suppose that Hypothesis 1 holds.

A, B ≥ 0.5

The solution N corresponds to the point (A,B), i.e. agent I receives item α and agent II obtains item β.

A < 0.5

The solution N assigns $U_I(N) = 0.5$ and $U_{II}(N) = \frac{0.5\,B}{1-A}$; note that by Hypothesis 1, B > 1-A so $U_{II}(N) > U_I(N)$. This corresponds to assign agent I item α plus the fraction $\frac{0.5-A}{1-A}$ of item β and the fraction $\frac{0.5}{1-A}$ of item β to agent II.

B < 0.5

Symmetrically, the solution N assigns $U_I(N) = \frac{0.5\,A}{1-B}$ and $U_{II}(N) = 0.5$, with $U_I(N) > U_{II}(N)$; this corresponds to assign agent I the fraction $\frac{0.5}{1-B}$ of item α and item β plus the fraction $\frac{0.5-B}{1-B}$ of item α to agent II.

1.7. Manipulation of the Nash Solution

Now, we suppose that an agent manipulates her/his declaration in order to improve her/his utility. Again, we profit of the symmetry of the situation and analyze only the behavior of agent I supposing that the false declaration is A* and the true evaluation is still A; we suppose that Hypothesis 1 holds.

A* ≥ 0.5

If B ≥ 0.5, then Agent I receives item α and agent II receives item β and their utilities are A and B, respectively.

If B < 0.5 then agent I receives the fraction $\frac{0.5}{1-B}$ of item α and agent II item β plus the fraction $\frac{0.5-B}{1-B}$ of item α; the utilities of the agents are $\frac{0.5\,A}{1-B}$ and 0.5, respectively.

In other words, agent I has no profit from a false declaration A* ≥ 0.5.

A* < 0.5

Agent I receives item α plus the fraction $\frac{0.5-A^*}{1-A^*}$ of item β; the fraction is decreasing in A*, so the optimal choice is to declare A* = 1-B + ε, if B > 0.5; the utility of agent I is $\frac{0.5\,A+B-\varepsilon-0.5}{B-\varepsilon}$. If B ≤ 0.5 Hypothesis 1 does not hold.

1.8. Conclusions

From the previous section, we see that the Nash solution is less profitable for a manipulation, so it can be seen as a point in favor of it. When agent I declares A* = 1-B + ε, with B > 0.5, comparing the utilities of the egalitarian solution $\frac{A+2B-1-\varepsilon}{2B-\varepsilon}$ and the utility of the Nash solution $\frac{0.5\,A+B-\varepsilon-0.5}{B-\varepsilon}$, we obtain a small advantage of a factor 0.5(1-A) ε of the egalitarian solution that goes to zero when ε goes to zero, i.e. the Nash solution offers a smaller gain.

But we can give a further interpretation to the above results. If we impose to each agent to declare at least 0.5 on the preferred item, the Nash solution assigns

item α to agent I and item β to agent II. This means that the Nash solution may reach the truthfulness when the agents have different preferences on the items.

Further research may consider a more general setting, i.e. more than two agents and more than two items, with the possibility for the agents to obtain no item and/or more than one item.

Another possibility is to analyze other solutions of the bargaining approach and/or other approaches from the fair division literature.

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Marco Dall'Aglio *

Fair Division Procedures for the CREA project

Abstract

We present two general purpose procedures that are simple and powerful enough to be used by specialists as well as common EU citizens without specific training on the subject matter.

In the first procedure, that we name "Name your price", available even without an estimated market value for the goods to preliminary agree upon is an adaptation of the Competitive/Nash solution for bargaining problems. Here, the users (agents) will simply have to express their preferences as bids on the items to divide. In the second one, named "Price and rate" users will have to express a rating on a simple discrete scale on which to express their satisfaction/dissatisfaction upon the possibility of receiving the items. The scale can be coarser or more refined, depending on the ability of the users to deal with such scales, and their familiarity with the goods to be distributed.

Regarding the solutions, we build upon very recent scientific findings that guarantee for the solution of the first procedure its (Pareto) efficiency (no other allocation is at least as good for every agent and strictly better for at least one agent) and its envy-freeness (no agent prefers the share of another agent to her own). Regarding the solution of the second procedure, we focus on the efficiency and the equality in the users' satisfaction, but also on another notion of high practical relevance, namely the comparison of the bundles' market values, a common condition often imposed by the law.

Both procedures satisfy invariance properties that discourage the parties from altering their preferences with the goal of gaining from this manipulation at the expense of the other participants in the division.

The procedures are applied to real instances of inheritances, divorces and liquidations presented within the project by the team of legal experts.

Keywords: *Fair Division, Algorithm, Competitive/Nash Solution, Egalitarian Solution, Dispute Resolution System*

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1.1. Introduction

In Dall'Aglio, Di Cagno and Fragnelli (2020) established and recent results in mathematical economics regarding procedures to allocate several objects to a finite number of entities (usually denotes as agents or players) were reviewed. After having described how to formally represent preferences, and what defines a satisfactory allocation, several procedures were listed. For each procedure we outlined the assumptions, the terminology specifically employed for the purpose, together with the most important results achieved.

In Dall'Aglio (2020) the algorithms were tested to provide solutions to many legal cases analyzed by the legal teams in WP1 in three sectors: Inheritances, Divorces and Company Law. Each example was commented from the viewpoint of the algorithm designers. Features to be shared by general-purpose procedures were outlined.

Originating from those works, we will develop two general purpose algorithms to solve fair division problems that form the computational core of the Dispute Resolution Systems (DRS onward) system that is the aim of the CREA project.

In Dall'Aglio, Di Cagno and Fragnelli (2020) many different models and methods are reviewed, and it was shown that many options are at the disposal of the applied mathematician to design optimal ways for allocating goods (concrete items or resources): Ordinal vs. cardinal utility representations, various optimality properties – often conflicting among them, divisible vs indivisible goods' procedures.

1.2. Making Choices

In order to proceed with the definition of clear and simple procedures to regulate the division of items, we have to select the most suited models and adapt them to our context, but we will also build new procedures specifically designed for the juridical setting we are exploring.

1.2.1. Ordinal vs. cardinal methods

In Dall'Aglio, Di Cagno and Fragnelli (2020) two classes of methods were broadly grouped in two classes: those involving the ordinal measurement of the utilities, and those relying on cardinal utility.

Ordinal preferences are certainly the ones that are easier to elicit: each individual ("agent" in the economic jargon) involved in a division should be able to rank items ("goods" or "bads" in economics) that he or she has some familiarity with, according to the personal preferences, even without being accustomed to any notion of economic theory. As a counterpart to this easiness in eliciting this information, we find several drawbacks:

- The reviewed methods work only for indivisible goods. Ordinal utility is not suitable for handling fractions of goods – because fractions themselves are quantitative in nature.
- Most methods in this group work only in the restricted case of two agents. Only three methods work for any number of agents. One of them, Picking Sequence (see Bouveret and Lang, 2011), is very simple, but to produce satisfactory result, it requires a piece of information that is very hard to extract from the agents, namely the probability distributions of the preference profiles. It is therefore impossible to design a general purpose ODR System based on this procedure. Another procedure in the restricted list, the Descending Demand Procedure (see Herreiner and Puppe, 2003), requires the agents to rank not only the goods, but all possible bundles of them. An agent should not only be able to rank a city apartment, a seaside apartment, some financial investment or a yacht, but should also be able to determine whether he or she prefers the city apartment AND the yacht to the seaside apartment AND the financial investment. This should hold for all possible bundles and for all possible agents. The task becomes daunting even with a moderate number of goods. There is only one method, the SA procedure by Brams, Kilgour, Klamler (2017) surviving.
- All the reviewed methods do not bring a guarantee of success, including the above-mentioned SA procedure. This is due to the coarseness of the information conveyed by these methods, together with their narrowness in dealing exclusively with indivisible items. Consider the following simple example where two agents have to divide two items, a luxury city apartment and a modest motorbike, with no possibility for sharing ownership or compensating the worst-off player with money. The analysis of these methods reveals that whenever two or more agents have the same profile over the goods, or profiles that differ by tiny differences, these methods easily fail to produce a satisfactory solution.

For all the above reasons, we turn our attention to cardinal methods. In what follows we review the most important optimality criteria for the cardinal methods.

1.2.2. Divisibile vs. Indivisible Goods

The same example involving a city apartment and a bike shows the inadequacy of methods that distribute goods that have to be assigned in their entirety to one of the parties without the possibility of dividing it, or its ownership, among the participants.

We will propose two different solutions to assign goods to players. In each solution, one or more goods may be split among several parties in different proportions (for instance, the solution may require that a house is divided among three heirs, with the first heir entitled to ½ of the house, the second heir 1/3, and the third heir 1/6).

In any case:

- The number of split goods or parties involved will never be too large: if there are p goods to distribute among n agents, at most n-1 goods may be split (see Sandomirskiy and Segal-Halevi, 2019).
- In practice, however, the number of split goods and involved agents is typically smaller in practice.
- As it will be made explicit later (see Section 2.6, later), there are ways of overcoming the actual splitting of the goods.
- Very often, and the procedures that we are presenting make no exception, a procedure for divisible goods can be easily adapted to incorporate indivisibility constraints for one or more items.

1.2.3. The Problem with Point Allocation Methods

Point allocation methods are the basis of many fair division procedures. Quoting the Wikipedia page for the Adjusted Winner procedure (Brams and Taylor, 1996 and 2000)

"Each player is given the list of goods and an equal number of points to distribute among them. He or she assigns a value to each good and submits it sealed to an arbiter."

When dealing with the division of a patrimony, however, we note that when parties allocate points to items to express their likes and dislikes, the market value of the disputed items involved cannot be ignored. This happens, among other causes, because if no agreement between the parties is found, items can always be sold to third parties, and the collected money shared between the parties.

Therefore, when an agent allocates points to a good, two pieces of information are conveyed:

- The degree of pleasantness/unpleasantness of the item for the agent
- The market value of the good.

We will try to disentangle to the two effects by proposing alternative ways for expressing cardinal utilities:

1. Utility is expressed by bids: How much an agent would be willing to pay for a good, or how much he would be willing to receive for a bad.
2. Since, the utility points convey two different pieces of information, two different types of measurement are used for each item: a) An objective market value agreed upon by all agents and b) an individual rating by each agent.

1.2.4. Building on Simple Data

The following preliminary steps are to be performed in order to initiate the procedure:

1. Goods to be divided are listed and parties involved, also called players or agents, are defined.
2. Each player is given a share of entitlement. Typically, if there are n players, the share will be 1/n for each one of them, but it could be different. For instance, shares could reflect the closeness of relatives to a deceased person, or the effort.

1.2.5. Finding a solution

How to find an optimal allocation of the goods? Research suggests that no single criterion is universally better than others. The recent literature shows that two criteria prevail:

1.2.5.1. Finding a solution

If agents are entitled to the same share, it makes sure that all agents receive goods (or parts of them) such that the sum of the goods' value according to his/her own bids is the same, and this value is as high as possible. In case of different entitlement quotas, equality is attained once values are weighted with the shares in order to attain equality. We refer to Moulin (2003) and to Lang and Rothe (2016) for further details. Some remarks on this solution:

- It was introduced in Pazner and Schmeidler (1978) and it is equivalent to the egalitarian solution in bargaining problems proposed by Kalai and Smorodinsky (1975) (see also Kalai, 1977).
- This is certainly the most intuitive notion. If agents are entitled to the same share, it makes sure that all agents receive goods (or parts of them) such that the sum of the goods' value according to his/her own bids is the same, and this value is as high as possible. In case of different entitlement quotas, equality is attained once values are weighted with the shares in order to attain equality.
- By construction, this solution is egalitarian. It turns out that this solution is also efficient: No other allocation, even a non-egalitarian one, can make all agents better off simultaneously.
- The allocation, however, fails to verify several interesting properties. In particular:
  1. The allocation may cause one or more players to be envious of the goods assigned to other players.
  2. The following problem may occur: Suppose that a solution gives two players some amount of the same good, and nothing else, while at the same time these two differ in their evaluations of the goods they do not get. The solution may favor one player with respect to the other, while, in principle they should get the same amount. We say that the Egalitarian solution suffers from domination.

Regarding the lack of envy, consider the following.

Toy Example: (Dall'Aglio and Hill, 2003) Suppose there are 3 players and 3 divisible items. In the following table, we describe the utility of each item by each player.

Table 1 – *Example where the egalitarian solution induces envy (for agent III)*

Player/Item	a	b	c
I	40	30	30
II	30	40	30
III	10	50	40

The egalitarian solution assigns item a to player I, item b to player II and item c to player III – without splits. Each one gets a utility of 40. However, player III is envious of player II, because he received item c (valued 40) but he would have preferred item b (valued 50 to him) and assigned to Player II.

1.2.5.2. The Competitive Equilibrium from Equal Income/Nash Allocation

It is an allocation where all players spend the same amount of budget, each player buys the goods according to their equilibrium prices, and all goods are bought in their entirety (the market is "cleared"). The budget here is the player's share of the total market value of the disputed goods. Typically, if all the agents have the same importance in terms of share of entitlement, this budget is simply the total market value of the goods, divided by the number of players. This Competitive Equilibrium allocation coincides with the solution proposed by John Nash (Nash 1950) for bargaining problems. More in details:

- We refer to A. Bogomolnaia et al. (2017 and 2019) for a thorough discussion.
- Suppose each agent is given the same budget (something similar to what happens in the present procedure, but the budget covers only the share to which the agent is entitled. For instance, if the whole asset is worth 600000 euros and there are 3 agents all equally entitled, each will have 200000 to spend). Each good is given a price and each agent spends the budget so to maximize his/her own satisfaction (in the economic jargon we speak of utility) buying goods at the fixed price. If goods are bought such that a) each player, independently of the others, makes the best choice: given the budget, he/she buys goods that maximize his/her own satisfaction and b) all goods are sold with no overlaps (for instance two agents buying the same good in its entirety) and no leftovers (no good remains unsold), the Competitive Equilibrium from Equal Income (CEEI) solution is reached. It can be shown that, in the given circumstances, such equilibrium always exists, and it is simple to compute.
- In the above cited references, it is shown that this allocation can be equivalently obtained as the solution of an optimization problem in which the sum of the

logarithms of the players' utility is maximized. This objective function was introduced by Nash (1950) and thus takes the name of Nash solution. This solution is [1]:

- **Pareto optimal or Efficient.** Informally, an allocation is Pareto-efficient if it cannot be improved to another allocation which is at least as good for every agent and strictly better for at least one agent. […] The terms Pareto efficiency and Pareto optimality are used synonymously.
- **Envy-Free** Informally, an allocation is envy-free if no agent prefers the share of another agent to her own.

However, the solution is usually not egalitarian, since the outcome may yield different utility levels for the agents involved. This may raise some questions if the evaluations concern money. We illustrate this fact with the same toy example that was used before

Toy example (continued). Considering again three players sharing three divisible items, with the same utility table considered before.

Table 2. – *The example in Table 1 shows that the CEEI/Nash solution is envy-free but not egalitarian*

Player/Item	**a**	**b**	**c**
I	40	30	30
II	30	40	30
III	10	50	40

The CEEI/Nash solution prescribes that

- Player I gets item a in its entirety.
- Player II gets 9/10 of item b.
- Player III gets item c and 1/10 of item b.

Player III will not be envious anymore, because in his evaluation item c in its entirety plus 1/10 of item b is no less (actually it equals) 9/10 of item b. They both have a utility of 45.

In terms of utility, we note that

- The utility of player I is 40.
- The utility of player II is 36.
- The utility of Player III is 45.

And therefore, the allocation is not equitable.

[1] The following two informal definitions are taken from Bouveret S, Chevaleyre Y, Maudet N (2016).

1.2.6. Adjusting the solution

The optimization problem returns a solution. Players receive the unsplit goods. For the goods that, according to the solution, must be split, negotiations should take place. A negotiation for each good should begin and should involve only the players entitled to some fraction of the good. Players may decide:

- To manage and enjoy the good together. This could occur, for instance, with a house or a piece of land.
- To sell the good and divide the proceeds according to the shares specified in the solution.
- To bargain so that one part receives the good in its entirety, and the others receive side payments. If possible, the party that gets the good should not recur to his/her own money, but the money already forming the estate should be used instead. In this way, instead of the classical solution where every player receives 1 n-th of the cash forming the estate, each player will receive a different amount of money, based on the outcome of the all the bargaining in which that player is involved.

1.3. Manipulability of the procedures

In Dall'Aglio and Fragnelli (2020), a comparison of the robustness against manipulation from one of the agents has been carried. Even in a simplified setting that consider two agents competing over two divisible goods, with one of the agents completely informed over the real preferences of the other agent, it turns out that none of the procedures is entirely immune from strategic fiddling of the informed agent's bid in order to get a larger share at the expenses of the other participant. The two procedures, however, behave differently with the CEEI/Nash solution outperforming the egalitarian one. In fact, it was proved that:

- The CEEI/Nash solution does not change at all for small alterations of the informed agent's bid. A change occurs only for larger deviations. In particular, a gain occurs only when the informed agent declares with his bids that his most preferred item is actually the least preferred.
- The gain that the informed agent can get from manipulating his preferences is always at least as great with the Egalitarian solution than with the CEEI/Nash one. This happens, in particular, at the maximum gain level that the informed agent can achieve.

The notion that the Egalitarian solution is more prone to manipulability is confirmed by the recent works by Bogomolnaia and Moulin (2016), where it is proved that when goods are divided among two or more agents, the CEEI/Nash solution satisfies the following properties, which are not verified by the Egalitarian solution

- CEEI/Nash is **Resource Monotonic**: More goods to divide should not be bad news for anyone;
- CEEI/Nash has **Responsive Shares**: If a player raises the bid on a certain good, he/she cannot end up with a smaller share of that good:
- CEEI/Nash is **Independent from Lost Bids**: If a player changes a bid on an unassigned good, this will have no consequences on the resulting allocation which will remain as it was before the change.

This adds to other problems of the Egalitarian solution. We refer, specifically, to the possible presence of envy and the possible negative effect of domination.

The above reasons suggest that the CEEI/Nash solution is better suited to prevent manipulation. However important, this factor is not the only one that we have to take into consideration to pick a solution criterion. We are going to see that, notwithstanding the higher robustness against manipulation shown by the CEEI/Nash solution, the Egalitarian solution may be chosen instead when goods' market values are introduced, and exact or approximate market value equality of the received goods has to be taken into consideration.

1.4. The CEEI/Nash procedure revisited

Sometimes the parties cannot agree on the monetary value of the item. Quoting Abrahams B, Bellucci E, Zeleznikow J (2012):

[…] mediators would reference standard objective tables and the like to reach a consensus. For example, if parties are arguing over the value of a car, then mediators may access websites that gave independent valuations.

Asset values could possibly be decided by an independent third party or mediator. However, it is quite conceivable that a major issue in dispute may involve determining the value of the item. For example, following a divorce, the husband may agree that the wife should be awarded the marital home. In this case it would be in his interests to overvalue the house (say he suggests it is worth $1,200,000) whilst it is in the wife's interest to undervalue it (say she suggests it is worth $800,000). One possible solution is to add an automated blind bidding feature where the program selects a dollar amount from several confidential offers or blind bids based on an agreed-upon settlement range. Cybersettle is one application that uses this strategy to resolve disagreements over monetary values.

In what follows, we describe a method for indicating a distribution of goods in case the market value of the goods has not been determined. This may happen for several reasons, the main ones being:

- The market value is itself matter of dispute among agents and there is no agreement over a value for some (or all) the contested goods.

- Evaluation from an external party (a valuator) has not been obtained. For instance, because it is considered too costly by the agents involved.

1.4.1. Utility Expressed as bids

Our experience suggests that people are more familiar with prices than they are with abstract numbers that arise the basis for point allocation methods. Even when a consensus has not been reached on the value of an item (that we define as the good's market value), the willingness to pay for an object reflects the satisfaction degree that an agent gets from receiving that item. We thus propose a special point allocation method in which:

1. Parties assign prices to items instead of absolute numbers.
2. Each party will assign a personal price to each item. These subjective prices usually differ (but not too much) from the market prices to reflect the personal preferences of the party.

To exemplify, consider two parties that must divide among themselves, house worth 100000 euros, a motorbike worth 10000 euros and other items. The allocation of points by all parties should reflect the fact that the first item is worth ten times more than the second one. Of course, the evaluations are personal and, therefore the ratio between the two evaluations does not need to be exactly equal to 10 but must be reasonably close to this value. Instead of assigning absolute points, we can attach prices, Thus it will be reasonable that the first party assigns 90000 (euros) to the house and 12000 (euros) to the motorbike, meaning that she is eager to receive the bike, less so to get the house, while the second party assigns 110000 to the house and 8000 to the motorbike, implying opposite preferences. Certainly, it is unreasonable to assign, say 55000 to both items.

1.4.2. Evaluation of preferences

We consider a simple method for making the parties involve express their preferences:

- A **budget** available to each player is computed. This could be obtained by summing up the estimated values of the items to be allocated. It is not important to have an exact and objective price for each item, because prices that are functional to the division will be automatically returned by the procedure.
- An equal budget for each player reflects the principle that all players should be treated equally. Only the share of entitlement could discriminate among players.
- Each player is asked to distribute the budget as **bids** over the unassigned goods.
- The idea is that the higher the bid, the more likely is for the agent to receive the good.

- The bids should reflect the utility of the good for that player – remembering that the good can be sold to the market, if no one bids higher than the lower bidding bound.
- It is reasonable to assume that no bid can be lower than a threshold, which we will call the **lower bidding bound**. The rationale behind this restriction is that in case of very low bids, a good can always be sold to the market. The lower bidding bound should be a low enough price to guarantee the selling of the good (or at least provide an extremely high probability for its selling). Below this price, an offer cannot be considered acceptable. Of course, the lower bidding value varies from item to item, but a reasonable general rule is to decrease the market price is decreased by a fixed percentage, say 20%. In a similar fashion, also **upper bidding bounds** may be considered.

Each player will participate to the allocation procedure with his/her own bids over the goods. A formula for the utility of an agent is then given by the following simple relationship

Utility of the bundle for an agent = Sum of the utility of the single goods for that agent

Utility of a single good for an agent I =
(Share of the good received by that agent) * (bid of that agent on the good)

For instance, if, according to the proposed solution, I should receive 1/3 of a house which I valued 90000 euros, my utility will be 90000 *(1/3) = 30000 euros.

1.4.3. The Procedure as Auction

The bids procedure defines each agent's utility over each good, and the utility of a bundle of goods as the sum of the single goods that make it. The next step requires the careful choice of a suitable optimality criterion to guide the automatic definition of the goods' allocation. As shown in the literature review, two optimality criteria stand out, the Egalitarian vs. the Competitive/Nash solution. We now weigh in the pros and cons of each, ending with a decision over our criterion of choice.

As mentioned in Dall'Aglio, Di Cagno and Fragnelli (2020), the Egalitarian solution returns an efficient solution that delivers the same utility for every agent. In this context, utility coincides with budget and, therefore, the procedure can be regarded as a first-price auction mechanism: Agents use their (equal) budget to bid on goods, and the item is assigned to the agent who bids highest. Ties can be dealt with according to some rule defined in advance that guaranties symmetry among the agents. On the other hand, the CEEI/Nash picks an allocation that maximizes the product of the agents' utilities. This solution guarantees envy-freeness and efficiency for the chosen allocation.

The Egalitarian solution stands out for its simplicity and ease of interpretation. A deeper look at the criteria, however, makes the choice much less obvious: As it will be made clear in the next section, the CEEI/Nash solution not only is efficient and envy-free, but it can be also explained in terms of auctions, with the prices paid by the auction winners defined by equilibrium prices which will assure every agent about the rationality of the received bundle of goods.

1.4.4. The Emergence of Equilibrium Prices

Envy-freeness and Efficiency are not the only properties guaranteed by the CEEI/Nash criterion. Among the other features of this solution, one is particularly suited in the context we are examining: The solution coincides with a market equilibrium allocation where agents spend a budget, equal for all of them, on the goods priced in such a way that no good is over- or under-demanded.

The striking coincidence dates to the works of Gale (1960) and Eisenberg (1961) This result has been recently elaborated on and extended to the cases of allocation of liabilities/chores, denoted as bads, or to the mixed allocation of goods and bads, by Bogomolnaia et al. (2017) and (2019). In the definition of the competitive equilibrium, it is assumed that all budgets are set to one, with prices defined accordingly. An explicit formulation for the equilibrium prices are given in the same reference. When utilities equate the bids, those prices can be suitable scaled to work with individual budgets amounting to the total budget that each agent is asked to distribute among the bids.

The setting of these prices gives an alternative explanation for the Nash solution as a market situation with clearing prices. In fact, once the prices have been posted, we may figure all agents acting as follows:

a) Each agent has one n-th of the total budget
b) Each agent compares his bids with the prices and
   i. Rules out all the goods with price higher than the bid
   ii. Among the goods with prices lower than the bid, he starts buying, starting from the good with highest rebate (discount)
   iii. Until the budget ends.

The results in Bogomolnaia et al. (2017) and (2019) allows us to conclude that, when prices are defined as prescribed by the main Theorem in that reference, all goods will be sold with no demand left unsatisfied, i.e. the market clears, and the allocation coincides with the Nash solution. It can be shown that, in the above procedure, no agent will ever buy goods at prices higher than the bids. Moreover, prices are consistent with the initial budget, that is, the sum of the prices equals the common agents' budget.

This procedure is particularly suitable in all the situations where a market value has not been agreed upon by the agents participating in the division. The emer-

gence of equilibrium prices provides a convincing answer to the issue of assigning a price tag to the goods to be divided. Choosing prices is, in principle, a dividing issue, because agents may have an interest in raising or lowering those prices in order to get a larger share of the asset. The mechanism defined here will bring the agent to an agreement, since each of them is shown that the assigned bundle coincides with the rational choice that would have been made by the same agent having a fixed budget to choose goods to but in the very same asset.

The results also show that the CEEI/Nash solution can be devised as an auction where the agent who is going to receive the good will pay a price which is always lower or equal to the announced bid. In this context the bid can be interpreted as the maximum price that the agent is willing to pay, given the assigned budget.

Returning to the comparison of the Egalitarian and the CEEI/Nash criteria we notice that the latter has many more ammunition on its side than previously stated. Furthermore, the results of the analysis on the manipulability of the two solutions, available in Dall'Aglio and Fragnelli (2020), shows a larger resilience of the CEEI/Nash solution against perturbations of the announced bids with respect to the true value of a good. The amount of evidence makes us lean towards using the CEEI/Nash solution to completely define the procedure.

1.4.5. The procedure's core

In the light of all the previous remarks, the core of the procedure simply becomes:

Core of Procedure 1 "Name your price":

- Each agent is asked to indicate the fair value of each good in the asset. The bids should stay within a specified range between a lower and an upper bound determined in advance.
- The CEEI/Nash solution, i.e. the solution that maximizes the weighted product of the utilities is sought.
- Equilibrium prices are computed, and a suitable diagnostic system explains to the agents the optimality of the allocation in relation to the announced bids. In particular:
  - the received bundle has higher value, in terms of the agent's bids, than all the bundles given to the other agents (the allocation is envy-free)
  - The equilibrium prices are revealed to each agent. It is shown that given the bids, the equilibrium prices and a fixed budget (corresponding to one n-th of the common total budget), the agent best behaviour coincides with the allocation prescribed by the CEEI/Nash solution.

1.5. A procedure for the division of goods with market value

In what follows, we propose a method that relies on the availability of market values for the goods to be divided. As previously suggested, this can be obtained by means of one or more impartial experts that are able to provide objective and reliable estimates of the goods that form the asset to be divided, or by a mediator, especially for items whose market value can be easily traced on websites or magazines.

The division has to take into consideration the following data set: for each good a market value is available, together with the evaluation of the good from each agent. Additional data such as the divisibility of the good, or priorities or feasibility of assignments may be present.

The only previous effort to combine preferences and market values is the ASSET_DIVIDER procedure by Bellucci (2008). In that context, however, the approach is very different, since a good's utility is given by the points allocated by each agent, while the market values are taken into consideration in the allocation phase in order to balance the allocated bundles of goods.

We substantially change the way in which the market value and the evaluations are processed in order to separate the pleasantness/unpleasantness of the goods from their market value. We then define a utility function which combines the two pieces of information. These utility functions will naturally lead to a new objective function that governs the asset's division which guarantees the maximization of a CEEI/Nash solution, constrained to the equality in the bundles' market value. This solution will inherit several properties of the original CEEI/Nash solution. In particular, the solution will be scale invariant with respect to the agents' utilities: If an agent declares that all the goods are extremely important to her, this will not bring any benefit with respect to an agent that declares all goods medium important, or all of them of no importance. What counts is the ability of an agent in distinguishing between the important and the negligible goods, according to her preferences.

A thorough comparison with the ASSET_DIVIDER procedure will be then performed for the case of two agents (the only case available for this procedure).

1.5.1. A utility function encompassing market values and preferences

We assume that each good has been assigned a market value that is agreed upon by all agents. These market values may be provided by the mediator or by experts appointed by her. Agents still maintain different view over the single goods. These are expressed as "distortions" of the market values, i.e., magnifications or reductions by factors which are specified in advance. Our aim is to provide a simple setting for the average user who might not be too familiar with a sophisticated system for the elicitation of preferences. Most users, however, deal with rating systems which are common in many internet services. Rating systems, typically

ranging in a scale of 1-to-5 stars are used to evaluate purchased products in large electronic goods retailers, such as Amazon, or to measure the user's satisfaction for the enjoyment of a restaurant or a hotel and provide advice for other customers which intend to repeat the same experience. This is the key to the success of internet services as TripAdvisor or Booking.com. These services typically assume that the rating system has 5 different levels. Note that other rating systems involving a different number of levels may be used, but an odd number of levels should be employed, so that the unique median rating naturally conveys an idea of neutral rating. If only three stars are used, 1 star will denote dissatisfaction, 2 stars will denote a neutral judgement and 3 stars will denote satisfaction. With the more common 5 stars rating system, 2 or 1 star will denote an increasing level of dissatisfaction, while 4 or 5 stars denote an increasing level of satisfaction, while 3 stars denote a neutral judgement. Using a 7 stars system, the palette of ratings is enriched with 3 different degrees of appreciation or depreciation: mild, medium and high.

It is worth noticing that this rating does not regard the monetary value of the good, but only the pleasantness of receiving a certain good. For instance, a player is involved in the allocation of a house worth 100000 euros and one Harley-Davidson motorbike worth 10000 euros. I know that the house is worth more, but I already own a beautiful house, and I know that managing a house is time and money consuming. On the other hand, I have always dreamt about riding that motorbike. On a 5 stars scale, I will give 2 stars to the house and 5 to the bike, overturning the monetary evaluation.

Also, the scale must be decided in advance and the agents' opinion may matter. If the agents are young and familiar with the internet services, they may opt for a 5 stars scale. Less digitally skilled agents may prefer a 3-star scale in which they only point out the goods they want and those that they wish to avoid.

The procedure to elicit the agents' preferences can be states in three simple steps

1. The scale is decided and agreed upon by the agents: 3-stars, 5-stars or 7-stars.
2. The monetary value of each good is determined by some external experts and agreed upon by the agents.
3. Each agent assigns a rating on each good.

The rating system reflects on the utility of goods according to the following rules:

If an agent assigns the median rating that is:

2-stars on a 3-stars rating system
3 stars on a 5-star rating system
4 stars on a 7-stars rating system

then the utility of a good coincides with its monetary value.

1. If the ratings of two agents on the same good differ by 1 star, then the utility ratio between the agents is the same, independently of the rating assigned by the two agents.
   To give an example, suppose that two agents are evaluating two goods: a house worth 100000 euros and a motorbike worth 10000 euros on a 5-star scale. Suppose that the first agent assigns 3 stars on the house and 4 stars on the motorbike, while the second agent assigns 2 stars on the house and 5 stars on the motorbike. Notice that the difference between the rating of the house by the first agent and the second agent is 1 star. Similarly, the difference in rating the motorbike between the second agent and the first agent is again 1 star. This enables us to write that the ratio between the agents' utility ratio has to remain constant for both goods. Let us denote as K this common value.

A general formula for the utility again relies on the fact that the utility of a bundle of goods for an agent is the sum of the utility of each good received.

Utility of the bundle for an agent = Sum of the utility of the single goods for that agent

This time the utility of a single good for an agent is given by

Utility of a single good for an agent I =
(Share of the good received by that agent)*(effect of rating by that agent)*(good's market value)

The rating will leave the market value unaltered if the rating coincides with the median value, it will increase the market value if the rating is above the median value and it will be proportional to the distance from this median value. Symmetrically, a rating below the median value will decrease the market value, and such decrease will be proportional to the distance from the median value.

More in detail, if a 3-stars scale is used, then the effect of rating will be given by the monetary value if the median rating (i.e. 2 stars) has been assigned by the agent. In case the rating is increased to 3 stars (decreased to 1 star, respectively), the effect of the rating is to appreciate (depreciate, resp.) the monetary value for that agent by a factor K_3. It is reasonable to assume $K_3 = 1.2$, so that the utility value can increase or decrease by 20%.

If a 5-star scale is used, then, starting from the median rate of 3 stars, which equates the utility of the agent to the good's monetary value, the effect of the rating may increase this value once, or twice, resp., by a factor K_5 depending on whether the rating one, or two stars resp., above the median rating. Symmetrically, the rating would decrease once or twice the good's value by the same factor, depending on a whether a low (2 stars) or very low (1 star) has been assigned by the agent to the good. Therefore, it is reasonable to set $K_5 = 1.1$.

Finally, if a 7-stars rating system is used, a procedure similar to that of the 5-

stars rating is used, except that in this case, starting from the median rating of 4 stars, the value may be increased or decreased once, twice or three times by a factor which we may assume to be $K_7 = 1.065$, in line with the procedure with smaller ranges.

Using this method, we can elicit the utility for every agent involved in the division. A fair division procedure can be started, just as it was done in the previous case.

1.5.2. Invariance by translation

Both optimality criteria that we are reviewing share an important property—that of scale invariance. This a very strong and appealing feature which can be stated in very simple words as follows "If an agent raises or decreases (i.e. scales) his ratings on all goods by a fixed constant, the outcome of the division will not change for him as well as for the other agents". This property avoids the behaviour of agents that may claim a higher than justified utility over the goods, in order to obtain a larger share. The benefits, however, are often hard to be appreciated essentially for two reasons: The procedure may bar the agents to scale the utilities properly. Think for instance about those point allocation methods in which an integer value must be attributed to every good. Raising the utility of all the goods by a fixed percentage may turn out to be impossible due to the integer value constraints. More importantly, however, is the fact that an exact scaling of the utility may require computational capabilities which an agent may fail to own. In our context, it turns out the scale invariance becomes another property which is easier to understand and implement, namely.

Invariance by translation: Adding or removing one star to the ratings of all of the goods will not change the outcome for that player as well as the others.

Put in even simpler terms, an agent will not be better off simply by adding more stars to her rating. To exemplify, the following rating profiles for a standard list of goods.

Table 3. – *An example of equivalent ratings*

Items	Profile A	Profile B	Profile C
Town House	**	***	****
Country House	***	****	*****
Car	*	**	***
Motorbike	***	****	*****
Garage	**	***	****

will yield the same outcome for an agent. Supposing that an agent's "true" profile is B. It would be natural for that agent to fake profile C with the hope of receiving

a larger share of the contended asset. Similarly, he will not switch to profile A, with the intent of claiming a mild interest for the division and receive some side compensation. A more extreme example derives from the following set of profiles.

Table 4. – *Another example of equivalent ratings*

Items	Profile D	Profile E	Profile F
Town House	*	***	*****
Country House	*	***	*****
Car	*	***	*****
Motorbike	*	***	*****
Garage	*	***	*****

Declaring option F will not bring more items, but it will only denote a complete indifference among them (as the other two profiles). Most likely, the agent will end up receiving goods that he simply does not want.

1.5.3. Does perfect equality in market values really matter?

When price tags are assigned to items in an asset that has to be divided among a certain number of agents, it seems natural to require a solution providing bundles of perfect equal values for the agents, if the agents have the same importance, or bundles that perfectly reflect the relative share of each agent. This goal is usually obtained at the cost of splitting one or more items among two or more agents. A division whose only goal is to provide bundles with market value proportional to the share of importance, may reveal totally unsatisfactory, so we turn to divisions which seek to maximize the agents' satisfactions, i.e. their utility, in a fair (and equal) way. Alternatively, a division may ignore any comparison among the agents' satisfaction and aim at a satisfactory division for every agent under his/her own perspective. With such divisions, the bundles' proportionality with respect to market values is typically lost, under the notion that bundles of goods having the same market values may bring different contentment among agents.

A natural way to guarantee proportionality of market value shares would be to impose such property by means of an additional constraint to the problem. The goal then becomes one of searching a division with the same objective function as the Egalitarian or the CEEI/Nash problem among the allocations that guarantee proportional market value shares.

While sound in its formulation, this approach presents two serious problems: first of all, the constrained problem may lose some optimality properties of the unconstrained problem. For instance, while with the unconstrained CEEI/Nash

solution we obtain an allocation which is simultaneously optimal according to the Nash criterion, and which delivers a competitive equilibrium, when we add a constraint, we may typically lose one of the two interpretations. Similarly, when we search for a constrained Egalitarian allocation, we may end up with an allocation which is not perfectly Egalitarian in the (normalized) utilities, but it is its closest proxy. There is, however, a more important problem that arises when we impose fairness in both the utilities and the market values: The simultaneous fulfilment of both criteria may yield a totally unacceptable solution because it delivers too many split items. The next example clarifies the matter.

A (Counter) Example

Consider two agents: A and B who are dividing 4 Marylin Monroe prints by Andy Warhol between themselves. The market value of the portraits is set to 100 dollars each, but each print has different background colours which modify the liking of the two agents. Here, a 5-stars rating scale is used with constant multiplying factor is set to $K_5 = 1.1$.

Table 5. – *Ratings and utilities for the Warhol-Monroe example*

Background colour	Market value	Rating A	Utility A	Rating B	Utility B
Green	100	*****	121	*	82.6
Blue	100	*****	121	*****	121
Pink	100	*	82.6	*	82.6
Grey	100	*	82.6	*****	121

Both the Egalitarian and the CEEI/NASH allocations yield a utility level of 222.8 for both agents. Such optimal level and an equal market value for the bundles received by the two agents is achieved by the following allocation:

Table 6. – *The equal-value constrained solution for the Warhol-Monroe example*

	Background colour	A	B
1	Green		
2	Blue	50%	50%
50%3	Pink	50%	50%
4	Grey		

This solution has two paintings equally split between the two agents. It can be shown that no arrangement with at most one split painting achieves the same level of utility for both agents, while maintaining an equal market value for both agents. The optimal allocation delivers two equally split items. This contradicts a well know fact in fair division theory that says that when two agents attend a division, any efficient (Pareto optimal) allocation can be obtained by splitting at most one item. In other words: there exist allocations that yare optimal in the Egalitarian or the CEEI/Nash sense and that return the same level of utility for both agents by splitting at most one item.

The example shows an important fact about divisions in the present setting. We may fail to have a division which is simultaneously:

1. Fair in a recognized criterion (Egalitarian or CEEI/Nash).
2. Proportional in the market shares.
3. Provides a minimal number of split items.

If we aim at an allocation which controls the first two factors in the above list, we may end up with an allocation which contains a higher number of split items than required by the attainment of optimal utility levels. The problem becomes more serious as the number of agents grows. A more analytical formulation of this issue can be found in Dall'Aglio (2019).

1.5.4. Our Proposal

The counterexample of the previous section shows that we cannot always obtain an allocation which guarantees simultaneously fairness, proportionality in market values, while keeping the number of split items to a minimum. A choice must be made about which feature to drop out.

We believe that minimizing the number of split items is a very important one, because any time that the division prescribes an item to be attributed to more than one agent, a plan must be sought to manage the item. The choice therefore becomes a dilemma: Should we privilege fairness, or should we keep perfect proportionality in the bundles' market values? It would be hard to justify an unfair solution, namely a division plan that treats some agent better than others, thus breaking an impartiality principle. We therefore solved this problem by privileging fairness while keeping differences in market values to a minimum and justifiable in terms of differences in the agents' satisfaction.

An empirical comparison of the fairness criteria revealed that, while we were not able to justify the differences in market values received by the agents according to the CEEI/Nash solution, we observed that the Egalitarian principle generates differences in market values shares that can be explained by differences in the average satisfaction arising from the goods. This intuition has been verified in a more rigorous setting. For simplicity, we compare two agents with the same

share of entitlement (for instance two divorcee, or two brothers in an inheritance). The market values of the bundles received by the two agents is usually different, but the difference can be explained quantitatively. If an agent received a bundle with higher market value, his average satisfaction measured in normalized number of stars per fraction of good corresponding to one unit of market value will be lower. For a more analytical treatment of this issue we refer to Dall'Aglio (2019).

The (Counter-)Example Revisited

An Egalitarian solution for the simple example just presented is the following:

Table 7. – *The Egalitarian solution for the Warhol-Monroe example*

	Background color	A	B
1	Green		
2	Blue	84%	16%
3	Pink		
4	Grey		

A symmetry argument shows that another solution is obtained if the Pink painting is assigned to agent A and the Blue painting is split so that a share of 16% (84%, resp.) goes to Agent A (Agent B, resp.).

If we compare market shares with average standardized ratings, we notice that the smaller market share received by Agent A is compensated by a larger average normalized rating for the same agent.

Table 8. – *Evaluating the division quality for the Counterexample*

Agent	A	B
Market value	184.15	215.85
Ave. rating	4.81	3.14

Empirical evidence shows that the difference in the bundles' market values is small. In any case, those small discrepancies can be explained by the fact that smaller (higher, resp.) share in terms of the latter principle may be justified by the higher (lower, resp.) contentment that that share may bring.

1.5.5. The procedure's core

The considerations in the previous sections lead us to enucleate the following procedure core:

Core of Procedure 2: "Price and rate"

1. The utilities are elicited in terms of market values and ratings by the agents, namely.
   a. The scale is decided and agreed upon by the agents: 3-stars, 5-stars or 7-stars.
   b. The monetary value of each good is determined by some external experts and agreed upon by the agents.
2. Each agent assigns a rating on each good.
3. The allocation (division) that maximizes the lowest normalized utility is chosen.

We remark that, differently from Procedure 1, there is no issue regarding the disclosure of information pertaining the agents' preferences after the division. Agents should announce their ratings independently of the others. Therefore, no one should access the ratings of others. But if an agent gets to know another agent's rating list after the solution announcement, this should not bring any discontent. On the contrary, a perfect knowledge of one's own and others' preferences will help explain the small differences in terms of market values that the division may generate.

1.5.6. A comparison with an existing procedure: ASSET_DIVIDER

To our knowledge, the only available method in fair division applied to a judicial setting that deals with both the preferences of the players and the goods' market values is given by the ASSET_DIVIDER procedure by Emilia Bellucci (2008). In the author's words (see Abrahams B, Bellucci E, Zeleznikow J, 2012)

> ASSET_DIVIDER […] uses a modified version of the Adjusted Winner algorithm developed by Brams and Taylor (1996), to divide *n* divisible goods between two parties as fairly as possible. AdjustedWinner starts with the designation of the items in a dispute. If either party says an item is in the dispute, then it is added to the dispute.
>
> Two sets of ratings are provided, one for each party in dispute. This rating (initially a numerical value between 0 and 100) does not represent the monetary value of the item, instead it symbolises how important the item is to the party. Whilst disputants can probably linearly order the significance to them of all items in dispute, it is unrealistic to expect them to give a numerical value to each item. But it is not unreasonable for the users to assign a linguistic variable to each item. A seven-point scale is used which can then be converted into points, according to the following table:

Table 9. – *Significance Rating Scale (Table 6 in Abrahams, Bellucci and Zeleznikow, 2012)*

Description	Points
Irrelevant	0
Little significance	10
Marginal	20
Moderate	30
Important	40
Very important	50
Essential	60

ASSET_DIVIDER also accepts the current monetary value of each item in dispute. We assume this dollar value has been negotiated (if necessary) before ASSET_DIVIDER is used. Hence, only one-dollar value is entered per item.
The proposed percentage split is also entered. This figure has to be derived from the mediator's knowledge in past cases or from computer systems such as SplitUp (Stranieri et al. 1999), which can provide a percentage split given certain characteristics and features of divorce cases.

A brief description of how this mechanism works follows

ASSET_DIVIDER's allocation strategy works by provisionally allocating an item to the party whose rating is the highest. It then checks the dollar value of items it has been allocated previously (that is, their current list of items), the dollar value of the item presently allocated, and the dollar amount permitted under the percentage split given by mediators. If by allocating the item in question the party exceeds its permitted amount, the item is removed from its allocation list and placed back into negotiation. [2]
In this case, the item has not been allocated to a party. If the dollar value of the item was within the limits of the amount permitted under the percentage split rule, then the allocation proceeds. Once an allocation has occurred the "losing party" is compensated by the trade-off equations modifying ratings (whereas in FAMILY_WINNER both winning and losing parties were affected).

Analogies can be traced between the ASSET_DIVIDER and our "Price and

[2] In a later version of the procedure, denoted Asset Divider agents modifies this rule as follows "When the allocation of an item breaches the agreed dollar value percentage split, rather than place the item back into the negotiation and continuing, if divisible, the item is now divided between the two parties in accordance with the required percentage split. If the item is not divisible, the cash variable is used to maintain the required split.

rate" procedure. Clearly, point allocation methods provide an environment richer in terms of information, but difficult to elicit in the presence of agents not too confident with numerical methods. Conversely, using the rating scales several assumptions on the agents' utilities have to be formulated. When a rating scale like the one described in table 7, the points of the asset divider procedure could be translated in ratings along the 7-stars scale of the price and rate procedure.

Here, we list the most relevant differences between the procedures. The comparison is not easy, due to the non-exhaustive descriptions of the ASSET_DIVIDER procedure provided in the published papers[3].

- While the ASSET_DIVIDER procedure deals exclusively with indivisible items, the price and rate procedure consider divisible items, but it can be adapted to consider one or more indivisible items.
- The price-and-rate procedure works for any number of agents, while the asset divider procedure works only for two agents.
- For the case of two agents, where we provide a more detailed description of its functioning, we highlight that:
  - The two methods have different criteria for allocating the goods. The asset divider allocates the goods to the agent that ranks them higher, and each allocation, once it fits the monetary constraints, is final. In the price and rate method it is the difference in the ratings that determines the initial allocation. Moreover, just like the Adjusted Winner method, allocations are temporary to assure equality in the market values.
  - The two methods provide different dynamics in the allocation of goods. The asset divider procedure allocates one good at-a-time, while the ratings for the goods still to allocate change according to ad-hoc rules. In the price and rate procedure, sets of goods move from one player to the other in a specified order (determined by the differences in the ratings between the groups) in order to seek equality in the market values – in a fashion similar to that of the Adjusted Winner procedure. The two procedures "Price and rate" and "Adjusted Winner", though, are inherently different, since in the former, items are arranged by increasing order of utility ratios and equality of utilities between the two agents is sought. In the latter, items are arranged according to the difference in ratings and equality in the market values is sought.
  - The "price and rate" procedure is inherently scale invariant, while preferences in the asset divider procedure have to be properly scaled before inputting them in the algorithm.

Above all, we wish to emphasize the different approach to the task that we are considering with this project. As Bellucci (2008) suggests,

[3] We could not find a step-by-step description of the algorithm in none of the cited references.

> Apart from AdjustWinner, most of the systems surveyed above do not make allowances for measuring the fairness or justness of the settlement. Further, most of the systems discussed are rarely based on theories derived from practice or empirical studies. […] Our goal is to provide feasible suggested solutions to the conflict that are acceptable to the user, which for our purposes does not involve searching for optimal solutions as in Pareto optimisation. We have found such techniques are difficult to use in our domain. The best we can arrive at computationally fair solutions is to ensure are solutions are acceptable (i.e. approximately optimal or fair solutions).

1.6. A detailed description of the proposed procedures

We now return on the algorithms, whose core was described in the previous sections, and we enrich them with the experience gained from the interaction with the Legal Workgroup of the current project. In Dall'Aglio (2020) many cases brought to us by units of legal experts in the European countries involved in the CREA project. From the analysis of 36 practical cases in the fields of Family Law (succession and divorce) and Company Law (liquidation), it turned out that a general-purpose procedure should encompass the following features:

a. It should avoid random outcomes;
b. It should be able to deal with agents having different shares of entitlement;
c. It should be able to consider allocations where items and/or money are preventively assigned to one of the agents;
d. It should take into account that certain items may have to remain indivisible, and, therefore, these have to be assigned to one of the agents in their entirety;
e. It should consider the need imposed by the Law or by the circumstances to certain agents for liquid assets
f. It should encompass liabilities, as well as assets.

First, we give some general principles for the implementation of these procedures. All the information should be provided to a web portal by means of registered access (userid and password). A simple information set to instruct the system should be provided. This is typically the task of a mediator that may help the agents. Alternatively, if the agents are on good terms, they could insert these pieces of information together. It must be noticed that none of the inserted information is sensitive and needs to be hidden to the agent.

In some situations, it may further restrictions may be imposed on the division. Here we list some instances

- **Indivisible goods**. The law or the court may require that an item is assigned in its entirety to one of the agents.

- **Assignment restrictions**. Not all the conceivable assignments may be acceptable as solutions because they may infringe some requirement of the law or some ruling of the court. The procedures must rule out such inadmissible results. We consider two notable classes of restricted admissibility:
  - **Simple assignment restrictions.** One or more goods may be assigned to a specific agent or one agent in a restricted group. For instance, if a child custody is given to one of the two parents involved in a divorce, some of the goods (for instance the family house) may be assigned to that parent.
  - **Joint assignment restrictions.** Two or more goods may not be assigned to the same agent. More in general, a combination of assignments among agents may be inadmissible. An example of this can be found in Bellucci and Zeleznikow (2005).

The issues of primary residency and visitation rights to children are mutually exclusive, since if one parent has residency, then the other, save for exceptional circumstances, is allocated visitation rights.

We notice that these are additional restrictions imposed on the solution. The resulting outcome may fail to satisfy one or more properties of the CEEI/Nash (Name your price) or the Egalitarian (Price and rate) solutions.

1.6.1. The procedures

In the following we describe in detail the procedure in which the utility of each item is given by the subjective bids of the agents and the CEEI/Nash solution is sought.

Procedure 1: Name your price.

a. PRELIMINARY PHASE. The mediator (or the agents, jointly) insert the following information:
   i. The number and names of the agents
   ii. The share of entitlement for each agent
   iii. The number and names of the goods
   iv. whether money is available in the division and whether it should be considered
      a) As an independent divisible item to be assigned to one or more agents
      b) As a separate resource that could be given to the agents in parts proportional to the shares of entitlement, and could help solve questions of joint ownership for the resulting division
   v. The mediator should determine the value of the whole asset. This will be the budget that the agents will spend on the bids.
   vi. For each good, a range of admissible bids should be specified. If the mediator has some idea of the good's market value, the interval should be built around this market value, the lower bound, resp. upper bound, should be

determined by subtracting, resp. adding, a fixed percentage, say 25%, of the estimated market value.

vii. whether there exist any constraints on the item, whether
   c) an item must be considered indivisible
   d) simple assignment restrictions must be enforced
   e) joint assignment restrictions must be enforced

b. BIDDING PHASE. Each agent is asked to make a bid on every item. If a range bracket is specified, the bids should stay within a specified range between a lower and an upper bound determined in advance. Moreover, the total amount of these bids must not exceed the budget determined by the mediator.

c. THE SOLUTION. The CEEI/Nash solution, i.e., a solution that maximizes the weighted (by the shares) product of the utilities is sought.
   i. In case no restriction has been imposed in step a-vii, the system should present the optimal solution, or one among the optimal solutions, and should explain the optimality properties of the solution:
      a) The solution is **proportional**, i.e. each agent receives a bundle with normalized utility higher than the agent due share.
      b) The solution is **efficient** (or Pareto-optimal), namely no allocation that globally improves the welfare of every agent, with at least one agent strictly better off, is possible
      c) The solution is **envy-free**, namely if any agent compares the goods received with those given to the other agents, he/she will verify that, according to the agents' own bids. he/she has received the larger share.
   ii. For each good, the system will compute an **Equilibrium price**. The system should then explain to each agent that comparing one's own bids and the equilibrium prices, the received goods (or parts thereof) coincide with the optimal purchase of an agent having a budget proportional to the relevance of the agent's entitlement. If all the agents have the same importance this fraction will equal to 1 n-th of the goods' total value, determined in step a-v.
   iii. In case restrictions have been imposed in step a-vii, the system is not able to compute the equilibrium prices and it should verify whether the computed optimal solution, satisfies any of the properties listed in step i.
      A. In the affirmative case, the system should explain the optimality properties shared by the solution.
      B. In the negative case, the system should quantify by how much a given property fails to be verify.
   iv. It should suggest ways of handling split items. One of the joint owners could outbuy the other participants with own money or, preferably, with money available in the asset.

d. REQUEST OF AGREEMENT. The system should then ask the agents if they are satisfied with the proposed solution. If any of them is dissatisfied, the solution could propose other optimal or suboptimal solutions, and repeat this step.

In step b., it is important that all the agents submit their bids independently, each one unaware of everybody else's bid. To this aim, it is necessary that agents access the web portal in separate sessions.

We move to a detailed explanation of the second procedure.

Procedure 2: Price and rate.

a. PRELIMINARY PHASE. The mediator (or the agents, jointly) insert the following information:
   i. The number and names of the agents
   ii. The share of entitlement for each agent
   iii. The number and names of the goods
   iv. The market value of each good.
   v. The rating scale for evaluating each item: 3-stars, 5-stars or 7-stars.
   vi. whether money is available in the division and whether it should be considered:
      A. As an independent divisible item to be assigned to one or more agents
      B. As a separate resource that could be given to the agents in parts proportional to the shares of entitlement, and could help solve questions of joint ownership for the resulting division
   vii. whether there exist any constraints on the item, in particular whether
      A. an item must be considered indivisible
      B. simple assignment restrictions must be enforced
      C. joint assignment restrictions must be enforced
b. RATING PHASE. Each agent is asked to indicate the degree of pleasantness for each good according to the rating scale fixed in advance.
c. THE SOLUTION. The Egalitarian solution, i.e., a solution that maximizes the normalized utility of the worst-off agent (weighted by the its entitlement) is computed.
   i. In case no restriction in step a-vii has been imposed, the system should explain the optimality properties of the solution:
      The solution is **proportional**, i.e. each agent receives a bundle with normalized utility higher than the agent due share.
      b) The solution is **efficient** (or Pareto-optimal), namely no allocation that globally improves the welfare of every agent, with at least one agent strictly better off, is possible
      c) The solution is **egalitarian**, namely all the agents should will receive the same amount of normalized utility
   ii. The system should compute the market value of the goods received by all agents and should explain possible differences in the market value of the received bundles by showing the differences in the average number of stars per fraction of good worth one unit of market value.
   iii. In case restrictions have been imposed in step a-vii, the system is not able

to compute the equilibrium prices and it should verify whether the computed optimal solution, satisfies any of the properties listed in step i.

C. In the affirmative case, the system should explain the optimality properties shared by the solution.

D. In the negative case, the system should quantify by how much a given property fails to be verify.

iv. The system should suggest ways of handling split items. One of the joint owners could outbuy the other participants with own money or, preferably, with money available in the asset.

d. REQUEST OF AGREEMENT. The system should then ask the agents if they are satisfied with the proposed solution. If any of them is dissatisfied, the solution could propose other optimal or suboptimal solutions, and repeat this step.

Just as in the previous solution, it is important that all the agents submit their bids independently, each one unaware of everybody else's bid.

1.7. Mathematical Formulation

In what follows we provide a description of the two procedures in mathematical terms.

1.7.1. The formulation for Procedure 1: Name your price

Input Phase 1 – The Mediator

The mediator should input

a. The set of agents $N = \{1,2,\cdots,n\}$

b. The entitlement (weight) of each agent $w_i > 0$, with $\sum_{i\in N} w_i = 1$

c. The set of items $M = \{1,2,\cdots,m\}$

d. A total budget B

e. A minimum bid $\underline{b}_j$ for each good $j \in M$

Input Phase 2 – The Agents

Each agent $i \in N$ should input

f. A bid b_{ij} for each good $j \in M$, making sure that

- Each bid does not fall below the minimum bid: $b_{ij} \geq \underline{b}_j$ for every $j \in M$
- The bids' sum does not exceed the budget: $\sum_{j\in M} b_{ij} \leq B$

Each agent should input the data privately. No agent should reveal the personal bids to the other agents

The optimization problem

Solve the following variables in the nm variables z_{ij} that indicate the part of good $j \in M$ allocated to agent $i \in N$.

$$\max \sum_{i \in N} w_i \log\left(\sum_{j \in M} z_{ij} b_{ij}\right)$$
$$\text{such that } \sum_{i \in N} z_{ij} = 1 \text{ for every } j \in M$$
$$\text{and } z_{ij} \geq 0 \text{ for every } i \in N \text{ and } j \in M$$

Output

g. The matrix $\{z_{ij}\}$ that indicates the allocation plan of the goods to the agents. This is public information.

h. The equilibrium prices computed as

$$p_j^B = B w_j \max_{i \in N}\left(\frac{b_{ij}}{\sum_{j \in M} z_{ij} b_{ij}}\right)$$

This is public information.

i. Each agent should be privately be informed about

- the envy-freeness of the allocation: the goods according to each agent's evaluation are more valuable than the goods given to the other agents.
- The equilibrium prices and the fact that, if agent $i \in N$ spends a budget $w_i B$ ion the goods with the equilibrium price tag on them, the returned solution shows that best bundle of items that each agent could get.

Remarks

- The system should provide an option (such as a tick box) to assign equal weight to every agent. In such a case $w_i = 1/n$ for every $i \in N$.

- The optimization problem is not linear in the variables, but concave. The first algorithm designed to solve this problem can be found in Devanur et al. (2008).

1.7.2. The formulation for Procedure 2: Price and rate

For simplicity of description, we assume that a 5-star range system is used.

Input Phase 1 – The Mediator

The mediator should input

j. The set of agents $N = \{1,2,\cdots,n\}$
k. The entitlement (weight) of each agent $w_i > 0$, with $\sum_{i\in N} w_i = 1$
l. The set of items $M = \{1,2,\cdots,m\}$
m. The market value for each item m_j, $j \in M$.
n. The revaluation factor K (typically $K = 1.1$)

Input Phase 2 – The Agents

Each agent $i \in N$ should input

o. A rating $r_{ij} \in \{1,2,3,4,5\}$

This information can be made public after all agents have communicated their choices.

Algorithm – Phase 1. Transformation

Rating are transformed into utilities by means of the following formula for every $i \in N$ and $j \in M$.

$$u_{ij} = K^{r_{ij}-3} m_j$$

Algorithm – Phase 2. The optimization problem

Solve the following variables in the nm variables z_{ij} that indicate the part of good $j \in M$ allocated to agent $i \in N$.

$$\max t$$

$$\text{such that } \sum_{j\in M} \left(\frac{u_{ij}}{w_i \underline{U_i}}\right) z_{ij} \geq t \text{ for every } i \in N$$

$$\sum_{i\in N} z_{ij} = 1 \text{ for every } j \in M$$

$$\text{and } z_{ij} \geq 0 \text{ for every } i \in N \text{ and } j \in M$$

$$\text{Here } \underline{U_i} = \sum_{j\in M} u_{ij}.$$

Output

p. The matrix $\{z_{ij}\}$ that indicates the allocation plan of the goods to the agents. This is public information
q. The market price of each bundle
r. The average rate of the items received

Remarks

- The system should provide an option (such as a ticking box) to assign equal weight to every agent. In such a case $w_i = 1/n$ for every $i \in N$.

- The optimization problem is now linear in the variables and can be solved using any linear solver.

1.8. Examples

We are now going to examine three different applications of the algorithm in different legal fields. We will apply procedure 1 to a case of inheritance. Procedure 2 will be applied to a divorce case and a company law case of liquidation. All examples are taken from the examples described in Dall'Aglio (2020).

1.8.1. Example 1

We start with Procedure 1 applied to inheritance.

Example A.8

During his life, X was the owner of a land plot in Zadar with a building and garden (180m2) with three flats: one on the ground floor (90m2, 180 000 Euros), one on the first floor (60m2, 120 000 Euros) and one on the second floor with a wonderful view of the shore and beach (60m2, 130 000 Euros). All flats were condominiums and were rented out. He also owned another land plot in Zagreb with a building with three flats; one on the ground floor (55m2, where his son A had a mechanic's workshop, 77000 euros not including equipment), one on the first floor (55m2, where X lived, 80 000 Euros) and one on the second floor (45m2, but needs full renovation, 45000 Euros). This second building was not condominium. After death of person X he is succeeded by his sons, A, B and C.

A is most interested in the ground floor because he operates a mechanic's workshop which is crucial for his livelihood. He wouldn't mind getting another apartment either in Zagreb or in Zadar.

B already had a house, so he was interested in the house in Zadar. He wants two flats, the one on the first floor but especially the one on the second floor (this is his mayor priority).

C has a tourist agency and he wants all flats in Zadar.

A lower bound for the items' prices is fixed in order to represent the minimal offer that each heir is allowed to present. The difference between the market price and the lower bound represents the amount of money that each heir is asked to allocate according to his or her preferences. Finally, the Egalitarian and Nash algorithms apply in order to fairly divide the items among the heirs.

1) Fix a lower bound for the bid: 20%. Then, the prices of the six apartments are

Table 10. – *Market prices and lower bounds for the Inheritance example*

	Zadar			Zagreb		
	GF	1	2	GF	1	2
Market price	180.000	120.000	130.000	77.000	80.000	45.000
Lower Bound	144.000	96.000	104.000	61.000	64.000	36.000

The Lower Bounds represent the minimum prices that each heir has to respect for the apartments in the Inheritance.

2) Let the heirs offer the amount of money they believe the most adequate for each item in the patrimony. The maximum that each heir may allocate when expressing his or her preferences is equal to the maximum value of the sum of all the items in the patrimony, that is euros 630.000 (The exact sum would be 632000, but for simplicity of communication we prefer a rounder digit)

Note that no offer can be below the minimum prices expressed by the lower bound.

For example, Mr. A may be willing to offer euros 100.000 for the ground floor in Zagreb, as he claims the apartment is crucial for his livelihood, and equally redistribute the remaining amount among the other apartments. Mr. B may translate his special preference for the 2nd floor of the building in Zadar with an offer 20% higher than the market price, and may offer 10% more than the market price for the 1st floor of the same building, he is not interested at all in the building in Zagreb. Similarly, Mr. C may distribute his preferences equally among the apartments in Zadar.

The allocation of the total offer will be as follows.

Table 11. – *A simulation of the bids compatible with the data of the Inheritance example*

	Zadar			Zagreb			Sum
	GF	1	2	GF	1	2	
A	170.000	112.000	123.000	100.000	80.000	45.000	630000
B	181.000	132.000	156.000	61.000	64.000	36.000	630000
C	200.000	129.000	140.000	61.000	64.000	36.000	630000

Here is the CEEI/Nash Allocation applied to the problem

- A gets all the flats in Zagreb.
- B gets the second floor in Zadar and a 68% share of the first flat in Zadar.

- C gets the ground floor in Zadar and a 32% share of the first flat in Zadar.

The allocation can be described by the following table

Table 12. – *The "Name your price" solution for the inheritance example*

	Zadar			Zagreb		
	GF	1	2	GF	1	2
A						
B		68%				
C			32%			

As previously explained, the proposed solution is Proportional, Efficient (Pareto optimal) and Envy-Free. The last property can be summarized by the following table, where the different valuations are described in the rows and the allocations are reported in the columns:

Table 13. – *The solutions satisfies Envy-Freeness*

		Allocations		
		A	B	C
Valuations	A	**225000**	191300	213700
	B	161000	**245800**	223200
	C	161000	218700	**250300**

The valuations of agent A are listed in the first row. That agent values the three flats received as the sum of the respective bids: 100000, 80000 and 45000, yielding 225000 Euros. Agent A values the bundles given to B (The second floor and 68% of the first floor in the Zadar) and to C (The ground floor and 32% of the first floor in Zadar), 191300 Euros and 213000 Euros, respectively. Agent A has no reason to envy agent B or agent C. A similar reasoning applies to the other two agents whose valuation of the received bundle (bold in the table) exceeds that of the bundles assigned to the other agents.

We remark that bids are personal and do not represent objective evaluations. For this reason, a comparison between the values in the main diagonal of Table 13 may induce some agents to complain over having obtained lower values than other agents. In order to avoid any complaint, we recommend that bids are kept private, and each agent has no access to everybody else's valuations.

To explain the solution as an equilibrium, we note that the procedure can compute the following prices for the properties:

Table 14. – *Equilibrium prices for the inheritance case*

	Zadar			Zagreb			Total
	Ground Fl	First Fl.	Second Fl.	Ground Fl	First Fl.	Second Fl.	
Prices	174000	113000	133000	93500	74500	42000	

To explain the relevance of those equilibrium prices, we consider the viewpoint of each agent. Since the asset total value is estimated as 630000 euros and all agents have the same relevance, we assume that each of them can spend 210000 euros.

We start with Mr. A, who will compare the equilibrium prices with the personal bids

Table 15. – *Comparison of bids with the equilibrium prices for Mr. A*

		Zadar			Zagreb	
	GF	1	2	GF	1	2
Bids of A	170k	112k	123k	100k	80k	45k
Prices	174k	113k	133k	93.5k	74.5k	42k
Discount	×	×	×	– 6.5%	– 6.87%	– 6.67%

Mr. A will spend his own budget, starting from the apartment with equilibrium price that presents the highest rebate if compared with the proposed bid: he will first buy the first floor in Zagreb, then the second floor and finally the ground floor. With the last purchase we will have spent all his budget of 210000 euros, so he stops.

Now it is Mr. B's turn to compare bids and prices.

Table 16. – *Comparison of bids with the equilibrium prices for Mr. B*

		Zadar			Zagreb	
	GF	1	2	GF	1	2
B's Bids	181k	132k	156k	61k	64k	36k
Prices	174k	113k	133k	93.5k	74.5k	42k
Discount	– 3.86%	– 4.39%	– 14.74%	×	×	×

Mr. B would first turn his attention to the second floor apartment In Zadar, whose equilibrium price is almost 15% lower than the proposed bid. So, he'd spend 133000 euros to buy that apartment. Then he would move on to buy the first-floor apartment in the same building, but the budget left, 97000 euros would allow to buy only a 68% quota of it. With no more budget left, Mr.B stops.

Finally, Mr. C's situation is described by the following table:

Table 17. – *Comparison of bids with the equilibrium prices for Mr. C*

		Zadar			Zagreb	
	GF	1	2	GF	1	2
Bids of C	200k	129k	140k	61k	64k	36k
Prices	174k	113k	133k	93.5k	74.5k	42k
Discount	– 13%	– 12.4%	– 5%	×	×	×

Mr. C would be most interested in buying the ground floor apartment in Zadar for 174000 euros, a price 13% lower than the corresponding bid. After that purchase, only 36000 remain in Mr. C's budget. This would allow the purchase of a 32% quota of the first floor flat in the same building. With no budget left, Mr.C would stop.

The three heirs would act independently to reconstruct the Competitive/Nash solution suggested by the "Name your price"procedure.

1.8.2. Example 2

Next, we provide an example of how the method works for one of the cases provided by the legal units of the project.

Example B.10

A, wife of B, asks for the statement of termination of the civil effects of the marriage, three years having passed since the judgment of personal separation.

The goods in common are:

1. an apartment, used as a family home, worth 1,500,000 euros;

2. an apartment in a seaside resort worth 1,250,000 euros;

3. a prestigious building, inherited by the couple through testamentary disposition, worth 1,750,000 euros;

4. valuable furniture (works of art) contained in the aforementioned buildings for a value of 550,000 euros;

5. two cars with a value of 60,000 and 50,000 euros, respectively;

6. a vintage car, worth 170,000 euros;

7. company equity investments for a value of 750,000 euros;

8. a sum of money equal to 1,500,000 euros.

The spouses exercise both professional financial activities in the risk capital market and are involved in several types of entrepreneurial activities. For this reason, both have an interest in retaining company holdings.

The wife also asks for the sub-4 assets for herself as part of her entrepreneurial activity involves the buying and selling of works of art. For its part B requires the assignment of works of art and vintage cars, as collectors.

The wife also requires the exclusive custody of the daughter of 7 years and a check for her maintenance amounting to 2,500 euros per month. Although not requiring a maintenance allowance for himself, she demands to keep the surname of the husband (Article 5, Law No. 898 of the Italian Civil Code), as this would allow her to continue more profitably her business, being many of the financial contacts she has made from her relationship with her husband.

The matters regarding the daughter's custody and her maintenance can be considered a separate issue from the division of the matrimonial asset. Regarding the latter, we make further assumptions in order to better define the agents' preferences. We thus suppose that the wife is interested in the family house and has some interest in the seaside resort apartment, while the husband has agreed to live in the inherited apartment

Money (item 8) be either considered as an item of the division or it can be distributed in equal parts between the parties. We opt for the second option and, therefore, it is distributed equally to the two parties prior to any other assignment. Based on the short description, we figured out the ratings of the two parties:

Table 18. – *Ratings and utilities for the divorce example*

Items		Market Price	A		B	
1	Family home flat	1.500.000	****	1.650.000	**	1.363.636
2	Seaside resort flat	1.250.000	***	1.250.000	**	1.136.364
3	Inherited flat	1.750.000	**	1.590.909	****	1.925.000
4	Furnitures	550.000	*****	665.500	****	605.000
5	Two cars	120.000	*	99.174	*	99.174
6	Vintage car	170.000	**	154.545	****	187.000
7	Equity Investments	750.000	*****	907.500	*****	907.500

The Egalitarian criterion yields the following solution:

Table 19. – *The "Price and rate" solution for the divorce example*

	Items	A	B
1	Family home flat		
2	Seaside resort flat	84%	16%
3	Inherited flat		
4	Furnitures		
5	Two cars		
6	Vintage car		
7	Equity Investments		

The following indices help define the quality of the division as perceived by the two agents:

Table 20. – *Evaluating the division quality for the divorce example*

Agent	A	B
Market value	3092000	2998000
Ave. rating	3.48	3.80

As expected, the larger market share obtained by A, is compensated by a slightly lower average standardized rating. We note that the sum of money given to the two agents at the beginning of the procedure could be used to assign the only split item to one of the agents. It seems reasonable for agents A to buy the smaller share originally assigned to B.

1.8.3. Example 3

Let us consider another example suggested by the legal workgroup. This time an instance of liquidation (company law) is considered.

Example C.2

A, B and C concluded a partnership contract in 2006, agreeing to contribute their work and/or property to achieve a common objective – a small carpentry factory and a store for selling goods. They had different stakes/contributions which would determine their shares as joint owners. A was a carpenter with experience especially in kitchens and bedrooms. He contributed equipment (valued at 35 000 euros) and of course with his "know-how" and experience. B had business premises large enough for the factory and for the store, and this was his contribution. C contributed in cash 30 000 euros.

After the financial crisis, the business began to deteriorate so person B proposed to change the purpose of their business to stocking and selling electronic appliances which would be directly imported from China. B still thinks that he is the only one who can decide about the purpose of the business premises. A was disappointed because they didn't need him or his work anymore. C only cares about profit. The content of their common asset (joint ownership) changed during the decade. They bought new machinery, but they also had a special website for selling furniture with the possibility of on-line interior design as an additional service. To set up this website they had to spend 4500 euros and they pay 1200 euros monthly for software licenses and website maintenance fees.

They decided to dissolve the joint ownership and the first step that court had to make was determining their shares. The court decided that A has 3/9, B has 5/9 and C 1/9 of the business. By determination of their shares joint-ownership was transformed into co-ownership.

At the dissolution of co-ownership (in May 2016) the assets consist of all of the above mentioned but also includes new machinery (valued at 20 000 Euros, store items valued at 30 000 euros, and a profit of 15 000 Euros). In the process of partitioning co-ownership, A wants all machinery, but also a part of the property where the factory was located because he wants to continue running the same business by himself. B wants a part of the profits to start with his idea and all business premises. He is also interested in the website because he wants to sell online. C is interested in money only and proposes to sell the business as a whole, but he might consider the possibility to continue working with A with slight changes he would additionally propose.

It is plausible that the former partners give the following evaluations in terms of stars: We assume that the website has been valued 25000 by an expert appointed by the court.

Table 21. – *Ratings and utilities for the liquidation example*

	Items	Market Price	A		B		C	
1	Equipment	35000	*	28926	**	31818	***	35000
2	Business Premises	70000	*****	84700	*****	84700	*	57851
3	Machinery	20000	****	22000	**	18181	***	20000
4	Items	30000	**	27272	**	27272	*	24793
5	Website	25000	*	20661	*****	30250	**	22727
6	Cash	15000	***	15000	****	16150	*****	18150

When running the algorithm, the percentages of the value of the Company Law that have to be assigned to the three shareholders are set equal to 3/9, 5/9 and 1/9, respectively, for shareholder A, B and C, as already decided by the Court of Law.

The Egalitarian allocation yields the following distribution:

Table 22. – *The "Price and rate" solution for the liquidation example*

	Items	A	B	C
1	Equipment		91%	9%
2	Business Premises	25%	75%	
3	Machinery			
4	Items			
5	Website			
6	Cash			
	Shares	3/9	5/9	1/9

The following indices characterize the division.

Table 23. – *Evaluating the division quality for the liquidation example*

Agent	A	B	C
Weight (W)	1/3	5/9	1/9
Market value (Mv)	67416	109651	17933
Mv/W	202248	197372	161397
Ave. rating	2.2560	2.4942	4.62

Whereas in the market value over the weight ratio A ranks first, B second and C third, this order is totally reversed when the gain over central rating is considered. In particular, C will discount the fact of receiving her most treasured item with a lower share of goods in terms market value.

1.9. Conclusive remarks

We present two general purpose procedures that are simple and powerful enough to be used by specialists as well as common EU citizens without specific training on the subject matter.

In the first procedure, that we name "Name your price", available even without an estimated market value for the goods to preliminary agree upon, the users (agents) will simply have to express their preferences as bids on the items to divide. In the second one, named "Price and rate" users will have to express a rating on a simple discrete scale on which to express their satisfaction/dissatisfaction upon the possibility of receiving the items. The scale can be coarser or more refined depending on the ability of the users to deal with such scales, and their familiarity with the goods to be distributed.

Regarding the solutions, we build upon the very recent findings of Bogomolnaia, Moulin, Sandomirskiy, and Yanovskaya (2017) and (2019) that guarantee for the solution of the first procedure its (Pareto) efficiency (no other allocation is at least as good for every agent and strictly better for at least one agent) and its envy-freeness (no agent prefers the share of another agent to her own). Regarding the solution of the second procedure, we focus, together with the efficiency, on another notion of high practical relevance, namely the guarantee that each agent will receive a bundle of items of equal utility and almost equal market price. There is however a feature that is common to both procedures: they are invariant in accordance to a specified criterion. This means, that raising the bids on all goods in a way specified by the criterion, thus altering a truthful profile of the preferences, will not make the agent any better off. On the contrary, the agent will incur the risk to get items she is not interest into and leave to others the items she really craves for.

A direction that awaits exploration is the inclusion of other features in the system, such as:

- handling liabilities ("bads") together with valuable items (goods) while maintaining scale invariant procedures;
- Include the evaluation of preferences for bads, or, more properly, the disposability in taking care of a bad, and freeing the others from the chore of dealing with it,
  - as "negative bids" in Procedure 1, which express the amount of money an agent is willing to get to take care of that bad.
  - as "negative ratings" in Procedure 2, as willingness to be in charge of the chore, in exchange of palatable goods, or fraction of them.

This important addition appears at a very close reach thanks to the recent work by Bogomolnaia et al. (2017, 2019), but a few details need to be investigated in order to translate brilliant research ideas into operating and efficient services for the citizens.

1.10. References

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Algorithms in conflict resolution: A lab experiment

Abstract

We design a laboratory experiment aimed at eliciting participants' preferences between two alternative algorithms stemming from two different theoretical models for resolution of conflicts in a sharing problem. We also investigate participants' willingness to appeal to the court in case they are not satisfied with the division suggested by the computer. Results show that the two algorithms, "Nash" and "Egalitarian", seem to be equally preferred by our participants. However, when we consider their gender, we find that women are more likely to prefer the "Nash" allocation while men prefer the "Egalitarian" one. Nevertheless, participants mostly accept the division proposed by the computer even when the preferred algorithm is not implemented: only 5% of the cases are brought to court. Lastly, we find that preferences between division rules are stable and persistent throughout repetitions of the game: participants show a clear individual preference for one of the two type of division.

Keywords: *Lab Experiment, Fair Division, Dispute and Conflict Resolution*

1.1. Introduction

The LUISS Rome Unit of the Horizon 2020 CREA project (Conflict Resolution Equitative Algorithms) has been in charge of providing experimental evidence of the ability of algorithms to address specific issues arising in legal disputes related to the division of divisible assets among people (e. g. distrust, emotions and conflict behaviors).

To this aim, we have experimentally tested in LUISS Laboratory participants'

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willingness to accept two theoretical methodologies of division based on different perceptions of fair distribution (i.e. either equitable or satisfactory) developed in the first part of the project (see Dall'Aglio, 2020).

Using the two algorithms proposed by the theorist and developed by the computer scientists of the team, we could assess which mechanism of division participants prefer.

In particular, considering homogenous, divisible and valuable goods we are able to compare the Egalitarian solution, in which the proposed allocation guarantees fair shares (1/n of the whole value) but are not envy free, to the competitive/Nash solution that implies both efficiency and envy freeness.

The experiment has been designed in order to tackle the following general research question addressed in the main project:

- *Can allocation algorithms increase the efficiency of courts*?

This question involves individual perception of the use of such instruments to solve division issues, practical aspects of their implementation, success of the instruments (both in terms of private and public costs of the procedures), length in time and accessibility.

- *Are they perceived as more «super partes»*?

The comparison between proposals of allocations coming from an algorithm and from a judge is linked also to the trust that subjects assign to the two different sources.

- *Are algorithms reducing conflicts among subjects*?

The use of algorithms in Courts disputes could reduce the role of emotions and face to face interactions. By giving the possibility to accept or to refuse (going to Court at a cost) the allocation proposed by the computer, we tested how subjects are happy to avoid such sometimes painful occurrences.

Overall this will allow us to assess if introducing "objective" methodologies for division could result in a reduced number of trials when people easily accept and seem more satisfied from the allocations suggested by the algorithm and therefore renounce to end up in the Court.

More specifically, the aim of the experimental part of the project is to answer to the following specific research questions:

i) Do people prefer computational algorithmic to human judgement in division problems, i.e. participants to the experiment accept to rely on division proposals made by an algorithm or they prefer a human decision maker, even if at a relevant cost?
ii) Which of the division proposals, stemming from the two algorithms, participants prefer most?

iii) How much trust and other individual characteristics affect such decisions and preferences? Moreover, are they stable through time?

1.2. Related literature

Fairness issues are a traditional subject within the economics discipline both from a theoretical and an empirical point of view. Nevertheless, there has recently been a growing interest in the role of equity concerns in the distribution of resources (Rabin, 1993; Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000).

Concerns about fairness arise in many different economic environments and the desire to achieve a "fair" outcome has been offered as an explanation for many outcomes that do not support the theoretical predictions of purely self-interested subjects aiming at maximising their own utility.

The theoretical literature on fairness makes clear that in any situation there are many allocations that could be considered as fair. However, which of these perceptions or definitions of fairness are actually considered by ordinary people?

Surveys show that a dominant concept of fairness does not exist and that fairness concerns could differ according to the context in which decisions are taken and across individuals, i.e. they are frame dependent. Consequently, the selection of a methodology to implement divisions that are perceived as "aequa" becomes an interesting and challenging field of research.

From the experimental point of view a large body of evidences generated a significant stream of behavioral literature addressing the nature and the determinants of fairness in non-hypothetical environments, since in many occasion subjects seems willing to sacrifice personal payoffs for other "fair" allocations (e.g. Guth et al., 2002; Kagel and Roth, 2016).

Experiments show that predictions stemming from the traditional theory of decision making fail to account for several others aspects that matter in individual allocation decision and which are related to "others' regarding preferences" (Cooper and Kagel, 2016).

Behavioral and experimental economics have shown the relevant role that fairness, inequity aversion, envy and trust have in affecting individual decision making when a division should be implemented in different settings: when preferences are influenced by such characteristics, the outcome of individual decisions is very far from that suggested by the traditional maximization of self-interests (see as example experimental results from investment or dictator games). Following other theoretical models, fairness has been defined on the basis of comparative payoff outcomes (Bolton and Ockenfels, 2000), reciprocity (Rabin, 1993) and trust (Berg et al., 1995).

Berg et al. (1995) experimentally found, via a simple trust game, the existence

of a large proportion of experimental subjects that trust others and receive reciprocity feedbacks, also if the theoretical predictions state no trust and no reciprocity. Fischbacher et al. (2001) found that in voluntary contribution to public good games a proportion of the population behaves as conditional cooperators, i.e. increase (decrease) her independent contribution if the other's independent contribution is larger (smaller) and does not adapt if it is equal to the own one, whereas the theory suggests free riding as optimal (i.e. payoff-maximizing) solution.

In many experiments addressing the division of a given endowment (i.e. dictator game and its modification), people accept division proposals only if those are not lower than a given threshold, while mere payoff-maximisation would dictate to accept any positive amount. This introduces the theme of envy and reciprocation in sharing situations.

Results from third-party experiments show that equality concerns and redistribution issues exist and depend on preexisting ownership and on the degree of inequality (Konow, 2000; Chavanne et al., 2009). Dickinson and Tiefenthaler (2002) run a battery of experiments focused on individual payoffs to other individuals, differently from the typical experimental bargaining game in which two individuals bargain over how to divide a monetary pie among themselves, finding that a wide range of allocations are viewed as fair distribution of resources among different subjects in a population.

The definition of fairness also differs across individuals: culture, background and other personal characteristics may influence individuals' notion and perception of fairness and the consequent acceptance of different methods of sharing a set of resources.

The gender of the decision maker plays quite a relevant role and becomes an important predictor of the allocation preferred. The result that gender significantly affects fairness perceptions is also consistent with the results from survey and experimental research in bargaining environments (e.g. Bolton and Katok, 1995; Eckel and Grossman, 1997; Andreoni and Vesterlund, 2001, Di Cagno et al. 2016, 2017).

The combined outcomes of those experimental results is that people trust, care about the others and does not like to be undervalued or not considered.

This is particularly relevant when we consider the high number of real-life situations in which we face the division problem of a good, an endowment, a firm, such as in a legacy, divorce or bankruptcy and should end up with a division unanimously accepted.

Given the existence of different methodologies to implement allocations, it is well possible that people are heterogeneous in which one they prefer. Therefore policy makers need, when stating a possible division rule, not only to consider its own welfare characteristics but also the degree of acceptance among "end users"

of the rule and the degree of consensus stemming from individual and aggregated fairness consequences of the proposal.

Another important question that follows the decision to adopt a general and mechanical rule to implement division procedures is whether the acceptance of a proposal of such a division between subjects could be affected by the way in which the proposal of division is presented to the parties of the bargaining process. In particular, we would like to investigate if the presence of an automata, instead of a human being, that applies a given well known and legally accepted algorithm, could help in reducing conflicts, inequity aversion, envy and fear of the other, ultimately increasing the overall welfare of participants and their feelings of satisfaction.

If this applies, it will reduce the number of litigations taken to Courts and also the queuing time in deliberation processes.

Our experimental setting allows to compare two alternative methodologies of division of a given number of goods between a pair of subjects and to evaluate if division proposals stemming from an algorithm that applies such methodologies are perceived fairer than traditional costly court procedures and are more likely to be accepted.

There exist a bunch of experiments testing the properties of different mechanisms of division and comparing alternative fairness criteria. Daniel and Parco (2005) implemented an experimental test of the Brams-Taylor Adjusted Winner Mechanism in a bilateral bargaining game of incomplete information for two divisible goods finding that this mechanism is quite effective in achieving both efficient, equitable and envy-free outcomes depending on the knowledge that each party has of the preferences of their bargaining party. Schneider and Kramer (2004) develop an experimental comparison among three different mathematical procedures of fair division in the lab finding that the adjusted Knaster procedure was largely preferred by participants to the proportional Knaster. Herreiner and Puppe (2007) analyse envy free models in experimental fair division problems in which inter- and intrapersonal criteria can be distinguished, finding that interpersonal comparisons play the dominant role: results provide strong evidence in support of inequality aversion as an empirically relevant fairness criterion.

Bouveret and Lamaitre (2016) investigate five different fairness criteria in the lab showing that they form an ordered scale that can be used to characterize how conflicting the agents' preferences are: for a given instance of resource allocation problem, the less conflicting the agents' preferences are, the more demanding criterion this instance is able to satisfy and the more satisfactory the allocation can be.

Also the relationship between trust and acceptance of algorithm proposals has been explored in our experiment. We hypothesize that the use of computer algo-

rithms automatically related to the bargaining and division decision process is beneficial because it reduces significantly the economic and psychological costs of litigations compared to more "traditional" judicial processes. This will also reduce the entry barriers that impede people lacking sufficient endowment and/or juridical literacy background to have access to Court. The implementation of an algorithm in the resolution of some specific and well identified problems of division could end up as welfare enhancing and perceived by the opponents as fairer. This is linked to the question related to the degree of reliability that individuals attach to proposals stemming from computers or prefigured algorithms: another issue to which our experiment tries to contribute.

There exists a restrict set of experimental literature that compares the different impact on individual economic decision making when facing automata versus human beings in different situations. Even if slightly indirectly, the majority of such experiments finds evidence supporting the idea that introducing robots in decision making allows to switch versus "better" equilibria not only in games with multiple equilibria (where the introduction of artificial subjects increases coordination) but also in bargaining and sharing interactions (where the introduction of artificial subjects reduces strategic behavior). Regarding the optimal behavior in EPA's emission trading auctions, Cason (1995) found that buyers facing human opponents compete more aggressively than the risk neutral predictions whereas bids do not differ significantly from the theoretical predictions when buyers face computerized Nash "robots". Walker et al. (1987) found the same behavior in bidding and attributed the overbidding against human opponents to strategic responses to a subset of aggressive high bidding subjects. Gode and Sunder (1993) and Cason and Friedman (1997) found that the presence of computerized subjects in markets facilitates the convergence towards the theoretical equilibrium. Houser and Kurzban (2002) and Cason and Sharma (2007) showed that introducing a robo-opponent substantially reduces the possible influence of social preferences such as subjects caring about the distribution between themselves and another person. Nishio et al. (2002) found that the number of fair proposals to agents playing an ultimatum game with the computer were higher than those with humans and that the number of rejections of unfair proposals are lower.

The different perception of having a human versus a robo-advisor and its effect on individual decision making has been recently investigated, especially in financial market after the surge of a relevant number of companies offering financial advices via computer (e.g. inter alia Money Farm). Evidence on "algorithm appreciation" is however still controversial. Logg, Minson and Moore (2019) show that people adhere more to advice when they think it comes from an algorithm rather than from a person. However, algorithm appreciation waned when people choose between an algorithm's estimate and their own and when they had expertise in forecasting. Moreover, many experiments involving computer (as

market participants or advisors) stress the aspect that the responsibility that subjects feel in decision making is slightly different when shared with a human dictator or a computer (Kirchkamp and Strobel, 2019).

Also lawyers applied experimental settings in order to evaluate how automated- and human-powered online dispute resolution affect procedural justice in mediation and arbitration. Sela (2018) run an experiment designed to examine whether disputants' experiences of procedural justice in Online Dispute Resolution (ODR) vary if the process is managed by a software or human third party, finding that participants had more positive procedural justice experiences in instrumental arbitration by a human arbitrator compared to principal arbitration by a software arbitrator.

1.3. Methodology and experimental design

1.3.1. Experimental methodology

In order to address our research questions, we design an "ad hoc" experiment in which real subjects face a division problem and are proposed two possible allocations, or have the option to appeal to a "Court". The problem concerns the division of either 6 or 8 items and the division proposals stem from the application of the algorithms developed and presented in the previous part of the project.

All subjects involved in the experiment are of legal age, participate on a voluntary basis and receive a final payment. All ethical requirements (privacy, data protection) are met through the recruitment system ORSEE (Online Recruitment System for Economic Experiments, see http://www.orsee.org/web/), comply the Ethical code of the University LUISS and are approved by its ethical committee.

The informed consent form complies with the provisions of Directive 2001/20/EC and national laws implementing the EC directive. A statement that participation is voluntary and informed. This will explicitly indicate that anyone has the right to refuse to participate or withdraw at any time, without any consequences.

The use of the experimental methodology could be preferred respect the use of interviews and/or on line questionnaires since it imposes incentives that connect individuals' decisions with their actual payment (instead of using fixed or time-related participation fees) and guarantees the elicitation of real preferences, as participants are asked to reveal them through their choices.

Moreover, an experiment is an effective methodology to study these questions because it allows to measure the effects of the independent variables to be ob-

served on the perception of fairness in different settings while holding constant potential confounding factors. As a comparison, field studies would have faced not only accessibility challenges, but also material differences in parties, case details, outcome, neutral performance, context and settings, that would have confounded our findings. The approach also allows to link those decisions to individual characteristics such as demographics and the individual propensity to trust.

The use of students as experimental subjects is not a problem for the external validity of the investigation since preferences on division methods and trust on human versus computers are general issues that everybody faces in real world situations. Moreover, the statistical reliability of data stemming from laboratory experiments is nowadays almost generally accepted by economists and experiments are becoming a standard methodology in procedural justice research and in online dispute resolutions (Brooke et al., 2012).

1.3.2. Experimental design and details

The experiment was carried out in CESARE Lab, the experimental laboratory of LUISS University, Rome. The procedure is fully computerized and run using the z-Tree software (Fischbacher, 2007). Experimental subjects are university students, who participate on a voluntary basis and are recruited via ORSEE (Greiner, 2015). Participants belong to undergraduate or postgraduate programs of LUISS University of Rome of three departments: Economics, Law or Political Science.

Recruited subjects were allowed to participate to a single session only, in order to avoid any effect due to experience and previous knowledge of the game and its instructions. Before each session begins, instructions are read aloud, subjects are informed that data is treated anonymously and questions are answered privately.

The experiment consists of three phases, which are independently incentivized. Participants are aware that they are (anonymously) matched in pairs at the beginning of every phase and that such pairs remain constant within each phase. Values and payoffs are expressed in ECU, Experimental Currency Units, throughout the experiment with an exchange rate of 200 ECU=1€.

In **Phase I** participants face the trust game proposed by Berg et al. (1995). Each pair member randomly receives the role of either Proponent or Responder. Proponents are given an initial endowment of 1000 ECU, while Responders have an initial endowment of 0 ECU. Proponents are asked if and how much ECU from their endowment they want to send to the matched Responders. Those latter will receive the triple of such amount and are then asked whether they want to send back a part of it to the Proposer.

Therefore, calling x the amount sent by the Proposer and y the amount sent back by the Responder, payoffs are:

Proposer's payoff = $1000 - x + y$

Responder's payoff = $3 * x - y$

This phase is aimed at eliciting participants' trust in (unknown) others before the more relevant division game. In order not to confound the result of this stage with the following one, participants receive feedback about the amount received and payoff earned only after Phase II finishes.

In **Phase II** participants face a series of cases regarding the division of several goods (either 6 or 8) between them and their matched partner. The Phase consists of 10 rounds and pairs remain constant throughout.

In each round, participants are asked to express their preference between two different proposals of division. More specifically, participants face two cases, referred to as Case A (6 goods) and Case B (6 goods), each for 5 rounds. In every round of the same Case, either A or B, market values of the goods are kept constant, i.e. each Case refers to the same set of goods. What changes from one round to another are the private evaluations of each good, which are exogenously imposed.

In every round, the computer reports two sets of evaluations, one numerical and another qualitative. The numerical set is based on the maximum willingness to bid in an auction for each good **("Offerte" in the screenshots presented in Appendix, Figure 1a and following)**. The second set of evaluations are based on preferences for having or not having the good and are expressed with stars.

Each subject, who is already informed about the market values, is first presented both his private evaluations and those of his partner. Subsequently, subjects are also presented the two proposed allocations, one based on the numerical set of preferences and the other based on the qualitative set of preferences. Proposed allocations stem, respectively, from the application of the Nash and Egalitarian algorithms (see Dall'Aglio, 2020). We randomize the order in which the two sets of preferences and corresponding solutions are presented.

In every round, participants are asked to state which method they prefer over the other ("METODO A" versus "METODO B" in Figure 1a). Once both participants select between the two possible divisions, the computer randomly selects with probability ½ which method is implemented and asks participants if they accept or reject the proposed division of the goods:

- If **both** components of the pair accept the proposed allocation, it will be implemented. Each partners' payoff for the round is the sum of the share of each good she obtained multiplied by the corresponding market value.
- If at **least one** of the two components of the pair rejects the proposed alloca-

tion, participants are forced to go to the Court. In that situation the judge will allocate to each of them half of the market value of all the goods. However, both group members have to pay the judicial expenses, which amount to the 30% of the overall value gained and their payoff will be reduced accordingly.

At the end of every round, participants are communicated their earnings for the round and their partner's acceptance decision. They therefore know whether the proposal was rejected by their partner and, in case it was, how much they paid for judicial costs.

At the end of the experiment, the computer randomly selects one of the 10 rounds of Phase II. The payoff earned in the selected round is effectively paid to participant (together with the payoff earned in Phase I and III).

In **Phase III** participants are randomly re-matched in pairs and face another round of the trust game presented in Phase I, knowing that they are playing with a (likely) different partner. This last phase was introduced in order to check whether receiving a high number of rejections has an effect on trust (compared to the trust level elicited before the division game, in Phase I). However, given the very low rejection rate, this effect could not be tested and therefore data from Phase III are not used in the analysis.

Only at the end of the experiment, the computer communicates to each participant the gains from the two trust games and the total payment for the experiment. After this, participants fill in a brief questionnaire with demographic data. Subjects are then immediately and anonymously paid in cash. On average they received a payment of 25 euro.

Finally, we briefly present how exogenous evaluations of the goods are determined. Market values of 6 or 8 goods are the same for all participants while private evaluation for each good differ between pair members. Private values in the 5 round of the same case (set of good A or set of good B) are selected in order to systematically vary the extent of "similarity" of preferences between the two contenders, from strong similarity to strong dissimilarity. In order to avoid any confounding or sequence effect, we randomized the order in which similarity is varied across rounds.

Private values for every item are selected by the computer among 5 different possible values, equal respectively to +/– 10%, +/– 20% and 0% of the market value of each good.

Table 1. – *Example of determination of private values, given the set of market value*

GOOD	MARKET VALUE	POSSIBLE PRIVATE VALUES
1	180	216,198,180, 144,162
2	120	144, 132, 120, 96, 108
3	130	156, 143, 130, 104, 117
4	77	92.4 , 84.7, 77, 64.6, 69.3
5	80	96,88, 80, 64, 72
6	45	54, 49.5, 45, 36, 41.6

Ranking according to star levels follow the same rationale as the private values. For each item, one-star level out of 5 possible is randomly selected. Star levels and their interpretations are as follows:

(*) = strongly prefers to leave the good [-2];
(**) = mildly prefers to leave the good [-1];
(***) = neutral about the good [0];
(****) = mildly prefers to take the good [+1];
(*****) = strongly prefers to take the good [+2].

1.4. Data characteristics and analysis

1.4.1. Sample characteristics

In total 68 subjects participated, in 3 distinct experimental sessions. Table 2 shows the composition of our sample: 38% of participants were female students, average age is between 22 and 23 years old and 57% of them are Economic students; 43 % of the sample has participated to more than 5 experiments (which is what we consider and *experienced* participant) and 94% of them found the experiment easy.

Table 2. – *Sample characteristics*

Variable	**Obs**	**Mean**	**Std. Dev.**	**Min**	**Max**
Female	68	.38	.49	0	1
Age	68	22.69	2.63	19	33
Economics	68	.57	.5	0	1
Experienced	68	.43	.5	0	1
Found experiment easy	68	.94	.24	0	1

1.5. Results

The focus of our analysis is first to show whether one of the two solutions, Egalitarian or Nash, is preferred to the other in absolute terms. We will then go deeper and try to assess which are the determinants of preferring a specific solution over the other. In this section we will show the results while the methods applied for the statistical validation of results are discussed in the methodological note.

Figure 1. – *Participants' preferences between Egalitarian Vs. Nash*

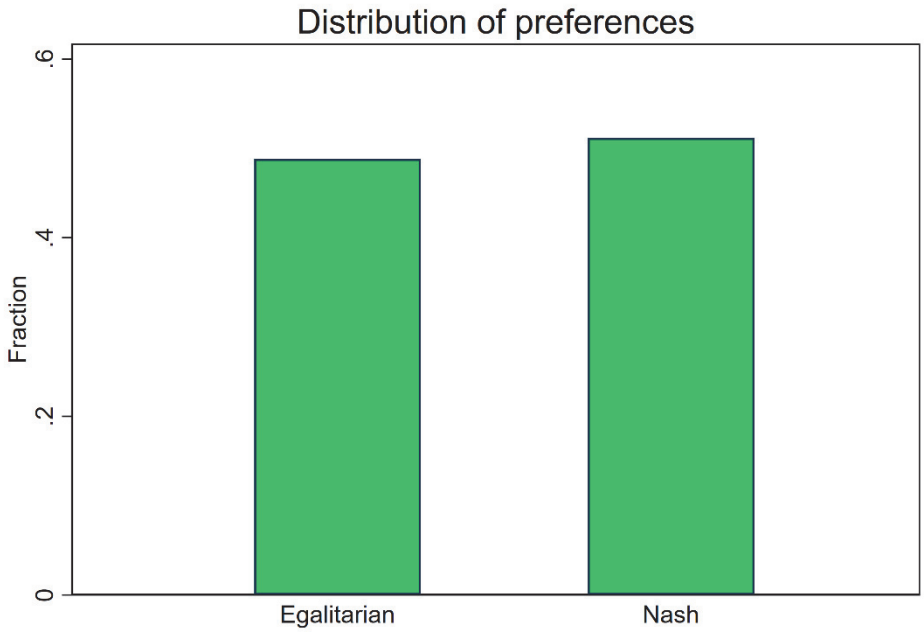


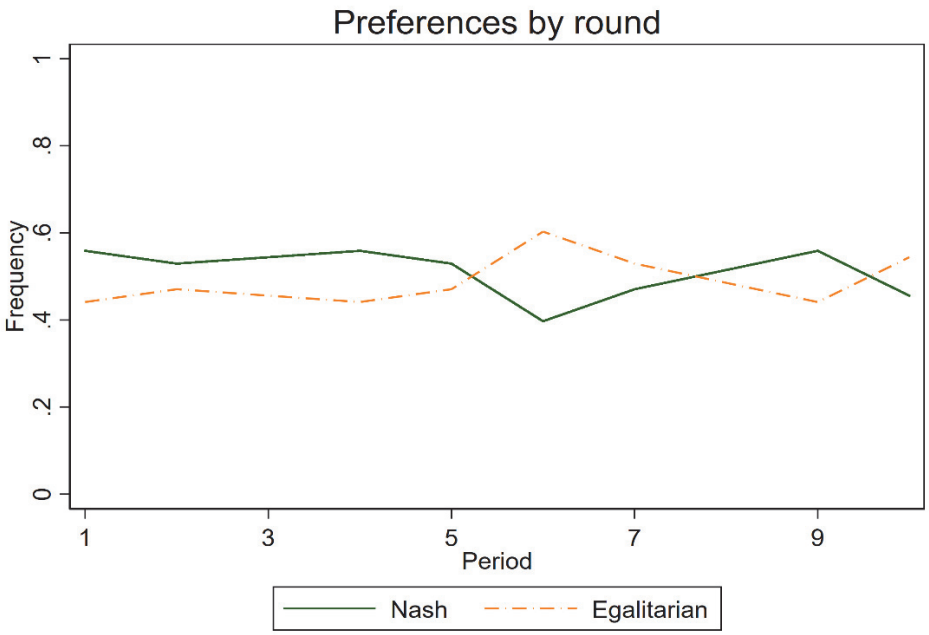
Figure 1 shows the distribution of participants' preferences between the Egalitarian or Nash solution. The exact frequencies and percentages are reported in Table 2, which shows how many times an Egalitarian or Nash solution is preferred in the 10 cases each subject is presented. Apparently, there is not a solution that is strongly preferred over the other.

Table 3. – *Tabulation of Choice*

	Freq.	Percent
Egalitarian	332	48.82
Nash	348	51.18
Total	680	100.00

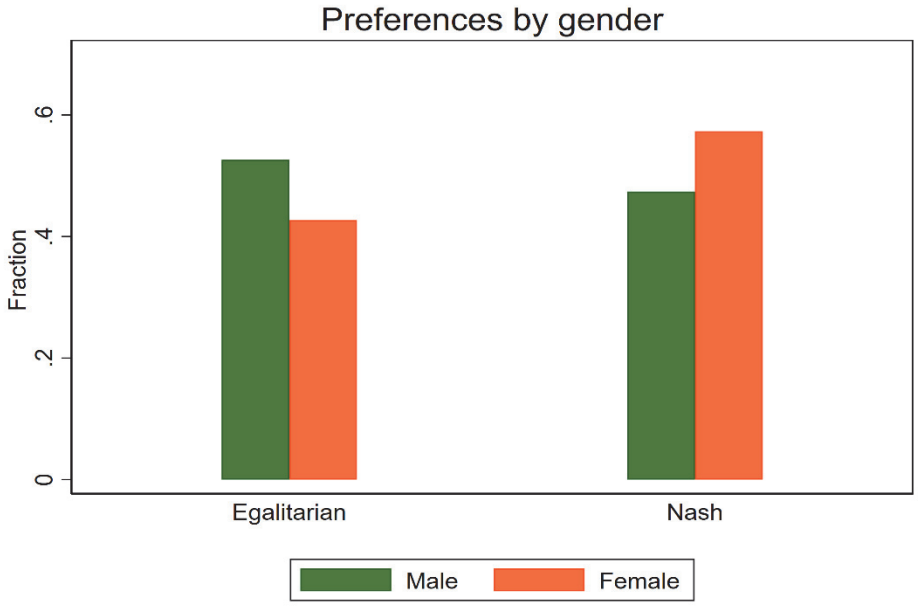
Figure 2 presents how preferences evolve throughout repetition of the game, i.e. in the 10 cases presented to participants. It shows that there are not sharp changes, which might be interpreted as a signal that stated choices are not case dependent. Such stability is therefore a first hint that subjects have a clear personal preference between a "willingness to pay" - based solution and a preference-based one, which is not sensible to the specific case considered.

Figure 2. – *Participants' preferences between Egalitarian Vs. Nash*



A substantial (and statistically significant at 10% level) difference is however observed when analysing preferences of female and male participant separately. Figure 3 shows that women seem to have a higher preference for Nash solution while men prefer the Egalitarian one.

Figure 3. – *Distribution of participants' preferences by gender*



After stating their preferences, our experimental subjects are proposed one of the two possible divisions and they are asked to state whether such allocation is for them acceptable or not. Figure 4 shows that 94.7% of divisions are accepted and therefore only 5.3% are rejected. Participants seem very unwilling to pay the cost of going to court and therefore accept the division even though this is not the preferred one. However, this could be interpreted also as their preference for allocations suggested by algorithms instead than by humans.

Although rejection decisions are observed quite rarely and the number of observations does not allow further inference, it is worthwhile to distinguish them by type of solution proposed. A focus on such rejected proposal of division (presented in Figure 5) shows that only one third of them were Egalitarian solutions while in two thirds of the cases the rejected allocation was a Nash one.

Figure 4. – *Distribution of participants' acceptance decisions*

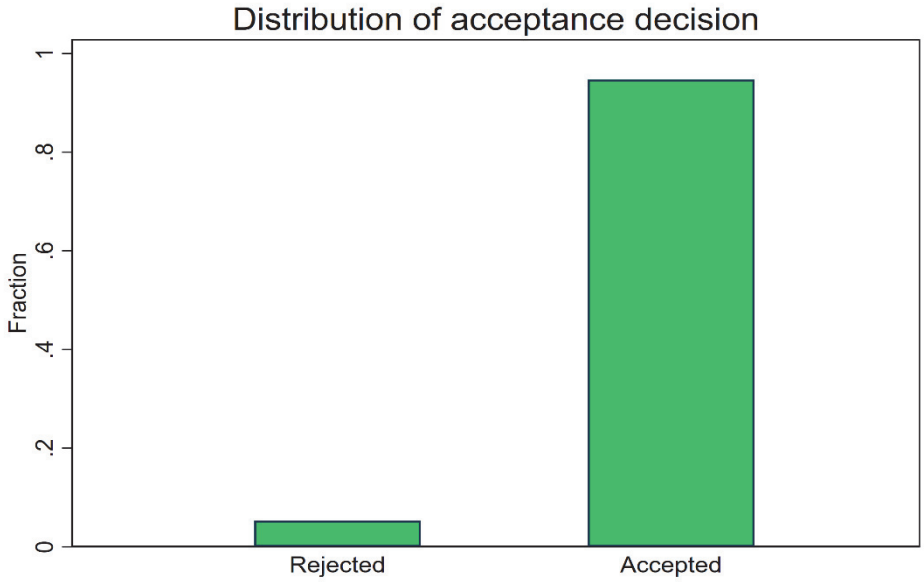


Figure 5. – *Percentage of rejected division by type of algorithm*

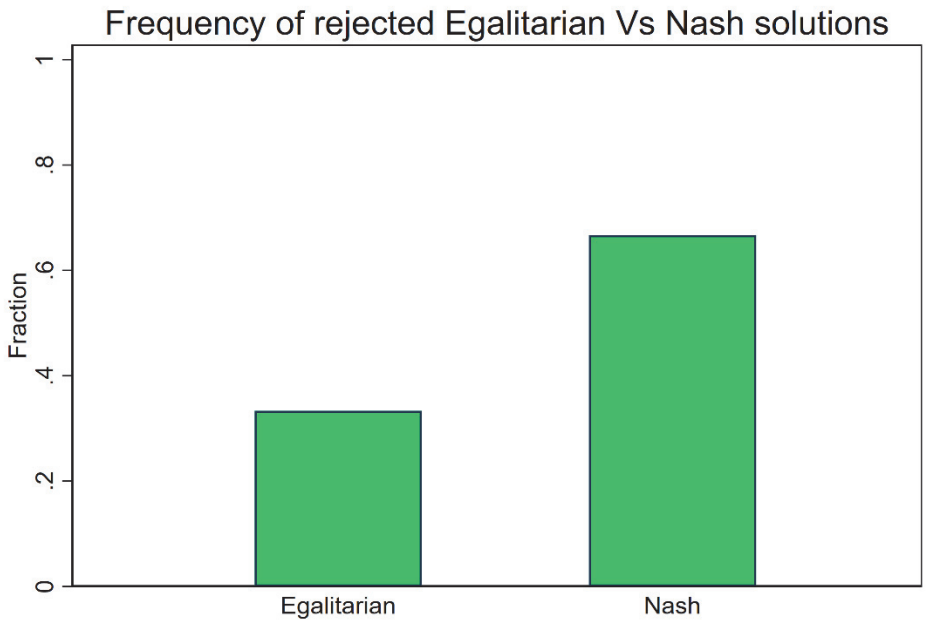


Table 3 shows, via static and dynamic logit regressions[1], which are the determinants of the probability of preferring a Nash allocation versus an Egalitarian one.

The dependent variable is both model is a dummy variable which is equal to 1 if in period *t* the subject has selected the Nash division as her preferred one and equal to 0 if she has instead selected the Egalitarian solution. We will refer to this dummy variable as *Nash (t)*. In order to increase the readability of results, we report both the coefficients and the corresponding odds ratios for each specification of the model.

Our static model highlights that individuals who have played as sender in the pre – experimental trust game and have sent an higher amount, showing a higher level of trust in the other participant, have a lower probability to prefer the Nash solution against the Egalitarian one.[2] On the other hand, having received more or less when playing the trust game as a Recipient has no effect on such preference. Also, having rejected past division proposals more often seems to decrease the probability of choosing the Nash solution as the preferred one, although the significance is admittedly weak.[3]

Furthermore, regression analysis confirms that female participants, on average, prefer more the Nash solution compared to male participants.

It is also worthwhile to note that being enrolled in a course in Economics has no effect on the stated preference, which allows us to state that being more or less accustomed to concepts such as willingness to pay, bid and market value does not seem push subjects in a specific direction.

Lastly, when we render our model dynamic, we find that past preferences are the main determinant of the solution preferred in the current round.

Therefore, having chosen the Nash division in the previous round is almost the unique predictor of also choosing again the Nash division in the current round.

This is the statistical confirmation that preferences are stable and persistent throughout repetitions of the game: subjects have a clear preference for one of the two type of division.

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[1] See the methodological note for further details concerning the empirical approach.

[2] The variables *TG – Sender* and *TG – Recipient* comes from the interaction between the role had in the pre-experimental game by the individual and the amount that the individual has sent or received in such game.

[3] The variable Rejections (*t – 1*) is the share of proposed divisions which were rejected by the individual until the last round.

Table 4. – *Determinants individual's preference for Nash division (Vs Egalitarian)*

	Static model		*Dynamic model*	
VARIABLES	Coeff	OR	Coeff	OR
Nash (*t -1*)			0.483**	1.622**
			(0.215)	(0.348)
Trust – Sender	-1.759**	0.172**	-1.284*	0.277*
	(0.825)	(0.142)	(0.771)	(0.214)
Trust – Recipier	0.270	1.310	0.329	1.389
	(0.868)	(1.138)	(0.800)	(1.112)
Rejections (*t-1*)	-1.471*	0.230*	-1.162	0.313
	(0.759)	(0.175)	(0.944)	(0.295)
Female	0.599**	1.820**	0.380	1.462
	(0.268)	(0.487)	(0.254)	(0.372)
Economics	0.373	1.452	0.384	1.468
	(0.256)	(0.371)	(0.237)	(0.348)
Nash (*initial*)			0.267	1.306
			(0.242)	(0.316)
Constant	-0.144	-0.144	-0.569*	0.566*
	(0.290)	(0.290)	(0.314)	(0.178)
Observations	680		612	
Number of id	68		68	
Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1				

1.6. Conclusion

The frequently assessed superior accuracy of algorithmic relative to human judgement (Dawes, Faust and Meehl, 1989) led organization to invest in the power of algorithms to produce better insights. In organization they are used to hire and to manage employees, individuals in need of financial and legal counsel can use "Robo-advisors" and, in general, decision makers could employ algorithms to take "automatic" and "aseptic" decisions. However, there is also a stream of judgement and decision making literature that demonstrates human distrust on algorithmic outputs (the so called "algorithm aversion", Dietvorst, Simmons and Massey, 2016).

Using algorithms in Courts represents a step further in this debate since it in-

troduces the problem of how ordinary people actually perceive algorithm verdicts in legal disputes. Concerning allocation disputes as divorce and heritage situations, it is important to assess if defendants perceive the implementation of algorithm solutions as fairer than human judges' deliberations and how much their utilization improves the resolution of legal conflict situations. This will allow to assess if introducing "objective" methodologies for division could result in a reduced rate of judicial requests.

The results of our experiment are encouraging in that direction since participants mostly accept the division proposed by the computer even when their preferred algorithm is not implemented: only 5% of the cases are brought to Court.

We did not find strong evidence of preference for one of the two theoretical methodology implemented for the division: results show that the two algorithms, "Nash" and "Egalitarian", seems to be equally preferred by our participants. However, when we consider their gender, we find that women are more likely to prefer the "Nash" allocation while men prefer the "Egalitarian" one. Being enrolled in a course in Economics instead has no effect on the stated preference.

As far as trust is concerned individuals who have played as sender in the first trust game and have sent a higher amount, showing a higher level of trust in the other participant, have a lower probability to prefer the Nash solution against the Egalitarian one. Having received more or less when playing the trust game as a Recipient has no effect on such preference; having rejected past division proposals more often seems to decrease the probability of choosing the Nash solution as the preferred one, although the significance is admittedly weak.

Finally, we find that preferences between division rules are stable and persistent throughout repetitions of the game: participants show a clear individual preference for one of the two type of division.

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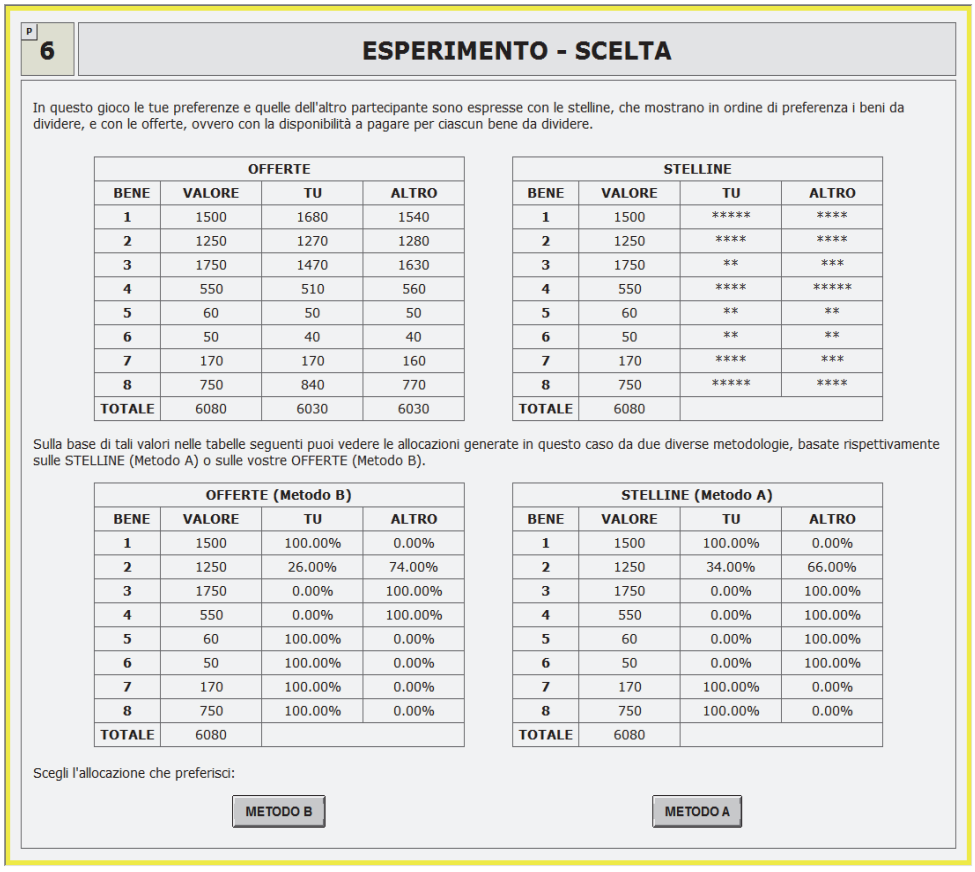
1.8. Methodological note

In presenting our descriptive statistics, when referring to *statistically significance difference* we refer to having performed a two – sample t test on the two groups of interest (e.g. female and male participants). Following Moffatt (2015), we perform such test on independent observations, meaning on average behavior at the individual level (aggregated across rounds) in order to eliminate any potential correlation between current and past choice of the same individual.

Concerning our regression analysis instead, we resort to random effect logit regression where the unit of observation is individual behavior in a given round. Following the approach proposed by Wooldridge (2005), our dynamic specification also includes among regressors the choice each subject has made in the very first round of play, in order to get rid of the so-called 'initial condition problem'.

1.9. Appendix

Figure 1a. – *Screenshot from the division game, expression of preferences*



P 6

ESPERIMENTO - SCELTA

In questo gioco le tue preferenze e quelle dell'altro partecipante sono espresse con le stelline, che mostrano in ordine di preferenza i beni da dividere, e con le offerte, ovvero con la disponibilità a pagare per ciascun bene da dividere.

OFFERTE			
BENE	VALORE	TU	ALTRO
1	1500	1680	1540
2	1250	1270	1280
3	1750	1470	1630
4	550	510	560
5	60	50	50
6	50	40	40
7	170	170	160
8	750	840	770
TOTALE	6080	6030	6030

STELLINE			
BENE	VALORE	TU	ALTRO
1	1500	*****	****
2	1250	****	****
3	1750	**	***
4	550	****	*****
5	60	**	**
6	50	**	**
7	170	****	***
8	750	*****	****
TOTALE	6080		

Sulla base di tali valori nelle tabelle seguenti puoi vedere le allocazioni generate in questo caso da due diverse metodologie, basate rispettivamente sulle STELLINE (Metodo A) o sulle vostre OFFERTE (Metodo B).

OFFERTE (Metodo B)			
BENE	VALORE	TU	ALTRO
1	1500	100.00%	0.00%
2	1250	26.00%	74.00%
3	1750	0.00%	100.00%
4	550	0.00%	100.00%
5	60	100.00%	0.00%
6	50	100.00%	0.00%
7	170	100.00%	0.00%
8	750	100.00%	0.00%
TOTALE	6080		

STELLINE (Metodo A)			
BENE	VALORE	TU	ALTRO
1	1500	100.00%	0.00%
2	1250	34.00%	66.00%
3	1750	0.00%	100.00%
4	550	0.00%	100.00%
5	60	0.00%	100.00%
6	50	0.00%	100.00%
7	170	100.00%	0.00%
8	750	100.00%	0.00%
TOTALE	6080		

Scegli l'allocazione che preferisci:

METODO B METODO A

Figure 2a. – *Screenshot from the division game, acceptance decision*

Figure 3a. – *Screenshot from the division game, end-of-period feedback in case of rejection*

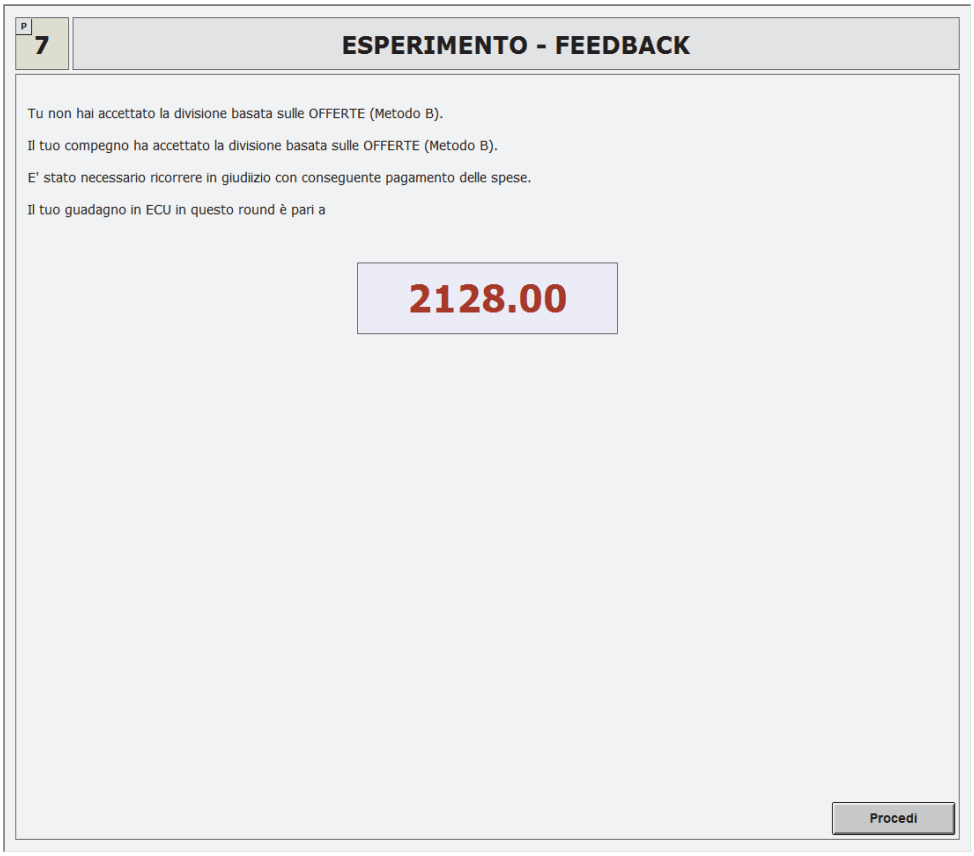


Figure 4a. – *Screenshot from the division game, end-of-period feedback in case of acceptance*